A novel technique to solve fully fuzzy nonlinear matrix equations

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Abstract. Several techniques are suggested in order to generate estimated solutions of fuzzy nonlinear programming problems. This work is an attempt in order to suggest a novel technique to obtain the fuzzy optimal solution related to the fuzzy nonlinear problems. The major concept is on the basis of the employing nonlinear system with equality constraints in order to generate nonnegative fuzzy number matrixes $\tilde{Y},\ \tilde{Y}^2,\ \ldots,\ \tilde{Y}^n$ that satisfies $\tilde{D}\tilde{Y}+\tilde{G}\tilde{Y}^2+\cdots+\tilde{P}\tilde{Y}^n=\tilde{Q}$ in which $\tilde{D},\ \tilde{G},\ \ldots,\ \tilde{P}$ and $\tilde{Q}$ are taken to be fuzzy number matrices. An example is demonstrated in order to show the capability of the proposed model. The outcomes show that the suggested technique is simple to use for resolving fully fuzzy nonlinear system (FFNS).

Keywords: Fuzzy solution, Fuzzy numbers, Fully fuzzy nonlinear system, Fully fuzzy matrix equations.

1 Introduction

An area of applied mathematics which contains many applications in different fields of science is resolving fuzzy nonlinear systems [1-8]. In [9] a numerical method is proposed for solving fuzzy systems. Theoretical aspects related to the fuzzy linear system are investigated in [10]. In [11] the Jacobi as well as Gauss Seidel techniques are suggested in order to find the solution of fuzzy linear system. In [12] the Conjugate gradient approach is suggested in order to resolve fuzzy symmetric positive definite system of linear equation. In [13] an iterative algorithm in order to resolve dual linear systems is proposed. In [14] LU decomposition technique is applied in order to solve fuzzy system of linear equation. In [15] a certain decomposition technique is applied in order to resolve fully fuzzy linear system of equations.

Generally, there is no approach on the basis of matrices which yields fuzzy solutions for FFNS. In this paper, a novel method is proposed in order to resolve the fully fuzzy nonlinear matrix equations (FFNME), $\tilde{D}\tilde{Y}+\tilde{G}\tilde{Y}^2+\cdots+\tilde{P}\tilde{Y}^n=\tilde{Q},$ in which $\tilde{D},\ \tilde{G},\ \ldots,\ \tilde{P}$ are $n\times n$ arbitrary triangular fuzzy number matrices, $\tilde{Q}$ is a $n\times 1$ arbitrary triangular fuzzy number matrix, also the unknown $\tilde{Y},\ \tilde{Y}^2,\ \dots,\ \tilde{Y}^n$ are matrices having n positive fuzzy numbers. The fuzzy matrices $\tilde{Y}^2,\ \dots,\ \tilde{Y}^n$ are defined with following elements:

If $\tilde{Y} = \begin{bmatrix} \tilde{y}_{1,1} \\ \tilde{y}_{2,1} \\ \vdots \\ \tilde{y}_{n,1} \end{bmatrix}$, then $\tilde{Y}^2 = \begin{bmatrix} \tilde{y}_{1,1}^2 \\ \tilde{y}_{2,1}^2 \\ \vdots \\ \tilde{y}_{n,1}^2 \end{bmatrix}, \dots,\ \tilde{Y}^n = \begin{bmatrix} \tilde{y}_{1,1}^n \\ \tilde{y}_{2,1}^n \\ \vdots \\ \tilde{y}_{n,1}^n \end{bmatrix}$.

A nonlinear system is applied in order to obtain nonnegative fuzzy number matrixs $\tilde{Y},\ \tilde{Y}^2,\ \dots,\ \tilde{Y}^n$ that satisfies $\tilde{D}\tilde{Y} + \tilde{G}\tilde{Y}^2 + \cdots + \tilde{P}\tilde{Y}^n = \tilde{Q}$.

This paper is organized as follows: Some basic definitions are given in Section 2. In Section 3, a novel technique in order to resolve FFNS is suggested with numerical example. Conclusion is given in Section 4.

2 Basic definitions and notations

In this section the essential notations utilized in fuzzy operations are given.

