

A late-accelerating universe with no dark energy – and a finite-temperature big bang

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(Dated: February 6, 2008)

Brane-world models offer the possibility of explaining the late acceleration of the universe via infra-red modifications to General Relativity, rather than a dark energy field. However, one also expects ultra-violet modifications to General Relativity, when high-energy stringy effects in the early universe begin to grow. We generalize the DGP brane-world model via an ultra-violet modification, in the form of a Gauss-Bonnet term in the bulk action. The combination of infra-red and ultra-violet modifications produces an intriguing cosmology. The DGP feature of late-time acceleration without dark energy is preserved, but there is an entirely new feature – there is no infinite-temperature big bang in the early universe. The universe starts with finite density and pressure, from a “sudden” curvature singularity.

1. INTRODUCTION

The standard cosmology based on General Relativity and inflation has been remarkably successful. But there remain deep puzzles left for theorists to resolve – what is the cause of the late-time acceleration of the universe (the “dark energy” problem)? how is the classical big bang singularity removed by quantum gravity effects? One approach to start tackling these problems is via the brane-world scenario, which is motivated by string theory. Most brane-world models, including those of Randall-Sundrum type [1], produce ultra-violet modifications to General Relativity, with extra-dimensional gravity dominating at high energies. However it is also possible for extra-dimensional gravity to dominate at low energies, leading to infra-red modifications of General Relativity [2, 3]. The Dvali-Gabadadze-Porrati (DGP) models [3] (see also [4]) achieve this via a brane induced gravity effect.

The generalization of the DGP models to cosmology lead to late-accelerating cosmologies, even in the absence of a dark energy field [5]. This exciting feature of “self-acceleration” may help towards a new resolution to the dark energy problem. But the models suffer from the short-coming that they do not modify 4D gravity at high energies, where we expect stringy corrections to start having an effect. How can we generalize the DGP models so that they also show ultra-violet modifications to General Relativity? One possibility is to introduce into the gravitational action a term that is associated with higher-energy stringy corrections – the Gauss-Bonnet term [6]. Indeed, in certain realizations of string theory, the ghost-free GB term in the bulk action may naturally lead to a DGP induced gravity term on the brane boundary [7].

We investigate what happens when a Gauss-Bonnet term is introduced in the 5D Minkowski bulk containing a Friedmann brane with DGP induced gravity. As we will show, this combination of infra-red and ultra-violet modifications leads to intriguing cosmological models.

2. FIELD EQUATIONS

The gravitational action contains the Gauss-Bonnet (GB) term in the bulk, as a correction to the Einstein-Hilbert term, and the Induced Gravity (IG) term on the brane,

$$S_{\text{grav}} = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g^{(5)}} \left\{ R^{(5)} + \alpha \left[R^{(5)2} - 4R_{ab}^{(5)} R^{(5)ab} + R_{abcd}^{(5)} R^{(5)abcd} \right] \right\} + \frac{r}{2\kappa_5^2} \int_{\text{brane}} d^4x \sqrt{-g^{(4)}} R^{(4)}, \quad (1)$$

where α (≥ 0) is the GB coupling constant [8] and r (≥ 0) is the IG “cross-over” scale, which marks the transition from 4D to 5D gravity. The DGP models are the special case $\alpha = 0$, and in this case the cross-over scale defines an effective 4D gravitational constant via $\kappa_4^2 = \kappa_5^2/r$.

