Assessing the Relation between Equity Risk Premium and Macroeconomic Volatilities in the UK

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\textbf{Abstract}
This paper uses the exponential GARCH-in-mean model to analyse the relationship between the equity risk premium and macroeconomic volatility. This premium depends upon conditional volatility, which is significantly affected by the long bond yield, acting as a proxy for the underlying rate of inflation.

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\textit{Keywords}: Asset pricing, risk premium, macroeconomic volatility, stochastic discount factor model, multivariate EGARCH-M model.

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1 Introduction

Finance theory predicts that risk premia - the extra returns that investors demand for holding risky assets - should reflect changing perceptions of risk. The UK has experienced considerable variation in macroeconomic volatility in recent decades and in this paper we examine the effect of this on the equity risk premium. We ask (i) whether macroeconomic volatilities significantly correlate with changes in inflationary expectations, proxied by the long-term government bond yield and (ii) whether the UK equity market investors significantly price in these macroeconomic volatilities. Our framework is based on the Stochastic Discount Factor (SDF), which rules out arbitrage. We use a modified trivariate exponential GARCH-in-mean (EGARCH-M) process to model the volatility in output growth, inflation and equity returns, and analyse the effect of macroeconomic volatilities upon ex ante expected returns, represented by the conditional mean returns on equity.

Following Scruggs (1998), we focus on the ‘convoluted’ (or two-stage) relation between the equity risk premium, macroeconomic risk and inflationary expectations. We use monthly data for the period 1964:1 - 2004:10. At the first stage, we find that the long bond yield exerts a significant effect on macroeconomic and financial volatilities. At the second stage, the covariance between output growth and equity return has a significant effect on the risk premium, although that between inflation and equity return does not. We find that the UK equity risk premium reflects the rise and subsequent fall in macroeconomic volatility. Specifically: the relatively low volatility period of the 1960s was followed by a more turbulent period in the 1970s, and then another low volatility period in the 1980s. Our research also suggests that the long-term government bond yield captures investor perceptions of UK stock market investment and macroeconomic risk. In addition, it suggests that investor perceptions are better represented by the long-term government bond yield than by the short-term interest rate.

As in the study of the US markets by Scruggs (1998), we find that volatility and risk premia are significantly affected by the level of the long-term bond yield, probably acting as a proxy for the underlying rate of inflation. This variable appears to provide a better explanation of volatility than the short-term interest rate used in an earlier study by Glosten, Jagannathan, and Runkle (1993). It explains a large part of the rise and subsequent fall in UK macroeconomic volatility.
volatility since the mid 1960s, consistent with the view that high levels of inflation increase macroeconomic uncertainty by confusing relative price signals and increasing the tension between fiscal and monetary policy (Okun, 1971, Friedman, 1977, Fischer, Hall and Taylor, 1981, Huizinga, 1993, among others).

Our findings are useful for practitioners and academics in several respects. First, they show how the risk-return relation can be analysed using a triangular-factorisation based multivariate EGARCH-M model, which has seldom been used in this literature. Second, they throw light upon the ‘convoluted’ relation between equity risk premia, macroeconomic and financial volatilities and long-term government bond yields for the UK, which has not yet been studied. Third, they suggest that the dramatic decline in macroeconomic volatility in the 1980s was followed by a fall in risk premia. Finally, our results may be useful for stock market investors who form expectations on the basis of macroeconomic information when evaluating their investment opportunities.

We organise our study as follows. In Section 2, we provide a literature review. In Section 3, we set up the SDF model of the equity risk premia. In Section 4, we formulate our empirical model. In Section 5, we describe the data. In Section 6, we report and discuss our empirical results and generate the implied risk premium. Finally, in Section 7, we offer some concluding remarks.

2 Literature Review

The relationship between equity market returns and inflation has been extensively studied in the financial literature and investigation of this topic has gained momentum recently. There are many ways in which the rate of inflation can affect excess returns. A number of authors have looked for a direct link between the mean of excess stock returns and inflation in the US and UK. Among these, Shiller and Beltratti (1992) reported a negligible or moderately negative relation. Lettau, Ludvigson and Wachter (2007) focused on the volatility of fundamentals in order to explain the decline in the long-term equity risk premium in the 1990s and found that the Sharpe ratio depends linearly on the volatility of consumption. However, Brandt and Wang (2003) claimed that news about inflation dominates news about consumption growth in accounting for time variation in relative risk aversion. They discarded the so-called ‘proxy
hypothesis’, but admitted that investors irrationally fear unexpected increases in inflation.\footnote{The ‘proxy hypothesis’ formulated by Fama (1981) suggests that the relation between risk aversion and inflation is misleading because it simply reflects the omitted variable bias, so long as inflation is correlated with an omitted real variable (such as future cash flows), which is in turn correlated with either risk aversion or real asset prices through a different channel.} Along similar lines, Campbell and Vuolteenaho (2004) extended the dynamic Gordon model to allow for both rational and irrational investors and found that inflation is positively correlated with rationally expected long-term real dividend growth, it is almost uncorrelated with the subjective risk premium and it is highly correlated with mispricing.

We build our study upon the methodology of Scruggs (1998), who used a modified bivariate EGARCH-M model in order to assess the two-tier risk-return (which he calls ‘convoluted’) relation embracing the equity risk premium, equity market volatility and interest rates. Our model departs from Scruggs (1998) in allowing the volatility of inflation and industrial production (as well as equity market volatility) to affect equity risk premium. Also, the information set used by investors to assess macroeconomic risk and price assets includes inflation and output growth.

Modelling EGARCH-M type heteroscedasticity of inflation and industrial production growth can be motivated by Friedman (1977), who argued that inflation uncertainty adversely affects the ability of price mechanisms to allocate resources efficiently. Fischer, Hall and Taylor (1981) and Huizinga (1993) explored this idea more formally. More recently, Grier et al. (2004) and Shields et al. (2005) have provided evidence that inflation and industrial production monthly data have a tendency to cluster in certain periods and thus exhibit conditional heteroscedasticity. In addition, the literature of empirical finance (see, e.g., Glosten, Jagannathan and Runkle, 1993, Perez-Quiros and Timmermann, 2000) reports a significant link between equity market volatility and short-term interest rate that is thought to embody investors' expectations about future inflation.