Definition 1. A fuzzy number is a fuzzy set $\tilde{z}: \mathbb{R}^1 \to [0,1]$ such that

i. $\tilde{z}$ is upper semi-continuous.

ii. $\tilde{z}(x) = 0$ outside some interval $[k, l]$.

iii. There exist real numbers l and m, $k \le l \le m \le n$, where

   1. $\tilde{z}(x)$ is monotonically increasing on $[k, l]$,

   2. $\tilde{z}(x)$ is monotonically decreasing on $[m, n]$,

   3. $\tilde{z}(x) = 1$, $l \le x \le m$.

The set of all fuzzy numbers is displayed by E^1 [16,17].

Definition 2. $\tilde{C} = (\tilde{c}_{ij})$ is named a fuzzy number matrix, if each element of $\tilde{C}$ be a fuzzy number. $\tilde{C}$ is named a positive (negative) fuzzy matrix, also is displayed by $\tilde{C} > 0$ ($\tilde{C} < 0$) if each element of $\tilde{C}$ be positive (negative). $\tilde{C}$ is named non-positive (non-negative), also displayed by $\tilde{C} \le 0$ ($\tilde{C} \ge 0$) if each element of $\tilde{C}$ is non-positive (non-negative).

Definition 3. Suppose $\tilde{p} = (p_m,\ p_l,\ p_u)$ as well as $\tilde{q} = (q_m,\ q_l,\ q_u)$ be two triangular fuzzy numbers. Hence:

1. $\tilde{p} \oplus \tilde{q} = (p_m + q_m,\ p_l + q_l,\ p_u + q_u)$,
2. $-\tilde{p} = (-p_u,\ -p_l,\ -p_m)$,
3. $\tilde{p} \ominus \tilde{q} = (p_m - q_u,\ p_l - q_l,\ p_u - q_m)$.

The fuzzy multiplication is displayed by $\hat{*}$ [18]. It is performed with the below mentioned equation:

$\tilde{p} \,\hat{*}\, \tilde{q} = (s_m,\ s_l,\ s_u)$,

where

$s_l = p_l . q_l$,

$s_m = \min(p_m.q_m,\ p_m.q_u,\ p_u.q_m,\ p_u.q_u)$,

$s_u = \max(p_m.q_m,\ p_m.q_u,\ p_u.q_m,\ p_u.q_u)$.

In a case that $\tilde{p}$ be a triangular fuzzy number as well as $\tilde{q}$ be a non-negative one, the following is concluded:

$$\tilde{p} \hat{*} \tilde{q} = \begin{cases} (p_m . q_m \ , \ p_l . q_l \ , \ p_u . q_u), & p_m \geq 0, \\ (p_m . q_u \ , \ p_l . q_l \ , \ p_u . q_u), & p_m < 0, \ p_u \geq 0, \\ (p_m . q_m \ , \ p_l . q_l \ , \ p_u . q_m), & p_m < 0, \ p_u < 0. \end{cases}$$

3 Fully fuzzy nonlinear matrix equation

Consider the below mentioned FFNME:

$$\begin{bmatrix} \tilde{d}_{11} \tilde{d}_{12} \dots \tilde{d}_{1n} \\ \tilde{d}_{21} \tilde{d}_{22} \cdots \tilde{d}_{2n} \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ \tilde{d}_{n1} \tilde{d}_{n2} \cdots \tilde{d}_{nn} \end{bmatrix} \begin{bmatrix} \tilde{y}_{11} \\ \tilde{y}_{21} \\ \vdots \\ \tilde{y}_{n1} \end{bmatrix} + \begin{bmatrix} \tilde{g}_{11} \tilde{g}_{12} \cdots \tilde{g}_{1n} \\ \tilde{g}_{21} \tilde{g}_{22} \cdots \tilde{g}_{2n} \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ \tilde{g}_{n1} \tilde{g}_{n2} \cdots \tilde{g}_{nn} \end{bmatrix} \begin{bmatrix} \tilde{y}_{11}^2 \\ \tilde{y}_{21}^2 \\ \vdots \\ \tilde{y}_{n1}^2 \end{bmatrix} + \cdots + \begin{bmatrix} \tilde{p}_{11} \tilde{p}_{12} \cdots \tilde{p}_{1n} \\ \tilde{p}_{21} \tilde{p}_{22} \cdots \tilde{p}_{2n} \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ \tilde{p}_{n1} \tilde{p}_{n2} \cdots \tilde{p}_{nn} \end{bmatrix} \begin{bmatrix} \tilde{y}_{11}^n \\ \tilde{y}_{21}^n \\ \vdots \\ \tilde{y}_{n1}^n \end{bmatrix} = \begin{bmatrix} \tilde{q}_{11} \\ \tilde{q}_{21} \\ \vdots \\ \tilde{q}_{n1} \end{bmatrix}$$