We assume mirror (Z_2) symmetry about the brane. The standard energy conservation equation holds on the brane,

$$\dot{\rho} + 3H(1+w)\rho = 0, \quad w = p/\rho. \quad (2)$$

The modified Friedmann equation was found in the most general case (where the bulk contains a black hole and a cosmological constant, and the brane has tension) in Ref. [9]. For a spatially flat brane without tension, in a Minkowski bulk,

$$4 \left[1 + \frac{8}{3}\alpha \left(H^2 + \frac{\Phi}{2} \right) \right]^2 (H^2 - \Phi) = \left[rH^2 - \frac{\kappa_5^2}{3}\rho \right]^2, \quad (3)$$

where Φ is determined by

$$\Phi + 2\alpha\Phi^2 = 0. \quad (4)$$

(In the most general case, the right-hand side of this condition is nonzero [9].) Equation (4) has solutions $\Phi = 0, -1/2\alpha$, but here we only consider $\Phi = 0$, since the

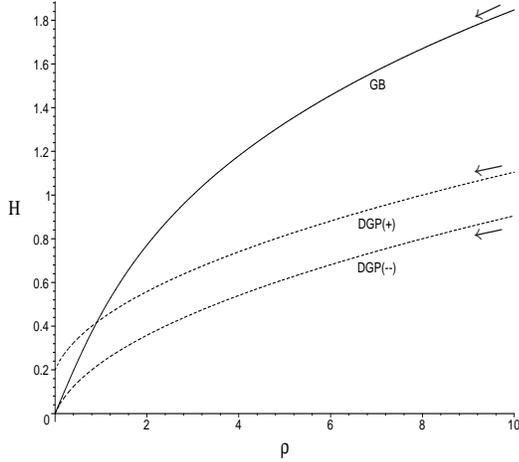


FIG. 1: DGP and GB solutions of the Friedmann equation (H vs ρ) for a Minkowski bulk. (Arrows indicate the flow of brane proper time t , with $t = \infty$ at $\rho = 0$.)

second solution has no IG limit and thus does not include the DGP model [10].

The DGP models have $\alpha = \Phi = 0$ and the Friedmann equation (3) reduces to a quadratic in H^2 , with solutions

$$H^2 = \pm \frac{2}{r}H + \frac{\kappa_5^2}{3r}\rho. \quad (5)$$

There are two branches DGP(\pm) for the two signs on the right (corresponding to different embeddings of the brane in the Minkowski bulk). Both branches have a 4D limit at high energies,

$$\text{DGP}(\pm): H \gg r^{-1} \Rightarrow H^2 \propto \rho, \quad (6)$$

while at low energies,

$$\text{DGP}(+): \rho \rightarrow 0 \Rightarrow H \rightarrow \frac{2}{r}, \quad (7)$$

$$\text{DGP}(-): \rho \rightarrow 0 \Rightarrow H^2 \propto \rho^2. \quad (8)$$

DGP(-) has a non-standard (and non-accelerating) late universe. The self-accelerating DGP(+) branch is of most interest for cosmology, and we focus here on this model and its generalization via a GB term.

The pure GB model with a Minkowski bulk has $r = \Phi = 0$ and the Friedmann equation (3) reduces to

$$\left(1 + \frac{8}{3}\alpha H^2\right)^2 H^2 = \frac{\kappa_5^4}{36}\rho^2, \quad (9)$$

which is a cubic in H^2 . This GB Friedmann equation has no 4D limit:

$$\text{GB high energy: } H \gg \alpha^{-1/2} \Rightarrow H^2 \propto \rho^{2/3}, \quad (10)$$

$$\text{GB low energy: } H \ll \alpha^{-1/2} \Rightarrow H^2 \propto \rho^2. \quad (11)$$

The Friedmann equations for pure DGP and pure GB models with a Minkowski bulk are compared in Fig. 1.

3. DGP BRANE WITH GB BULK GRAVITY: COMBINING UV AND IR MODIFICATIONS

The DGP(+) models are attractive for cosmology since they accelerate at late times, without the need for dark energy, when gravity begins to leak off the brane, i.e., when the 5D Ricci term in Eq. (1) begins to dominate over the 4D Ricci term. At early times, the 4D term dominates and General Relativity is recovered (in the background – note that the perturbations are not General Relativistic [11]). The DGP models are in some sense “unbalanced”, since they do not include ultra-violet modifications to cosmological dynamics. In order to modify 4D gravity at high energies as well as low energies, we can include a GB term in the action.