Our work builds on four previous papers. Methodologically, it builds upon Smith, Sorensen and Wickens (2007a,b) and Cappiello and Guene (2005). In Smith, Sorensen and Wickens (2007a), the authors revisit the general equilibrium-based SDF models in the context of the UK and US equity markets. The SDF is a very general pricing model, which simply rules out arbitrage. Smith, Sorensen and Wickens (2007a) find that the conditional variance between
equity return and CPI inflation is significantly priced by equity market investors. In Smith, Sorensen and Wickens (2007b), the authors, using the SDF approach, seek to identify and explain the potential asymmetries in the volatility of equity returns, inflation, industrial production growth rate and money growth rate. They find that the inflation risk premium is significantly priced by equity market investors. Although the conditional variances of equity market return and industrial production growth exhibit notable asymmetries, unexpected inflation appears to exert no asymmetric effect on the conditional variance of inflation. We follow Smith, Sorensen and Wickens (2007b) and use the SDF approach with a volatility model that contains inflation and industrial production growth rate as rewardable macroeconomic volatility factors. However, we allow their volatilities to be conditioned by the long-term government bond yield.

Ideologically, our paper is also motivated by Cappiello and Guene (2005). They used the VAR-MGARCH-M to model the inflation risk premium in bond and equity market returns in Germany and France using a more specific model - the intertemporal CAPM of Merton (1973). In Merton's intertemporal world, there is scope for hedging demands against unfavourable shifts in the investment opportunity set. Because of this hedging need, equilibrium expected equity returns on assets will depend not only on ‘systematic’ or ‘market’ risk (as in a traditional static CAPM), but also on ‘intertemporal’ risk. The intertemporal risk premium involves the covariance of equity returns with the state variables driving future returns. Because inflation can be thought to bring about unfavourable shifts in the investment opportunity set, the intertemporal risk premium can be proxied by the inflation risk premium. Cappiello and Guene (2005) find that the inflation risk premium may explain a significant proportion of the variability in the excess equity returns. It is also worth noting that in Cappiello and Guene (2005) the inflation risk premium is larger for long-term government bonds than short-term government bonds. This result is consistent with the notion that inflation is a more important macroeconomic source of risk in the long run than in the short run or, put differently, is a long-run phenomenon. For this reason, we argue that it is the long-term government bond yield that should be used to capture inflationary expectations, rather than the short-term government bond yield.

Motivated by the above literature, we ask whether macroeconomic volatilities significantly correlate with changes in inflationary expectations and whether investors significantly price in
these macroeconomic volatilities. As in Scruggs (1998), we focus on the ‘convoluted’ relation between the equity risk premium, macroeconomic risk and inflationary expectations.

3 The SDF Model of the Equity Premium

To study the relation between the equity risk premium and macroeconomic volatilities, we use the SDF model. The SDF model provides a general framework to asset pricing and is based on the no-arbitrage condition. The advantage of the SDF model is that it does not require knowledge about investors’ preferences. The use and usefulness of the SDF model in macrofinance is surveyed by Smith and Wickens (2002).

The stochastic discount factor (SDF) model is based on the notion that the price of an asset at the beginning of period \( t \), \( P_t \), is given by the expected (stochastically) discounted value of its payoff at the beginning of period \( t+1 \), \( X_{t+1} \):

\[
P_t = E_t[M_{t+1}X_{t+1}],
\]

where \( M_{t+1} \) is the stochastic discount factor and \( X_{t+1} \) is defined as

\[
X_{t+1} = P_{t+1} + D_{t+1},
\]

where \( X_{t+1} \) is a dividend payment to be received at the beginning of period \( t+1 \). Dividing equation (1) by \( P_t \) gives:

\[
1 = E_t \left[ M_{t+1} \frac{X_{t+1}}{P_t} \right] = E_t \left[ M_{t+1} R_{t+1} \right],
\]

where \( R_{t+1} = 1 + r_{t+1} \) is the gross equity return (\( r_{t+1} \) is the net equity return) and is defined as

\[
R_{t+1} = 1 + r_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t}.
\]

Assuming log-normality and taking logarithms of equation (3) gives:

\[
0 = \ln E_t \left[ M_{t+1} R_{t+1} \right] = E_t \left[ \ln (M_{t+1} R_{t+1}) \right] + \frac{1}{2} V_t \left[ \ln (M_{t+1} R_{t+1}) \right],
\]

where \( V_t \) denotes the variance conditional on time \( t \). Further operating yields:

\[
0 = E_t \left( m_{t+1} \right) + E_t \left( r_{t+1} \right) + \frac{1}{2} V_t \left( m_{t+1} \right) + \frac{1}{2} V_t \left( r_{t+1} \right) + \text{COV}_t \left( m_{t+1}, r_{t+1} \right),
\]
where $m_{t+1} = \ln M_{t+1}$ and $COV_i$ denotes the covariance conditional on time $t$.

From equation (6) and the no-arbitrage condition for a risk-free asset we obtain the risk premium:

$$E_t \left( r_{t+1} - r_t' \right) + \frac{1}{2} V_t \left( r_{t+1} \right) = -COV_t \left( m_{t+1}, r_{t+1} \right),$$

where $r_t'$ is the rate of return on a risk-free asset. Equation (7) tells us how the risk premium on an asset satisfies the no-arbitrage condition when its return and the SDF are log-normally distributed. The right-hand side is the equity premium, and $\frac{1}{2} V_t \left( r_{t+1} \right)$ is the time-varying Jensen effect arising from the assumed log-normality of the above variables.

Our main objective is to study the role of macroeconomic volatilities and the risk premium. In general, the SDF model incorporates any potential source of risk into an explanation of the risk premium as long as the no-arbitrage condition is satisfied (Smith and Wickens, 2002). One way to introduce macroeconomic volatilities in our framework is to assume that the SDF can be expressed as a linear combination of macroeconomic factors:

$$-m_{t+1} = \beta' z_{t+1},$$

where $z_{t+1}$ denotes a vector of $N$ macroeconomic factors. Therefore, the no-arbitrage condition can now be written as:

$$E_t \left( r_{t+1} - r_t' \right) + \frac{1}{2} V_t \left( r_{t+1} \right) = \sum_{i=1}^{N} \beta_i COV_t \left( z_{i,t+1}, r_{t+1} \right).$$

Assuming that the only macroeconomic factors that affect the equity risk premium are the real industrial production growth rate $\Delta y_t$ and inflation $\pi_t$, the unrestricted version of equation (9) can be expressed as:

$$E_t \left( r_{t+1} - r_t' \right) = \beta_0 V_t \left( r_{t+1} \right) + \beta_1 COV_t \left( \Delta y_{t+1}, r_{t+1} \right) + \beta_2 COV_t \left( \pi_{t+1}, r_{t+1} \right).$$