In which $\tilde{d}_{ij}$, $\tilde{g}_{ij}$ as well as $\tilde{p}_{ij}$ $(for\ 1 \leq i,\ j \leq n)$, are arbitrary triangular fuzzy numbers, the elements $\tilde{q}_{i1}$ as well as the unknown elements $\tilde{y}_{i1}$ are nonnegative fuzzy numbers. Utilizing matrix notation, the following is extracted

$$\tilde{D} \hat{*} \tilde{Y} + \tilde{G} \hat{*} \tilde{Y}^2 + \cdots + \tilde{P} \hat{*} \tilde{Y}^n = \tilde{Q}. \quad (1)$$

The fuzzy number matrices $\tilde{Y} = (\tilde{y}_1\ ,\ \tilde{y}_2\ , \dots ,\ \tilde{y}_n\ )^T$, $\tilde{Y}^2 = (\tilde{y}_1^2,\ \tilde{y}_2^2, \dots ,\ \tilde{y}_n^2)^T$, …, $\tilde{Y}^n = (\tilde{y}_1^n,\ \tilde{y}_2^n,\ \dots ,\ \tilde{y}_n^n)^T$ demonstrated by $\tilde{y}_i = (\tilde{u}_{i1},\ \tilde{y}_{i1},\ \tilde{v}_{i1})$, $\tilde{y}_i^2 = (\tilde{u}_{i1}^2,\ \tilde{y}_{i1}^2,\ \tilde{v}_{i1}^2)$, …, $\tilde{y}_i^n = (\tilde{u}_{i1}^n, \tilde{y}_{i1}^n, \tilde{v}_{i1}^n)$, (*for* $1 \leq i \leq n$), are the solutions of the fuzzy matrix system Eq. (1) if

$$\tilde{d}_i \hat{*} \tilde{Y} + \tilde{g}_i \hat{*} \tilde{Y}^2 + \cdots + \tilde{p}_i \hat{*} \tilde{Y}^n = \tilde{q}_i\ ,\ 1 \leq i \leq n, \quad (2)$$

where

$$\tilde{q}_i = \left(\tilde{a}_{i1}, \tilde{q}_{i1}, \tilde{c}_{i1}\right),$$
$$\tilde{d}_i = \left(\left(\tilde{b}_{i1}, \tilde{d}_{i1}, \tilde{e}_{i1}\right), \left(\tilde{b}_{i2}, \tilde{d}_{i2}, \tilde{e}_{i2}\right), \dots, \left(\tilde{b}_{in}, \tilde{d}_{in}, \tilde{e}_{in}\right)\right),$$
$$\tilde{g}_i = \left(\left(\tilde{f}_{i1}\ , \tilde{g}_{i1}, \tilde{h}_{i1}\right), \left(\tilde{f}_{i2}\ , \tilde{g}_{i2}, \tilde{h}_{i2}\right), \dots, \left(\tilde{f}_{in}\ , \tilde{g}_{in}, \tilde{h}_{in}\right)\right),$$
$$\tilde{p}_i = \left(\left(\tilde{r}_{i1}\ , \tilde{p}_{i1}, \tilde{s}_{i1}\right), \left(\tilde{r}_{i2}\ , \tilde{p}_{i2}, \tilde{s}_{i2}\right), \dots, \left(\tilde{r}_{in}\ , \tilde{p}_{in}, \tilde{s}_{in}\right)\right).$$

Definition 4. In the nonnegative FFNME Eq. (1), with new notations $\tilde{D} = (B, D, E)$, $\tilde{G} = (F,\ G,\ H)$, …, $\tilde{P} = (R, P, S)$ in which *B, D, E, F, G, H, …., R, P, S* are crisp matrices, we say that $\tilde{Y}, \tilde{Y}^2$, …, $\tilde{Y}^n$ are the solutions if:

$$\begin{cases} BU + FU^2 + \cdots + RU^n = A, \\ DY + GY^2 + \cdots + PY^n = Q, \\ EV + HV^2 + \cdots + SV^n = C. \end{cases} \quad (3)$$

Moreover, if $U \geq 0,\ Y - U \geq 0,\ V - Y \geq 0,\ Y^2 - U^{\ 2} \geq 0,\ V^2 - Y^2 \geq 0, \dots ,$ $Y^n - U^{\ n} \geq 0,\ V^n - Y^n \geq 0$, so it can be denoted that $\tilde{Y}$, $\tilde{Y}^2$, …, $\tilde{Y}^n$ are consistent solutions of the nonnegative FFNME.