The combined Gauss-Bonnet Induced Gravity (GBIG) model has a DGP brane in a Minkowski bulk with Einstein-Gauss-Bonnet gravity. The GBIG Friedmann equation follows from putting $\Phi = 0$ in Eq. (3). Defining dimensionless variables,

$$\gamma = \frac{8\alpha}{3r^2}, \quad h = Hr, \quad \mu = \frac{r\kappa_5^2}{3}\rho, \quad \tau = \frac{t}{r}, \quad (12)$$

the GBIG Friedmann equation becomes

$$4(\gamma h^2 + 1)^2 h^2 = (h^2 - \mu)^2, \quad (13)$$

while the conservation equation becomes

$$\mu' + 3h(1+w)\mu = 0, \quad (14)$$

where a dash denotes $d/d\tau$, and $h = a'/a$.

Combining Eqs. (13) and (14), we find the modified Raychaudhuri equation,

$$h' = \frac{3\mu(1+w)(h^2 - \mu)}{4(\gamma h^2 + 1)(3\gamma h^2 + 1) - 2(h^2 - \mu)}. \quad (15)$$

The acceleration $a''/a = h' + h^2$ is then given by

$$\frac{a''}{a} = \frac{4h^2(\gamma h^2 + 1)(3\gamma h^2 + 1) - (h^2 - \mu)[2h^2 - 3(1+w)\mu]}{4(\gamma h^2 + 1)(3\gamma h^2 + 1) - 2(h^2 - \mu)}. \quad (16)$$

The GB correction, via a non-zero value of γ , introduces significant complexity to the Friedmann equation, which becomes cubic in h^2 , as opposed to the quadratic DGP(\pm) case, $\gamma = 0$, for which

$$h^2 = \mu + 2 \pm 2\sqrt{\mu + 1}. \quad (17)$$

This additional complexity has a dramatic effect on the dynamics of the DGP(+) model, as shown in Fig. 2. The contribution of GB gravity at early times removes the infinite density big bang, and the universe starts at finite maximum density and finite pressure (but, as we show below, with infinite curvature). Furthermore, there are two such solutions, each with late-time self-acceleration, marked GBIG1 and 2 on the plots. Since GBIG2 is accelerating throughout its evolution (actually super-inflating,

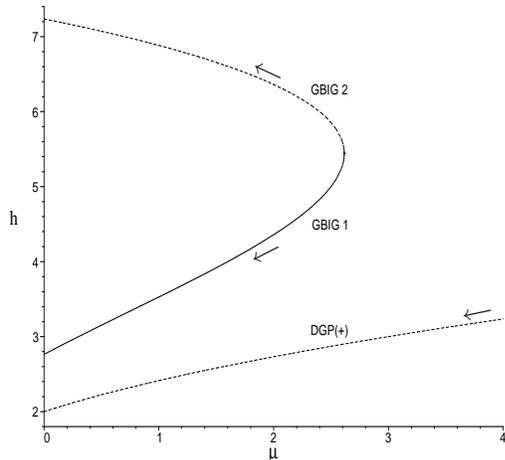


FIG. 2: Solutions of the Friedmann equation (h vs μ) for the DGP(+) model and its Gauss-Bonnet corrections, GBIG1 and GBIG2. The curves are independent of the equation of state w . Arrows indicate the flow of normalized brane proper time τ , with $\tau = \infty$ at $\mu = 0$. (Here $\gamma = 0.05$.)