In equation (10), the equity risk premium consists of two parts: the output growth risk premium defined by $\beta_1 COV_t \left( \Delta y_{t+1}, r_{t+1} \right)$ and the inflation risk premium $\beta_2 COV_t \left( \pi_{t+1}, r_{t+1} \right)$. The inflation risk premium was modelled by Cappiello and Guené (2005) in bond and equity market returns in Germany and France by means of the intertemporal CAPM of Merton (1973). Smith, Sorensen and Wickens (2007a,b) used the SDF to model both the inflation risk premium and the output risk premium in the UK and the US.
The exact direction of the relation between the equity risk premium and macroeconomic factors is determined by the signs of the parameters $\beta_1$ and $\beta_2$. The SDF model does not place any restriction on these parameters. In the literature of macro-finance, a consensus has not yet emerged on what sign the relation between equity risk premium and macroeconomic volatilities should take. Although conventional wisdom suggests that equity market investors will require a higher reward or a higher inflation risk premium, Chen, Roll and Ross (1986) argued that since changes in inflation have the general effect of shifting wealth among investors, there is no prior presumption that would sign the risk premia for inflation. The negative signs on equity risk premia would probably mean that equity market assets are generally perceived to be hedges against the adverse influence on other assets that are, presumably, more fixed in nominal terms.

4 The Econometric Model

In order to estimate the time-varying risk premium in equation (10), we seek a specification which allows us to estimate jointly a time-varying variance and covariance matrix of excess equity return, inflation and industrial production growth. We employ the multivariate VAR-EGARCH-M model in which the conditional mean equation for excess equity return is restricted by the no-arbitrage condition.

The conditional mean equation is written in a VAR form augmented with the EGARCH-M effects:

$$Y_t = A + BY_{t-1} + \Gamma \Sigma_t j_t + u_t,$$

where $Y_t = (\Delta y_t, \pi_t, r_t - r_t^f)$ is a vector of the following variables: production growth rate, inflation, and excess return; $u_t$ is a normally distributed zero-mean error vector, $\Sigma_t$ is a (time-varying) variance-covariance matrix, and $j_t$ is a selection vector that selects the third column of $\Sigma_t$. The no-arbitrage condition requires that the third element of the intercept parameter vector $A$ and the elements of the third row of the parameter matrix $B$ equal zero. In other words, in order to rule out arbitrage opportunities, the constant term in the excess equity return equation is constrained to zero. By constraining the third row elements of $B$ to zero, we rule out lagged effects of the variables contained in the VAR. The third row of the coefficient
matrix $\Gamma$ contains the time-varying Jensen effect, shown in equation (10), and the time-varying covariances, whereas the parameters in the two other rows are constrained to zero.

We now consider the time-varying variance-covariance matrix $\Sigma_t$. In order to ensure the positive definiteness of $\Sigma_t$, a number of useful parameterisations have been proposed in the literature. A parameterisation we adopt in this research is the triangular factorisation. This parameterisation has several advantages over other parameterisations. On the one hand, the triangular decomposition can be used to identify the sequence of residuals of the structural VAR. It underlies the identification scheme proposed by Sims (1980), who suggested obtaining a unique triangular factorisation of residuals of the reduced-form VAR by imposing a specific ordering of the endogenous variables included in the VAR. Moreover, it requires no parameter constraints for the positive definiteness of $\Sigma_t$. In addition, the triangular factorisation is an orthogonal transformation, so that the resulting likelihood function is extremely simple. Because of the positive definiteness of $\Sigma_t$, there exists a lower triangular matrix $L$ with unit diagonal elements and a diagonal matrix $G$ with positive diagonal elements such that

$$\Sigma_t = LG \cdot L'.$$

As stated in Tsay (2002), an attractive feature of this decomposition is that the lower off-diagonal elements of $L$ and the diagonal elements $G$ have attractive interpretations. In particular, in the three-dimensional case, in which

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}, G_t = \begin{pmatrix} g_{11,t} & 0 & 0 \\ 0 & g_{22,t} & 0 \\ 0 & 0 & g_{33,t} \end{pmatrix},$$

the triangular decomposition of $\Sigma_t$ in equation (12) implies

$$\Sigma_t = \begin{pmatrix} \sigma_{11,t} & \sigma_{21,t} & \sigma_{31,t} \\ \sigma_{21,t} & \sigma_{22,t} & \sigma_{32,t} \\ \sigma_{31,t} & \sigma_{32,t} & \sigma_{33,t} \end{pmatrix} = \begin{pmatrix} g_{11,t} & l_{21}g_{11,t} & l_{31}g_{11,t} \\ l_{21}g_{11,t} & l_{21}^2g_{11,t} + g_{22,t} & l_{31}^2g_{11,t} + l_{22}g_{22,t} \\ l_{31}g_{11,t} & l_{31}^2g_{11,t} + l_{32}g_{22,t} & l_{31}^2g_{11,t} + l_{32}^2g_{22,t} + g_{33,t} \end{pmatrix}.$$  

Henceforth, we call the elements $g_{ii,t}$ $(i = 1, 2, 3)$ structural conditional variances.\(^2\) Using the

\(^2\) By the same token, we call the elements $\sigma_{ii,t}$ $(i = 1, 2, 3)$ reduced-form conditional variances.
triangular decomposition to parameterise $\Sigma_t$ has several convenient features. The most important feature is that $\Sigma_t$ is positive definite if $g_{ii,t} > 0$ for each $t$. Consequently, to yield the positive definiteness of $\Sigma_t$ all we have to do is to restrict $g_{ii,t}$ to being positive for each $t$. We assume here that the time-varying structural variances are driven by the lagged long-term government yield that proxies for inflationary expectations (see, for instance, Kim and Nelson, 1989, Glosten, Jagannathan and Runkle, 1993, Perez-Quiros and Timmermann, 2000 among others).