3.1 The proposed technique

Here, a novel technique in order to obtain fuzzy solutions of an FFNME is suggested. Take into consideration the FFNME Eq. (2) in which all the parameters $\tilde{d}_{ij}$, $\tilde{g}_{ij}, \dots, \tilde{p}_{ij}, \tilde{y}_{i1}$ as well as $\tilde{q}_{i1}$ are demonstrated as (b_{ij} , d_{ij} , e_{ij}), (f_{ij} , g_{ij} , h_{ij}), …, (r_{ij} , p_{ij} , s_{ij}), (u_{i1}, y_{i1} , v_{i1}) and (a_{i1} , q_{i1} , c_{i1}) respectively. Hence the FFNME is written as below

(B, D, E)(U, Y, V) + (F, G, H) (U^2, Y^2, V^2)+…+(R, P, S)(U^n, Y^n, V^n) = (A, Q, C), (3)

Assuming $(b_{ik}, d_{ik}, e_{ik}) \hat{*} (u_{k1}, y_{k1}, v_{k1}) + (f_{ik}, g_{ik}, h_{ik}) \hat{*} (u_{k1}^2, y_{k1}^2, v_{k1}^2) + \cdots + (r_{ik}, p_{ik}, s_{ik}) \hat{*} (u_{k1}^n, y_{k1}^n, v_{k1}^n) = \left(k_{k1}^{(j)}, o_{k1}^{(j)}, x_{k1}^{(j)}\right), 1 \le i,\ j,\ k \le n$, in which each (u_{k1}, y_{k1}, v_{k1}) is a nonnegative triangular fuzzy number. The FFNME (2) can be displayed as below:

$$\sum_{k=1}^{n} \left(k_{k1}^{(j)},\ o_{k1}^{(j)},\ x_{k1}^{(j)}\right) = (a_{i1},\ q_{i1},\ c_{i1}), \quad 1 \le i \le n. \qquad (4)$$

Utilizing arithmetic operations, described in section 2, the following nonlinear programming is obtained in which, the artificial variables r_i, $i = 1, 2, \dots, n^2$ is added.

Minimize $r_1 + r_2 + \dots. + r_{n^2}$,

$$subject\ to \begin{cases} \sum_{k=1}^{n} w_{k1}^{(1)} + r_1 = d_{11}, \\ \sum_{k=1}^{n} w_{k1}^{(2)} + r_2 = d_{21}, \\ \vdots \\ \sum_{k=1}^{n} w_{k1}^{(n)} + r_n = d_{n1}, \\ \sum_{k=1}^{n} q_{k1}^{(1)} + r_{n+1} = b_{11}, \\ \vdots \\ \sum_{k=1}^{n} u_{k1}^{(n)} + r_{3n} = f_{n1}. \end{cases}$$

4 Example

In order to explain the suggested approach, the below mentioned example is given.

Example 4.1. Take into consideration the below mentioned FFNME:

$$\begin{bmatrix} (2,3,5) & (2,4,5) \\ (1,2,3) & (3,4,6) \end{bmatrix} \begin{bmatrix} \tilde{x}_{11} \\ \tilde{x}_{21} \end{bmatrix} + \begin{bmatrix} (1,2,3) & (3,5,6) \\ (3,4,5) & (1,3,4) \end{bmatrix} \begin{bmatrix} \tilde{x}_{11}^2 \\ \tilde{x}_{21}^2 \end{bmatrix} = \begin{bmatrix} (19,140,467) \\ (14,136,436) \end{bmatrix},$$