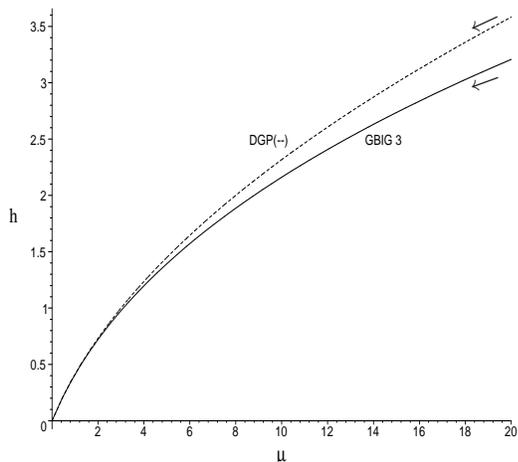


FIG. 3: The DGP(-) model and its GB correction, GBIG3.

$h' > 0$), the physically relevant self-accelerating solution is GBIG1.

The cubic in h^2 , Eq. (13), has three real roots when $0 < \gamma < 1/16$ (see below). Two of these roots correspond to GBIG1–2, which are modifications of the DGP(+) model. The third root GBIG3 is a modification of the DGP(-) model, as illustrated in Fig. 3. Note that the curves in these figures are independent of the equation of state w of the matter content of the universe – w will determine the time evolution of the universe along the curves, via the conservation equation (14).

The plots show that GBIG3 starts with a standard

big bang, $\rho = \infty$, in common with the DGP(\pm) and GB models in Fig. 1. By contrast, GBIG1–2 have a finite-temperature big bang, since the density is bounded above,

$$\mu \leq \mu_i, \quad (18)$$

where μ_i (which is positive only for $\gamma < 1/16$), is found below, in Eq. (32).

The finite-density beginning was pointed out in Ref. [9], where the cubic for the general case (i.e., with tension, bulk cosmological constant and bulk black hole) was qualitatively analyzed. Here, we focus on the simplest generalization of the DGP models, and give a detailed quantitative analysis of the cosmological dynamics. In particular, our analysis shows that one solution is not bounded, which was not noticed in Ref. [9]. The numerical plots of the Friedmann equation in Figs. 2 and 3 are crucial to a proper understanding of the algebraic analysis of the cubic roots.

A detailed analysis [12] of the cubic equation (13) confirms the numerical results, and shows that (for $\mu > 0$):

$$0 < \gamma < \frac{1}{16} : 3 \text{ real roots, GBIG1–3}, \quad (19)$$

$$\gamma \geq \frac{1}{16} : 1 \text{ real root, GBIG3}. \quad (20)$$

The real roots are given as follows:

- For $0 < \gamma < 1/16$: the roots GBIG1–2 are

$$4\gamma^2 h^2 = \frac{1-8\gamma}{3} + 2\sqrt{-Q} \cos\left(\theta + \frac{n\pi}{3}\right) \text{ for } \mu \leq \mu_i, \quad (21)$$

where $n = 4$ for GBIG1, $n = 2$ for GBIG2, and the root GBIG3 is

$$4\gamma^2 h^2 = \frac{1-8\gamma}{3} + \begin{cases} 2\sqrt{-Q} \cos \theta & \text{for } \mu \leq \mu_i, \\ S_+ + S_- & \text{for } \mu \geq \mu_i. \end{cases} \quad (22)$$

- For $\gamma \geq 1/16$: the only real root GBIG3 is

$$4\gamma^2 h^2 = \frac{1-8\gamma}{3} + S_+ + S_-. \quad (23)$$

In the above, S_{\pm}, Q, R, θ are defined by

$$S_{\pm} = \left[R \pm \sqrt{R^2 + Q^3} \right]^{1/3}, \quad (24)$$

$$Q = \frac{8\gamma^2}{3}(\mu + 2) - \frac{1}{9}(1 - 8\gamma)^2, \quad (25)$$

$$R = 8\gamma^4 \mu^2 - \frac{4\gamma^2}{3}(1 - 8\gamma)(\mu + 2) + \frac{(1 - 8\gamma)^3}{27}, \quad (26)$$

$$\cos 3\theta = R/\sqrt{-Q^3}. \quad (27)$$

The explicit form of the solutions can be used to confirm the features in Figs. 2 and 3. Equations (22)–(26)

show that GBIG3 starts with a big bang, $h, \mu \rightarrow \infty$, with $h^2 \sim \mu^{2/3}$ near the big bang. This is the same as the high-energy behaviour of the pure GB model, Eq. (10) – the GB effect dominates at high energies in GBIG3. This is not the case for GBIG1–2.