In order to model the time-variation in the conditional variance-covariance matrix $\Sigma_t$, we adopt a multivariate EGARCH-M specification, a univariate version of which was developed by Nelson (1991). As Scruggs (1998) noted, the EGARCH model constitutes a significant refinement of the GARCH model. Unlike the other functional forms of conditional heteroscedasticity, the exponential form of conditional variance ensures its positive-definiteness and thus requires placing no constraints on parameters capturing GARCH effects. Furthermore, in the last decade, the literature of empirical finance has strongly advocated using an EGARCH specification for volatility modelling, rather than square-root or affine volatility models (see, e.g., Scruggs, 1998, Perez-Quiros and Timmermann, 2000, Adrian and Rosenberg, 2005, to mention just a few). Chernov et al. (2003) compared a number of stochastic volatility models and found that exponential models perform better than affine models. In addition, EGARCH models seem to better accommodate the existence of extreme values in the financial data.\(^3\)

Compared to Scruggs (1998), our model allows for richer volatility dynamics and provides scope for efficiency gains. In fact, we estimate a three-factor CAPM model within a restricted VAR with exogenous terms and conditionally heteroscedastic errors. As in Glosten, Jagannathan and Runkle (1993), Scruggs (1998), Perez-Quiros and Timmermann (2000), our volatility model accounts for the observed relation between equity market volatility and the level of the nominal risk-free interest rate. It includes a long-term bond yield as exogenous variable, which is thought to capture long-term inflationary expectations. For the long-term bond yield we use the consol (the UK government perpetual) yield. In this model, the

\(^3\) As an alternative specification, we also use Glosten, Jagannathan, and Runkle (1993) with the lagged long-term bond yield as exogenous variable.
conditional variances of output growth, inflation and excess equity return are governed by (Model 1): \[ g_{ii,t} = \exp \left( \alpha_{i0} + \alpha_{ii} \ln(g_{ii,t-1}) + \alpha_{i2} \frac{v_{ii,t-1}}{\sqrt{g_{ii,t-1}}} + \alpha_{i3} \left( \frac{v_{ii,t-1}}{\sqrt{g_{ii,t-1}}} - \sqrt{2} \right) + \alpha_{i4} i_{t-1} \right), i = 1, 2, 3; \] (15)

where \( i_{t-1} \) denotes the long-term government bond yield, and \( v_{ii,t-1} \) denotes the structural shock. The leverage effect can be decomposed into the sign effect, captured by the parameter \( \alpha_{i2} \) and the size effect, captured by the parameter \( \alpha_{i3} \). This is consistent with the three stylised facts documented by Engle and Ng (1993). In addition, the long-term bond yield is thought to capture long-term inflationary expectations. The use of the lagged level of the long-term government bond yield is intuitively appealing. Glosten, Jagannathan and Runkle (1993) argued that, to the extent that short-term nominal interest rate embodies expectations about future inflation, it could be a good predictor of future volatility in excess return. Along similar lines, as a sole predictor of the conditional variance of excess return the short-term nominal interest rate was also used by Perez-Quiros and Timmermann (2000), which also entered exponentially in the conditional variance equation. Increasing inflation raises the riskiness of investment. Modelling inflation and output growth volatilities as a function of inflationary expectations is suggested by the Friedman (1977) hypothesis and was further supported by Fischer, Hall and Taylor (1981) and Huizinga (1993). The rationale of Friedman's hypothesis is two-fold. First, a failure of coordination of monetary and fiscal policies leads to an increased variability of inflation when a central bank attempts to counter the increased level of inflation as a consequence of loose fiscal policy. Second, the increased variability of the level of inflation distorts the allocative efficiency of the price system, causing a decrease in the natural level of output. The former hypothesis implies that the volatility of inflation may depend upon the level of inflation, supported by Spencer (2005). The latter implies that output decreases when the variability of inflation increases, corroborated by Fountas, Karanasos and Kim (2006). Therefore, we would expect inflationary expectations to exert a positive effect on macroeconomic and financial volatilities.

In Model 2, \( \alpha_{i4} \) captures the effect of the nominal short-term interest rate, whereas in Model 3, we estimate both the effect of the long-term government yield and the short-term interest rate.
Using the level of inflation is not entirely new in the literature of finance. Researchers often include the level of inflation in the investors' information set in order to account for adverse shifts in the investment opportunity set and thus the source of investment risk, as argued by Chen, Roll and Ross (1986). Also, Merton (1973) and Cappiello and Guene (2005) argued that the time-varying risk premium measures two components: (a) a stochastic volatility component and (b) a hedging component. Hedging against adverse shifts in the investment opportunity set provides scope for the consumption-smoothing behaviour of investment. For instance, if an asset provides a good hedge against inflation, intertemporally maximising investors will attempt to smooth consumption over time by holding that asset. As a result, the price of an asset will go up and investors will be willing to accept a lower rate of return.

Modelling inflation and output growth uncertainty is supported by the theoretical and empirical literature. Very recently, the literature of empirical macroeconomics (see, e.g. Grier et al., 2004; Shields et al., 2005) has come up with some evidence on the asymmetric behaviour of output growth and inflation. For instance, unanticipated inflation tends to increase inflation uncertainty more than unanticipated deflation of equal magnitude. Therefore, for the conditional variance of inflation, we would expect $\alpha_{i_2}$ to be positive. With regard to a differential size effect, the estimated model in Grier et al. (2004) provides no indication, but the Positive Size test performed by Shields et al. (2005) suggests the existence of important positive size asymmetries in the post-war data of US inflation. As for output growth uncertainty, Grier et al. (2004) found that an unexpected decline in output growth raises output uncertainty more than an unexpected increase, which would imply a negative sign for the $\alpha_{i_2}$. The estimates in Grier et al. (2004) have no implication for the differential size effect, but the analysis in Shields et al. (2005) suggests that both negative and positive size biases are present in the post-war data of industrial production growth rate. This suggests $\alpha_{i_3}$ should be positive.

We do not explicitly model conditional covariances in this research. Instead, we choose to model the lower triangular matrix $L$ that is subsequently used to obtain time-varying correlations between the residuals of the VAR. One alternative is to use the constant-correlation assumption to estimate a multivariate GARCH model (see Bollerslev, 1990). Although the constant-correlation assumption gives rise to a convenient multivariate GARCH model for estimation, many empirical studies have found that this assumption is not supported by financial data, as noted by Engle (2002). In our framework, as Tse and Tsui (2002) argued, this
assumption implies a strong restriction on data.\(^5\)

5 The Data

In order to model equity risk premium in the UK, we use a number of different sources for macroeconomic data. We obtained monthly FTSE All Share Index from the Institute of Actuaries. David Miles at Morgan Stanley kindly provided us with the UK consol yield data. Industrial production data, the 3-month Treasury bill rate and the retail price index (RPI) data were obtained from IMF IFS. As dictated by data availability, we use data sample spanning 1964:1 - 2004:10. The data are depicted in Figure 1.