Assuming $\tilde{x}_{11} = (y_{11},\ x_{11},\ z_{11})$, $\tilde{x}_{21} = (y_{21}, x_{21},\ z_{21})$, $\tilde{x}_{11}^2 = (y_{11}^2,\ x_{11}^2,\ z_{11}^2)$ *and* $\tilde{x}_{21}^2 = (y_{21}^2,\ x_{21}^2,\ z_{21}^2)$. The FFNME is displayed as follows:

$$\begin{cases} (2,3,5) \hat{*} (y_{11},\ x_{11},\ z_{11}) + (2,4,5) \hat{*} (y_{21},\ x_{21},\ z_{21}) + (1,2,3)(y_{11}^2,\ x_{11}^2,\ z_{11}^2) + \\ \qquad (3,5,6)(y_{21}^2,\ x_{21}^2,\ z_{21}^2) = (19,140,467), \\ (1,2,3) \hat{*} (y_{11},\ x_{11},\ z_{11}) + (3,4,6) \hat{*} (y_{21},\ x_{21},\ z_{21}) + (3,4,5)(y_{11}^2,\ x_{11}^2,\ z_{11}^2) + \\ \qquad (1,3,4)(y_{21}^2,\ x_{21}^2,\ z_{21}^2) = (14,136,436). \end{cases}$$

Wherein

$$\begin{cases}\left(2y_{11}+2y_{21}+y_{11}^2+3y_{21}^2, 3x_{11}+4x_{21}+2x_{11}^2+5x_{21}^2, 5z_{11}+5z_{21}+3z_{11}^2+6z_{21}^2\right)\\ \qquad = (19,140,467),\\ \left(y_{11}+3y_{21}+3y_{11}^2+y_{21}^2, 2x_{11}+4x_{21}+4x_{11}^2+3x_{21}^2, 3z_{11}+6z_{21}+5z_{11}^2+4z_{21}^2\right)\\ \qquad = (14,136,436).\end{cases}$$

Applying the suggested approach, the FFNME is transformed into the below mentioned crisp system:

$$\begin{cases}2y_{11}+2y_{21}+y_{11}^2+3y_{21}^2=19,\\ 3x_{11}+4x_{21}+2x_{11}^2+5x_{21}^2=140,\\ 5z_{11}+5z_{21}+3z_{11}^2+6z_{21}^2=467,\\ y_{11}+3y_{21}+3y_{11}^2+y_{21}^2=14,\\ 2x_{11}+4x_{21}+4x_{11}^2+3x_{21}^2=136,\\ 3z_{11}+6z_{21}+5z_{11}^2+4z_{21}^2=436.\end{cases}$$

Minimize $r_1+r_2+\cdots+r_6$

$$\begin{cases}2y_{11}+2y_{21}+y_{11}^2+3y_{21}^2+r_1=19,\\ 3x_{11}+4x_{21}+2x_{11}^2+5x_{21}^2+r_2=140,\\ 5z_{11}+5z_{21}+3z_{11}^2+6z_{21}^2+r_3=467,\\ y_{11}+3y_{21}+3y_{11}^2+y_{21}^2+r_4=14,\\ 2x_{11}+4x_{21}+4x_{11}^2+3x_{21}^2+r_5=136,\\ 3z_{11}+6z_{21}+5z_{11}^2+4z_{21}^2+r_6=436.\end{cases}$$

where $r_1+r_2+\cdots+r_6\geq 0$. The optimal solution is $y_{11}=1,\ y_{21}=2,\ y_{11}^2=1,\ y_{21}^2=4,\ x_{11}=4,\ x_{21}=4,\ x_{11}^2=16,\ x_{21}^2=16,\ z_{11}=6,\ z_{21}=7,\ z_{11}^2=36,\ z_{21}^2=49$. Hence the fuzzy solution is given by $\tilde{x}_{11}=(1,4,6),\ \tilde{x}_{21}=(2,4,7),\ \tilde{x}_{11}^2=(1,16,36)\ and\ \tilde{x}_{21}^2=(4,16,49)$.

5 Concluding remarks

The fuzzy nonlinear systems are extremely significant in numerical analysis. In this paper, a novel technique in order to extract the nonnegative fuzzy optimal solutions of FFNME, $\tilde{D}\tilde{Y}+\tilde{G}\tilde{Y}^2+\cdots+\tilde{P}\tilde{Y}^n=\tilde{Q}$ is suggested, in which $\tilde{D},\tilde{G}$, ... , $\tilde{P}$ are $n\times n$ arbitrary triangular fuzzy number matrices, $\tilde{Q}$ is a $n\times 1$ arbitrary triangular fuzzy number matrix, also the unknown $\tilde{Y}$, $\tilde{Y}^2$, ... , $\tilde{Y}^n$ are matrices having n positive fuzzy numbers. A nonlinear system with equality constraints to FFNME is utilized in order to resolve FFNME. The suggested technique is validated with a numerical example.

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