The maximum density feature of GBIG1–2 is more easily confirmed by analysing the turning points of μ as a function of h^2 . The Friedmann equation (13) gives

$$\frac{d\mu}{d(h^2)} = \frac{h^2 - \mu - 2(\gamma h^2 + 1)(3\gamma h^2 + 1)}{h^2 - \mu}. \quad (28)$$

Substituting $d\mu/d(h^2) = 0$ into Eq. (13), we find that

$$h_i = \frac{1 \pm \sqrt{1 - 12\gamma}}{6\gamma}, \quad (29)$$

$$\mu_i = \pm \frac{1}{3} h_i^2 \left(2\sqrt{1 - 12\gamma} \mp 1 \right). \quad (30)$$

The second equation shows that positive maximum density only arises for the upper sign and with $\gamma < 1/16$, in agreement with the cubic analysis. Thus the initial Hubble rate and density for GBIG1–2 are

$$h_i = \frac{1 + \sqrt{1 - 12\gamma}}{6\gamma}, \quad (31)$$

$$\mu_i = \frac{1}{3} h_i^2 \left(2\sqrt{1 - 12\gamma} - 1 \right). \quad (32)$$

If $\gamma = 0$, then GBIG1–2 reduce to DGP(+), and $h_i = \mu_i = \infty$. Note that

$$h_i > 4. \quad (33)$$

The case $\gamma = 1/16, \mu_i = 0$ corresponds to a vacuum brane with de Sitter expansion, and $h = h_i = 4$, generalizing the DGP(+) vacuum de Sitter solution [5].

The late-time asymptotic value of the expansion rate, as $\mu \rightarrow 0$, is

$$h_\infty = \frac{1}{2\sqrt{2}\gamma} \left[1 - 8\gamma \mp \sqrt{1 - 16\gamma} \right]^{1/2}, \quad (34)$$

where the minus sign corresponds to GBIG1 and the plus sign to GBIG2. In the limit $\gamma \rightarrow 0$, GBIG1 recovers the DGP(+) case, $h_\infty = 2$, while for GBIG2, $h_\infty \rightarrow \infty$; the parabolic GBIG1–2 curve in Fig. 2 “unwraps” and transforms into the DGP(+) curve. Equations (19) and (34) show that

$$2 \leq h_\infty < 4 \text{ for GBIG1,} \quad (35)$$

while $4 < h_\infty < \infty$ for GBIG2.

The behaviour of the key GBIG1–2 parameters is illustrated in Figs. 4 and 5.

4. COSMOLOGICAL DYNAMICS

The GBIG1 model, which is the physically relevant generalization of the DGP(+) model, exists if Eq. (19)

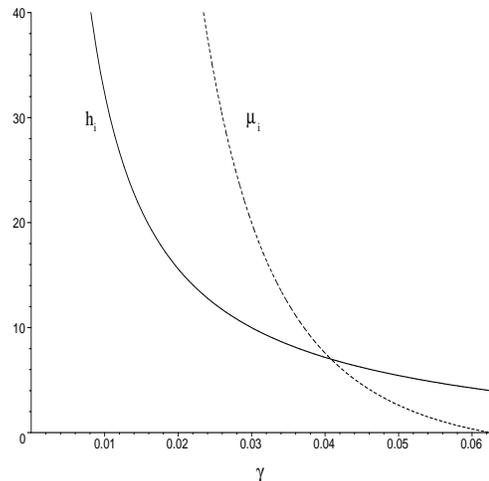


FIG. 4: The dependence in GBIG1–2 of the initial expansion rate and density on γ .