6 Estimation Results

The estimation results are available in Tables 1 through 3. In these tables we report estimates of restricted modified multivariate EGARCH-M models (corresponding p-values in brackets). Models are estimated using monthly data spanning the period 1964:1 – 2004:10. The triangular factorisation of the variance and covariance matrix is performed in order to identify structural innovations. Industrial production growth is ordered first, inflation is ordered second, and

\[\rho_{21,t} = \frac{\sigma_{21,t}}{\sigma_{11,t} \sqrt{\sigma_{22,t}}}.
\]

Using the triangular factorisation of the variance and covariance matrix, we obtain:

\[\rho_{21,t} = \frac{\sigma_{21,t}}{\sigma_{11,t} \sqrt{\sigma_{22,t}}} = \frac{l_{21} \sigma_{11,t}}{\sqrt{\sigma_{11,t} \sigma_{22,t}}} = l_{21} \frac{\sqrt{\sigma_{11,t}}}{\sqrt{\sigma_{22,t}}}.
\]

Observe that, although the elements of matrix \(L\) are constant, \(\rho_{21,t}\) is necessarily time-varying. The time-varying correlation between variables 1 and 2 in the system can be recovered from the structural quantities:

\[\rho_{21,t} = l_{21} \frac{\sqrt{g_{11,t}}}{\sqrt{l_{21} g_{11,t} + g_{22,t}}}.
\]

One can show that \(\rho_{21,t}\) can only take values between \(-1\) and 1.

\(^5\) To see this, consider the time-varying correlation between the first and second variables in the system

\[\rho_{21,t} = \frac{\sigma_{21,t}}{\sigma_{11,t} \sqrt{\sigma_{22,t}}}.
\]
excess equity return is ordered third. The Schwarz Bayesian Information Criterion (BIC) is used to determine the optimal lag length for vector autoregressive (VAR) models, and selects VAR(1).

**Figure 1**

Macroeconomic and financial variables

Notes: This figure depicts monthly time series of the UK macroeconomic and financial variables that we use in our study. For all the variables, data are available for the sample 1964:1-2004:10. All variables are measured as monthly percentages.
Model 1 sets the long-term government bond yield as an exogenous explanatory variable in the conditional variance equation, giving the estimation results summarised in Table 1.

**Table 1: Model 1C (restrictions in conditional mean and conditional variance).**

<table>
<thead>
<tr>
<th>Variable</th>
<th>IP Growth</th>
<th>Inflation</th>
<th>Excess Return</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Conditional mean</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>const</td>
<td>0.2082 (0.0000)</td>
<td>0.2777 (0.0000)</td>
</tr>
<tr>
<td>2</td>
<td>Δyt-1</td>
<td>-0.1578 (0.0001)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>πt-1</td>
<td>-0.1311 (0.0139)</td>
<td>0.4106 (0.0000)</td>
</tr>
<tr>
<td>4</td>
<td>r²t-1</td>
<td></td>
<td>0.0016 (0.6005)</td>
</tr>
<tr>
<td>5</td>
<td>VTt-1(r)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>COVt-1(r, Δyt)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>COVt-1(r, πt)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Risk Premium (monthly %)</td>
<td>0.5549</td>
<td></td>
</tr>
</tbody>
</table>

| **Conditional variance** | | | |
| 9 | const | -1.3138 (0.0000) | -4.3047 (0.0000) | 0.6617 (0.0000) |
| 10 | GARCH | 0.2521 (0.0003) | -0.5555 (0.0000) | 0.6480 (0.0000) |
| 11 | Sign ARCH | | -0.1571 (0.0004) | |
| 12 | Size ARCH | 0.8625 (0.0000) | 0.5408 (0.0000) | 0.2347 (0.0006) |
| 13 | Long Rate | 2.0253 (0.0000) | 3.0220 (0.0000) | 0.6576 (0.0000) |

| **Conditional correlations** | | | |
| 14 | Chol \ Corr | 1 | 0.0839 (0.0190) | -0.0020 (0.0124) |
| 15 | Chol \ Corr | 0.0359 (0.0013) | 1 | 0.0667 (0.0004) |
| 16 | Chol \ Corr | -0.0087 (0.0103) | 0.6429 (0.0181) | 1 |

17 LogL=−1265.7328

Notes: The restricted conditional variance model uses the nominal long-term government yield as exogenous explanatory variable. In rows 1-7 we report estimates of the conditional mean model. Row 8 depicts average monthly risk premium (in percentage terms). In rows 9-13 we report estimates of the conditional variance model. In rows 14-16 we report estimates of the off-diagonal element $ij$ of the Cholesky factor matrix (lower triangular matrix) and the implied correlations (upper triangular matrix) with the corresponding asymptotic p-values in brackets. Row 17 shows the log-likelihood value that is obtained upon MLE estimation. Bollerslev and Wooldridge (1992) robust QML estimation of variance-covariance matrix of parameter estimates is calculated.

We analyse model adequacy by means of a number of diagnostics (not reported). The Engle and Ng (1993) Sign Bias, Negative Size Bias, Positive Size Bias and Joint tests suggest no evidence of predictable components left over in the squared standardised residuals that are related to volatility sign and size asymmetries. Likewise, Nelson (1991) specification tests suggest that the orthogonality conditions are not, with few exceptions, significantly different from zero at 5% significance level. Nevertheless, robust quasi-maximum likelihood (QML) estimation of the variance and covariance matrix of the parameters (Bollerslev and Wooldridge, 2008, 50-77).
1992) produces consistent standard errors when the model is possibly misspecified. Overall, the trivariate modified EGARCH-M model seems to be reasonably well specified.

Model 2 sets the short-term interest rate as an exogenous variable in the conditional variance equation (see footnote 4).

Table 2: Model 2C (restrictions in conditional mean and conditional variance).

<table>
<thead>
<tr>
<th>Variable</th>
<th>IP Growth</th>
<th>Inflation</th>
<th>Excess Return</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Conditional mean</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 const</td>
<td>0.2011 (0.0001)</td>
<td>0.2696 (0.0000)</td>
<td></td>
</tr>
<tr>
<td>2 Δyt</td>
<td>-0.1756 (0.0006)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 πt</td>
<td>-0.1117 (0.1283)</td>
<td>0.4286 (0.0000)</td>
<td></td>
</tr>
<tr>
<td>4 re</td>
<td>0.0034 (0.3528)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 Vt(rt)</td>
<td></td>
<td></td>
<td>0.0058 (0.7166)</td>
</tr>
<tr>
<td>6 COVt(rt, Δyt)</td>
<td></td>
<td></td>
<td>-4.0309 (0.8951)</td>
</tr>
<tr>
<td>7 COVt(rt, πt)</td>
<td></td>
<td></td>
<td>0.2598 (0.8769)</td>
</tr>
<tr>
<td>8 Risk Premium (monthly %)=0.2645</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| **Conditional variance** |           |           |               |
| 9 Const              | -0.6650 (0.0000) | -3.8391 (0.0000) | 0.4144 (0.1448) |
| 10 GARCH             | 0.3308 (0.0004)  | -0.5433 (0.0000) | 0.8260 (0.0000) |
| 11 Sign ARCH         |           |           | 0.0984 (0.0000) |
| 12 Size ARCH         | 0.9494 (0.0000)  | 0.5709 (0.0000)  | 0.2670 (0.0156) |
| 13 Short Rate        | 1.2840 (0.0000)  | 2.6856 (0.0000)  | 0.2440 (0.0000) |

| **Conditional correlations** |
| 14 Chol \ Corr       | 1          | 0.0944 (0.0320) | -0.0055 (0.0494) |
| 15 Chol \ Corr       | 0.0398 (0.0019) | 1          | 0.0765 (0.0010) |
| 16 Chol \ Corr       | -0.0230 (0.8939) | 0.7409 (0.0814) | 1 |