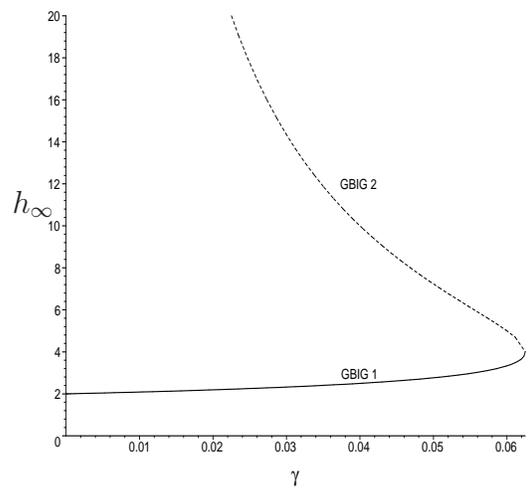


FIG. 5: The GBIG1–2 late-time asymptotic expansion rate as a function of γ .

holds. By Eq. (12), this means that the GB length scale $L_{\text{gb}} = \sqrt{\alpha}$ must be below a maximum threshold determined by the IG cross-over scale:

$$\gamma < \frac{1}{16} \Leftrightarrow L_{\text{gb}} \equiv \sqrt{\alpha} < \frac{1}{8} \sqrt{\frac{3}{2}} r. \quad (36)$$

If the GB term is taken as the correction term in certain string theories, then $L_{\text{gb}} \sim L_{\text{string}}$, while $r \sim H_0^{-1}$, so that this bound is easily satisfied.

When Eq. (36) holds, the universe starts with a maximum density ρ_i and maximum Hubble rate H_i , and

evolves to an asymptotic vacuum de Sitter state:

$$0 < \rho < \rho_i = \frac{rH_i^2}{\kappa_5^2} \left(2\sqrt{1 - \frac{32\alpha}{r^2}} - 1 \right), \quad (37)$$

$$H_\infty < H < H_i = \frac{r}{16\alpha} \left(\sqrt{1 - \frac{32\alpha}{r^2}} + 1 \right). \quad (38)$$

At any epoch τ_0 , the proper time back to the beginning is

$$\tau_0 - \tau_i = \int_{a_i}^{a_0} \frac{da}{ah}. \quad (39)$$

Since a and h are nonzero on the interval of integration, the time back to the beginning is finite.

The current Hubble rate can be approximated by the final de Sitter Hubble rate, $H_0 \sim H_\infty$, so that by Eq. (35), the cross-over scale obeys

$$2H_0^{-1} \lesssim r \lesssim 4H_0^{-1}. \quad (40)$$

In the DGP(+) limit, $r \sim 2H_0^{-1}$. The effect of GB gravity is to increase r but not beyond $r \sim 4H_0^{-1}$.

However, there is a UV-IR “bootstrap” operating to severely limit the GB effect at late times. The key point is that appreciable late-time GB effects require an increase in γ , whereas the primordial Hubble rate H_i is suppressed by an increase in γ – as shown in Figs. 4 and 5. Equations (31) and (34) imply that

$$H_i \gg H_0 \Rightarrow \gamma \ll \frac{1}{16}. \quad (41)$$

Thus the GBIG1 model does not alleviate the DGP(+) fine-tuning problem of a very large cross-over scale, $r \sim H_0^{-1} \sim (10^{-33} \text{ eV})^{-1}$.