LogL=-1279.9528

Notes: The restricted conditional variance model uses the nominal long-term government yield as exogenous explanatory variable. In rows 1-7 we report estimates of the conditional mean model. Row 8 depicts average monthly risk premium (in percentage terms). In rows 9-13 we report estimates of the conditional variance model. In rows 14-16 we report estimates of the off-diagonal element $l_{ij}$ of the Cholesky factor matrix (lower triangular matrix) and the implied correlations (upper triangular matrix) with the corresponding asymptotic p-values in brackets. Row 17 shows the log-likelihood value that is obtained upon MLE estimation. Bollerslev and Wooldridge (1992) robust QML estimation of variance-covariance matrix of parameter estimates is calculated.

Lastly, we estimated Model 3, which uses both the long-term government bond yield and the short-term interest rate as exogenous explanatory variables in the conditional variance equation (see footnote 4).
Table 3: Model 3C (restrictions in conditional mean and conditional variance).

<table>
<thead>
<tr>
<th>Variable</th>
<th>IP Growth</th>
<th>Inflation</th>
<th>Excess Return</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Conditional mean</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>const</td>
<td>0.2076 (0.0000)</td>
<td>0.2742 (0.0000)</td>
</tr>
<tr>
<td>2</td>
<td>Δyt-1</td>
<td>-0.1612 (0.0023)</td>
<td>0.4097 (0.0000)</td>
</tr>
<tr>
<td>3</td>
<td>πt-1</td>
<td>-0.1260 (0.3980)</td>
<td>0.0021 (0.5478)</td>
</tr>
<tr>
<td>4</td>
<td>rt-1</td>
<td>0.0021 (0.5478)</td>
<td>0.6540 (0.0002)</td>
</tr>
<tr>
<td>5</td>
<td>Vt-1(rt)</td>
<td>-1.3010 (0.0000)</td>
<td>-4.4569 (0.0000)</td>
</tr>
<tr>
<td>6</td>
<td>COVt-1(rt, Δyt)</td>
<td>0.2562 (0.0031)</td>
<td>-0.5552 (0.0000)</td>
</tr>
<tr>
<td>7</td>
<td>COVt-1(rt, πt)</td>
<td>0.8727 (0.0000)</td>
<td>0.5338 (0.0000)</td>
</tr>
<tr>
<td>8</td>
<td>Risk Premium (monthly %)=0.4497</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Conditional variance</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Const</td>
<td>-1.3010 (0.0000)</td>
<td>-4.4569 (0.0000)</td>
</tr>
<tr>
<td>10</td>
<td>GARCH</td>
<td>0.2562 (0.0031)</td>
<td>0.5552 (0.0000)</td>
</tr>
<tr>
<td>11</td>
<td>Sign ARCH</td>
<td>-1.5300 (0.0050)</td>
<td>0.2409 (0.0126)</td>
</tr>
<tr>
<td>12</td>
<td>Size ARCH</td>
<td>0.8727 (0.0000)</td>
<td>0.5338 (0.0000)</td>
</tr>
<tr>
<td>13</td>
<td>Long Rate</td>
<td>1.7667 (0.0000)</td>
<td>2.0939 (0.0163)</td>
</tr>
<tr>
<td>14</td>
<td>Short Rate</td>
<td>0.2618 (0.3512)</td>
<td>1.2204 (0.1381)</td>
</tr>
<tr>
<td><strong>Conditional correlations</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>Chol \ Corr</td>
<td>1</td>
<td>0.0870 (0.0188)</td>
</tr>
<tr>
<td>16</td>
<td>Chol \ Corr</td>
<td>0.0371 (0.0370)</td>
<td>1</td>
</tr>
<tr>
<td>17</td>
<td>Chol \ Corr</td>
<td>-0.0053 (0.6630)</td>
<td>0.6396 (0.0000)</td>
</tr>
<tr>
<td>18</td>
<td>LogL= -1263.6680</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: The restricted conditional variance model uses the nominal long-term government yield and nominal short-term interest rate as exogenous explanatory variables. In rows 1-7 we report estimates of the conditional mean model. Row 8 depicts average monthly risk premium (in percentage terms). In rows 9-13 we report estimates of the conditional variance model. In rows 14-16 we report estimates of the off-diagonal elements $l_{ij}$ of the Cholesky factor matrix (lower triangular matrix) and the implied correlations (upper triangular matrix) with the corresponding asymptotic p-values in brackets. Row 17 shows the log-likelihood value that is obtained upon MLE estimation. Bollerslev and Wooldridge (1992) robust QML estimation of variance-covariance matrix of parameter estimates is calculated.

Models 1C through 3C refer to the models with restrictions in the equations for conditional mean and conditional variance, discussed in Section 6.2. Because Models 1C through 3C are our preferred models, we report the estimation results for these models only.

6.1 Conditional Mean Equation

We first consider the conditional mean model. Because in Models 1A through 3A the effect of excess equity return on output growth, and the effect of output growth on inflation are insignificant, we reestimate these models excluding these insignificant variables. Models that
restrict the conditional mean dynamics are henceforth referred to as Models 1B through 3B, respectively. We use a standard likelihood ratio test to test the restricted Models 1B through 3B against Models 1A through 3A, respectively. With regard to Model 1, the test statistic \( \chi^2(2) = 0.6134, p = 0.7539 \) can not reject the restricted Model 1B. Similarly, we can not reject Model 2B against Model 2A \( \chi^2(2) = 0.4950, p = 0.7808 \) or Model 3B against Model 3A \( \chi^2(2) = 0.6542, p = 0.7210 \). Therefore, our preferred models, which best characterise the conditional mean dynamics, are Models 1B through 3B (not reported in the paper, but available upon request).

Notably, the parameter estimates in the industrial production growth and inflation equations appear to be relatively more stable than the estimates of the EGARCH-M effects in the conditional mean equation of excess equity return. We observe that industrial production growth and inflation are essentially determined by their own lagged terms. In addition, the lagged rate of inflation has a significantly negative effect on the industrial production growth rate.