The GBIG1 Friedman equation (21) gives

$$H^2 = \frac{1 - 8\gamma}{12\gamma^2 r^2} + \frac{\sqrt{8\gamma^2(r\kappa_5^2\rho + 6) - (1 - 8\gamma)^2}}{6\gamma^2 r^2} \cos \left[\theta(\rho) + \frac{4\pi}{3} \right] \quad (42)$$

where

$$\cos 3\theta(\rho) = \frac{216\gamma^4\mu^2 - 36\gamma^2(1 - 8\gamma)(\mu + 2) + (1 - 8\gamma)^3}{[(1 - 8\gamma)^2 - 24\gamma^2(\mu + 2)]^{3/2}} \quad (43)$$

A more convenient form of the Friedmann equation follows from solving Eq. (13) for μ ,

$$\mu = h^2 - 2h(\gamma h^2 + 1), \quad h_\infty \leq h < h_i. \quad (44)$$

By expanding to first order in $h^2 - h_\infty^2$, we find that at late times,

$$h^2 = h_\infty^2 + 2 \left(\frac{h_\infty}{4 - h_\infty} \right) \mu + O(\mu^2). \quad (45)$$

Taking the DGP(+) limit $h_\infty \rightarrow 2$, and comparing with Eqs. (5) and (17), we find that the effective Newton constant in GBIG1 is

$$G = \left(\frac{h_\infty}{4 - h_\infty} \right) \frac{G_5}{r}, \quad (46)$$

where $G_5 = \kappa_5^2/8\pi$ is the fundamental, 5D gravitational constant. In the DGP(+) case $G = G_5/r$.

Equation (46) gives a relation for the fundamental Planck scale M_5

$$M_5^3 \sim \left(\frac{rH_0}{4 - rH_0} \right) \frac{M_p^2}{r}, \quad (47)$$

where M_p is the effective 4D Planck scale, and we used $H_\infty \sim H_0$. As $r \rightarrow 4H_0^{-1}$ (its upper limit), so M_5 increases. This is very different from the DGP(+) case, where $M_5^3 = M_p^2/r$, so that M_5 is constrained to be very low, $M_5 \lesssim 100 \text{ MeV}$. In principle, GB gravity allows us to solve the problem of a very low fundamental Planck scale – but in practice the UV-IR bootstrap, Eq. (41), means that $\gamma \sim 0$ so that M_5 is effectively the same as in the DGP(+) case.

What is the nature of the beginning of the universe in GBIG1? We can use Eq. (28) in Eq. (15), for matter with $w > -1$, to analyze the initial state, $d\mu/d(h^2) \rightarrow 0+$. We find that $h'_i = -\infty$, i.e., infinite deceleration,

$$a''_i = -\infty. \quad (48)$$

(For GBIG2, with $d\mu/d(h^2) \rightarrow 0-$, we have $h'_i = +\infty$.) The initial state has no infinite-temperature big bang, but it has infinite deceleration, and thus infinite Ricci curvature. The brane universe is born in a “quiescent” singularity. (Although similar singularities may be found in Induced Gravity models [13], they arise from the special extra effect of a bulk black hole or a negative brane tension.) The key point is that neither the DGP(+) model nor the GB model avoid the standard big bang, as shown in Eqs. (6) and (10). But together, the IG and GB effects combine in a “nonlinear” way to produce entirely new behaviour. If we switch off either of these effects, the infinite-temperature big bang reappears.

This singularity is reminiscent of the “sudden” (future) singularities in General Relativity [14] – but unlike those singularities, the GBIG1–2 singularity has finite pressure. The initial curvature singularity signals a breakdown of the brane spacetime. The (Minkowski) bulk remains regular, but the imbedding of the brane becomes singular. Higher-order quantum-gravity effects will be needed to cure this singularity. The fact that the matter is regular at the singularity indicates that the singularity is weaker than a standard big bang singularity, and may be easier to “cure” with quantum corrections.