### 6.2 Conditional Variance Equation

The focus of this research concerns the two-tier relation between the equity risk premium, financial and macroeconomic volatilities and inflationary expectations. This can be decomposed into two parts. The first tier involves the relation between the conditional volatilities and the inflationary expectations captured by the long-term government bond yield. The second tier concerns the relation between the equity risk premia and financial and macroeconomic volatilities.

In this subsection, we consider the first tier structure. In this relationship, the lagged conditional variance is found statistically significant in all three equations. The asymmetric sign effect, captured by the parameter \( a_{12} \), is significantly negative for the equity market volatility, as expected. With regard to the inflation equation, we report a negative, albeit imprecisely estimated, inflation volatility sign effect. The inflation volatility sign effect is dominated by the volatility size effect, which is significantly positive. This result is consistent with our previous
discussion in Section 4 and the results of Grier et al. (2004) and Shields et al. (2005). As in the case of equity market volatility, the finding that large innovations of either sign to inflation (industrial production growth) impact upon the conditional variance of inflation (industrial production growth) is not unreasonable.

Within the first tier structure, we are specifically interested in the effect that the long-term government yield exerts on the conditional variances. Section 4 suggests that the long-term government bond yield should exert a significantly positive influence. The long yield has a significantly positive effect on the three conditional variances, as expected.

We would expect the long-term government bond yield to have played a more important role in affecting macroeconomic and financial risk than the short-term interest rate. To test this proposition, we estimate a multivariate EGARCH-M model, in which we use the nominal short-term interest rate in financial and macroeconomic volatility modelling (Model 2). We observe a strong correlation of the nominal short-term interest rate with the conditional variance of industrial production growth and inflation, but not with the conditional variance of equity return. Moreover, as Model 3 indicates, when the long-term government bond yield is also included, the effect that the short-term interest rate exerts on the conditional variances of industrial production growth rate and inflation becomes insignificant. We conclude that the UK equity market assessments of macroeconomic volatility are better represented by long-term financial yields than by short-term rates.

Because the asymmetric sign effects in the volatility models for inflation and output growth turn out to be insignificant, we reestimate Models 1B through 3B, excluding these effects. This gives us Models 1C through 3C, respectively. We use a likelihood ratio test to test Models 1C through 3C against Models 1B through 3B. With regard to Model 1, the test statistic 
\( \chi^2(2) = 0.2902, p = 0.8649 \) cannot reject the restricted Model 1C. Similarly, we can not reject Model 2C against Model 2B \( \chi^2(2) = 0.0628, p = 0.9691 \) or Model 3C against Model 3B \( \chi^2(2) = 0.3168, p = 0.8535 \). Thus the models that characterise best the conditional mean and conditional variance dynamics are Models 1C through 3C. Because the long-term government yield outperforms the short-term interest rate, our model of reference is Model 1C.

Finally, we also calculate Ljung-Box Q-statistics for the standardised residuals from Model 1C (results are not reported, but are available upon request). Because the Q-statistics suggest...
some evidence of serial correlation in the residuals of the equation for inflation \( Q(12) = 278.902, p = 0.0000 \), we reestimate this model adding more lags to the mean equation for inflation. We find that using 13 lags of inflation eliminates serial correlation from the residuals of the equation for inflation \( Q(12) = 15.8777, p = 0.1969 \), but that the results do not change. The average monthly risk premium for Model 1C when 13 lags are used in the conditional mean equation for inflation is estimated at 0.52\% (6.24\% per annum). Also, the t-test suggests that risk premium implied by the model with 13 lags is not significantly different from the risk premium for the model with one lag in the mean equation for inflation. However, because Lutkepohl (1991) argued that using less rather than more lags improves predictability of a VAR, our preferred model is Model 1C with one lag.

### 6.3 The Risk Premium

We next focus on the second tier relation. The estimation results of Model 1C indicate that there is some evidence that the UK equity risk premium reflects the behaviour of macroeconomic volatilities. More specifically, we find that the output growth risk premium has a significant effect on the UK excess equity return.

The implied equity premium is given by

\[
RP_{t+1} = \gamma_{31} \hat{COV}_t(\Delta y_{t+1}, r_{t+1}) + \gamma_{32} \hat{COV}_t(\pi_{t+1}, r_{t+1}),
\]

where \( \gamma_{31} \) and \( \gamma_{32} \) are the (3,1)th and (3,2)th elements of the parameter matrix \( \Gamma \), respectively, and correspond to the parameters \( \beta_1 \) and \( \beta_2 \) in equation (10). \( \hat{COV}_t(\Delta y_{t+1}, r_{t+1}) \) and \( \hat{COV}_t(\pi_{t+1}, r_{t+1}) \) are estimated time-varying conditional covariances of the equity return with industrial production growth and inflation, respectively. Having estimated the model, we generate the implied equity premium series over the sample period. To yield a better representation for the implied equity risk premium, we remove unnecessary noise by taking a 12-month moving average of the series. The average monthly risk premium is 0.55\% (6.60\% per annum) for the UK. The implied risk premium series is drawn in Figure 2.
Figure 2
Equity risk premium (Model 1C)

Notes: This figure depicts a 12-month moving average of the time-varying equity risk premium for the UK (Model 1C), as a monthly percentage. Top (bottom) panel shows the contribution of output (inflation) volatility to the equity risk premium.

Figure 2 shows a rise in the UK equity risk premium in the early 70s followed by a gradual decline. At the beginning of the sample, the risk premium is just slightly higher than at the end of the sample. The risk premium features a sharp increase in February 1974, in the aftermath of the first oil price shock, but then it steadily decreases towards the end of the sample. To pursue a deeper analysis of this risk premium pattern, we also generate the equity risk premium shares due to the time-varying covariance between industrial production growth rate and excess equity return (output growth risk premium) and covariance between inflation and excess equity return (inflation risk premium), depicted in the top and bottom panels of Figure 2, respectively. Because this outlier alone appears to have shaped the time variation in the risk premium, we
analyse whether this sharp increase is due to inflation or output growth. The data indicate that in January 1974 industrial UK output slumped by 7.7% in comparison with the previous month and the month-to-month inflation increased to 1.6\% in January 1974, reaching 2.0\% in February and peaking at 3.4\% in May of the same year. Therefore, the first oil price shock appears to have simultaneously decreased industrial production and increased consumer prices, a phenomenon commonly described by macroeconomists as an ‘adverse supply-side shock’ or ‘stagflation’. This decomposition indicates that the output growth risk premium experienced a larger, albeit less persistent, increase than the inflation risk premium. There was also a rise, albeit much smaller in magnitude, in the UK risk premium in 1979, in the aftermath of the second oil price shock, which gradually worked itself out.