By performing an expansion near the initial state, using Eq. (44), we find that the primordial Hubble rate in GBIG1, after the infinite deceleration at the birth of the

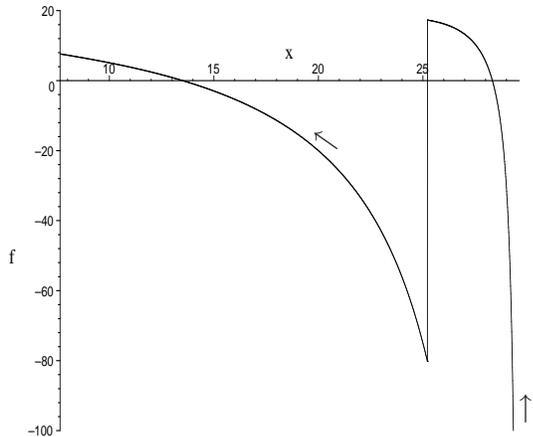


FIG. 6: The acceleration $f = a''/a$ vs $x = h^2$, for a GBIG1 cosmology with inflation, followed by radiation domination, followed by late-time self-acceleration. Arrows indicate the flow of brane proper time. Here $\gamma = 0.05$, and $n = 0.8$ in Eq. (51).

universe, is given by

$$H^2 \approx H_1^2 - H_i \left[\frac{2\kappa_5^2}{3\sqrt{r^2 - 32\alpha}} \right]^{1/2} (\rho_i - \rho)^{1/2}. \quad (49)$$

This is independent of the equation of state w . If there is primordial inflation in the GBIG1 universe, then the acceleration a'' will become positive. For a realistic model (satisfying nucleosynthesis and other constraints), a'' must subsequently become negative again, so that the universe decelerates during radiation- and early matter-domination. Finally, a'' will become positive again as the late universe self-accelerates.

We can simplify the expression (16) for the acceleration in GBIG1 via Eq. (44),

$$f = \frac{2x(3\gamma x + 1 - \sqrt{x}) + 3(1+w)[x\sqrt{x} - 2x(\gamma x + 1)]}{2(3\gamma x + 1 - \sqrt{x})}, \quad (50)$$

where $f \equiv a''/a$, $x \equiv h^2$. For a given $w(x)$, we can plot $f(x)$. We show an example in Fig. 6 of a simple model, with primordial inflation followed by radiation domination, followed by late-time self-acceleration. We

have used the effective equation of state

$$w = \begin{cases} -0.9 & n(h_i^2 - h_\infty^2) + h_\infty^2 < x < h_i^2, \\ 1/3 & h_\infty^2 < x < n(h_i^2 - h_\infty^2) + h_\infty^2. \end{cases} \quad (51)$$

Here $0 < n < 1$ is a parameter determining the time of reheating (with $n = 0$ corresponding to no inflation and $n = 1$ to no reheating/ radiation).

5. CONCLUSIONS

In summary, the GBIG1 model provides an intriguing generalization of the DGP(+) model – the Gauss-Bonnet (ultra-violet) correction to the (infra-red) Induced Gravity preserves the late-time self-acceleration of the universe, but leads to striking new behaviour in the early universe. Although there is still a curvature singularity at the beginning, the density, pressure and temperature are finite. This model deserves further investigation as a viable cosmological model. Future work will impose constraints on the model parameters from nucleosynthesis and Supernova redshifts (compare Ref. [15]). We expect that these constraints will not differ appreciably from the DGP(+) case, given the very small value of the GB parameter γ that is imposed by the UV-IR bootstrap. However, a nonzero γ , no matter how small, leads to dramatic and nonperturbative changes at high energies in the primordial universe.

From a theoretical viewpoint, it will also be important to investigate how the GB term affects the issues of strong coupling and ghosts in the DGP(+) cosmological model [16]. Can the GB term provide a lowest-order ultra-violet completion of the DGP(+) theory? The analysis of perturbations about a Minkowski brane with GB and IG terms [17] gives a starting point for tackling the Friedmann brane case.

Acknowledgements: RB and RM are supported by PPARC, EP and VZ by the NTUA research program Protagoras and by (EPEAEK II)-Pythagoras (co-funded by the European Social Fund and National Resources). We thank Mariam Bouhmadi-Lopez, Georgios Kofinas, Kazuya Koyama and Alexey Toporensky for very helpful discussions.

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