Conditional variances are an important constituent of the risk premium. To see this, consider the conditional mean equation for the excess equity return:

\[ r_{t+1} - r_t = \beta_0 V_t(r_{t+1}) + \beta_1 COV_t(\Delta y_{t+1}, r_{t+1}) + \beta_2 COV_t(\pi_{t+1}, r_{t+1}) + u_{5,t+1}, \]

(17)

Where the first component represents the Jensen effect, and the equity risk premium is defined by the sum of the second and third components in the right hand side of equation (17). The reparameterisation we use implies:

\[ V_t(r_{t+1}) = l_{11}^2 g_{11,t} + l_{22}^2 g_{22,t} + g_{33,t}; COV_t(\Delta y_{t+1}, r_{t+1}) = l_{31} l_{21} g_{11,t} + l_{32} g_{22,t}; COV_t(\pi_{t+1}, r_{t+1}) = l_{31} l_{21} g_{11,t} + l_{32} g_{22,t}. \]

(18)

This suggests that the conditional mean is a linear function of the conditional variances.

The conditional variances are depicted in Figure 3. The implied risk premium plot in Figure 2 shows some of the features of the time variation in the conditional variances depicted in Figure 3. Remarkably, the UK risk premium closely resembles the time variation in the conditional variance of industrial production growth rate. What is noteworthy is the time variation in conditional correlations, depicted in Figure 4.

We observe a positive time-varying correlation between output growth and inflation. Thus the sign of this correlation is as predicted by the conventional Phillips curve, although not necessarily supported by the empirical evidence. Moreover, as Smith, Sorensen and Wickens (2007b) argue, this is only true when a given business cycle phase is due to a demand shock. However, a recession due to a supply shock is likely to have higher rather than lower inflation, which would imply a negative relation between output growth and inflation. We further observe a small negative time-varying correlation between industrial production growth rate and equity return, the sign of which is difficult to interpret. Finally, because low returns and
low inflation are expected in a recession, we observe a positive correlation between these two variables, which is in line with the implications of procyclical monetary policy (Boyle and Peterson, 1995).

**Figure 3**

Conditional variances (Model 1C)

Notes: This figure depicts conditional variances of industrial production growth rate (top panel), inflation (middle panel) and stock return (bottom panel) implied by the model (12-month moving average).
Figure 4
Time-varying correlations (Model 1C)

Notes: This figure illustrates time-varying correlations implied by the model (12-month moving average). The top panel depicts the correlation between output growth and inflation, the middle panel depicts the correlation between output growth and equity return, and the bottom panel depicts the correlation between inflation and equity return.
6.4 Discussion

This evidence indicates that macroeconomic and financial volatility may be driven by inflationary expectations captured by the nominal long-term government yield rather than by the nominal short-term interest rate, the view advocated by Glosten et al. (1993), Scruggs (1998), Perez-Quiros and Timmermann (2000), among others. To our knowledge, such evidence is largely new and it is consistent with findings reported in Cappiello and Guene (2005) that the inflation risk premium is larger for long-term government bonds than short-term government bonds. The long-term government yield has a positive sign in the three equations governing macroeconomic and financial volatility. The statistically positive effect in the conditional variance of inflation can be reconciled with Friedman (1977), Fischer, Hall and Taylor (1981) and Huizinga (1993). This effect in the conditional variance of output growth stems from Grier et al. (2004), whereas the positive effect in the conditional variance of equity returns is emphasised in Shanken (1990), Glosten et al. (1993), Scruggs (1998), Perez-Quiros and Timmermann (2000). Further, consistent with the findings of Shields et al. (2005), the impact coefficient of large macroeconomic and financial shocks on macroeconomic and financial volatility is larger than that for small shocks, of either sign. However, contrary to implications in Grier et al. (2004) and Smith, Sorensen and Wickens (2007b), we do not find evidence that a negative macroeconomic shock has a different effect on macroeconomic volatility than a positive shock.

We find that the UK equity risk premium reflects the rise and subsequent fall in macroeconomic volatility. More specifically, a relatively low premium in the 1960s was followed by a more turbulent period and a larger premium in the 1970s, but the risk premium has fallen since the early 1980s. Using macroeconomic volatilities to explain equity risk premium is consistent with Lettau, Ludvigson and Wachter (2007), who argued that the declining US equity risk premium can be explained by the behaviour of macroeconomic volatilities and with Brandt and Wang (2003), who showed that news about inflation dominates news about consumption growth in accounting for time variation in relative risk aversion. In contrast to findings in Schwert (1989), but consistent with Smith, Sorensen and Wickens (2007b), who studied the US equity risk premium, the output growth risk premium exerts a significant effect on the UK excess equity return.
7 Conclusions

In this paper, we have used a multivariate EGARCH-M model to study the two-tier risk-return relation between the equity risk premium, macroeconomic and financial volatilities and inflationary expectations. To rationalise this structure, we built our empirical study upon the SDF model. One of the distinctive features of our empirical model is the triangular-factorisation based modelling of time-varying structural variances and dynamic conditional correlations. Another distinctive feature of our empirical model is using a long-term government bond yield, which can be thought of as capturing inflationary expectations, to condition macroeconomic and financial volatilities in the first tier relation. We also use both the long-term government bond yield and the short-term interest rate and the short-term interest rate alone in modelling macroeconomic and financial volatilities.

Our research suggests that the long-term government bond yield does capture investor perceptions of UK stock market investment opportunities and risk. In addition, it suggests that these investor perceptions are better represented by the long-term government bond yield than by the short-term interest rate. In fact, we no longer find a significant relation between conditional variances and the nominal short-term interest rate when both the nominal long-term government yield and nominal short-term interest rate are included in the volatility model.

Within the second tier relationship, we allow the volatility of inflation and industrial production growth, as well as equity market volatility, to affect the equity risk premium. At this stage, we find that the covariance between output growth and equity return has a significant effect on the risk premium, although that between inflation and equity return does not.

The implied risk premium reflects the rise and subsequent fall in macroeconomic volatilities. Specifically, the relatively low volatility period of the 1960s was followed by a more turbulent period in the 1970s, but in the 1980s volatility gradually decreased towards the end of the sample. These fluctuations were clearly reflected in the equity risk premium.

Our research contributes to the ongoing analysis of the relationship between risk and return. The finding that the long-term nominal yield is an important influence on volatility suggests that these effects are more persistent than documented in earlier studies of short-term interest rate and volatility-clustering effects.
References


