LMI Based Robust PID Controller Design for PWR with Bounded Uncertainty using Interval Approach

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***Abstract*—Pressurized Water Reactors are popular in the nuclear power generation industry due to their self-regulating and self-stabilizing features which in turn help them to perform load-following operation. However, fast power maneuvering is a challenging task due to inherent nonlinearity in a reactor. It leads to changes in the behavior with variation in reactor power. Further, the heat transfers from fuel to coolant and the reactivity changes due to variation in fuel and coolant temperatures introduces uncertainty in the system. Thus, it is essential to design a robust controller for load-following operation. In this paper, the point kinetics model of PWR coupled with the Mann's thermal-hydraulic model has been considered in addition to power sensor model and actuator model. This model has been identified as an interval system with bounded parametric uncertainties in measurements (fuel and coolant). The work then formulates a methodology to design a single robust PID controller using linear matrix inequalities by varying the weighting matrices. The outcomes have been validated using MATLAB simulations and discreetly exemplified in the result section.**

Keywords-Interval Approach, LMI, PWRs, Robust PID Controller, Uncertain System.

I. Introduction

Designing a controller for Pressurized Water Reactor (PWR) is a challenging task and needs ample attention due to many reasons. The most important reason is the inherent non-linear and time-variant nature of the system that depends on the operating power of the reactor [1]. In another case, reactivity defect (i.e. change in the reactivity) caused by the changes in reactor power is variable with respect to the fuel condition of the core (fresh or equilibrium core), adding to the uncertainties in reactivity feedback. Further, uncertainties also exist in the estimation of the heat-transfer coefficient and heat transfer area. Due to these difficulties, the control of PWR has always instilled deep interest into the contemporary researchers.

Various researches have been attempted to address the problem of load-following control of PWR during power maneuvering. Model Predictive Control (MPC) has been studied extensively for addressing load-following problems. Kim et al. [2] used MPC to automatically control power distribution and power level. In another work, Eliasi et al. [3] proposed robust MPC by imposing constraints on states during optimization. State estimators based MPC design has been considered in [4]. Apart from MPC, many other controller design techniques exist in the load-following literature. Dong et al. [5] used feedback dissipating controller with nonlinear observer having a Hamiltonian structure for single input single output system and extended it to design controller for a low temperature PWR. Torabi et al. [6] designed a controller for power regulation of a PWR using quantitative feedback theory approach. Adaptive controller design methodology has been adopted for power level control of PWR, where the designed controller guarantees global asymptotic stability [7]. Bose et al. [8, 9] has proposed load following operation for small PWR using nonlinear dynamic inversion technique which efficiently tracks the set-point but could not guarantee stability of the system.

A robust controller for a bounded uncertain system by various means ranging from basic optimal Full State Feedback Controllers (FSFC) to optimal FSFC is proposed in [10]. Optimal FSFCs are known to procure robust designs [11] although they do not necessarily guarantee desired time response under bounded parametric uncertainties. It may be noted that Fuzzy Logic Controllers (FLC)s are well-known

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for their uncertainty handling capacities and some researchers [12] have also proposed the use of FLCs for control of uncertain system. However, the use of such controllers for the control of different systems is difficult to validate from stability point of view.

Kharitonov theorem to design a robust controller for a nuclear reactor has been introduced by Lee et al. in [13]. An inexact transfer function model of a nuclear reactor is obtained using the methodology presented in [13] and then used to derive a controller which can take care of typical parametric uncertainties in measured reactor power, reactivity defect and heat-transfer coefficient. It shows a transfer function with bounded parametric uncertainties as an interval system which can be represented in the form.

$$G(s,\mathbf{v}) = \frac{r_0(s)+\alpha}{q_0(s)+\sum_{i=1}^{2}\varepsilon_i s^i} :\ \alpha \in \left[\underline{\alpha},\ \bar{\alpha}\right]; \varepsilon_i \in \left[\underline{\varepsilon}_i,\ \bar{\varepsilon}_i\right] \quad (1)$$

where the parameter vector is defined as

$$\mathbf{v} = \left[\varepsilon_1, \varepsilon_2, \alpha\right]^{\mathrm{T}} \quad (2)$$

Thus a second order interval system can be written as

$$G(s,\mathbf{v}) = \frac{L}{s^2+as+b}; L = \left[\underline{L},\ \bar{L}\right], a = \left[\underline{a},\ \bar{a}\right], b = \left[\underline{b},\ \bar{b}\right] \quad (3)$$

In (1) and throughout the rest of this paper a bounded interval variable N is defined as $N \in \mathbb{R} \mid \underline{N} \le N \le \bar{N}$ where $\underline{N} = N - \Delta N$ represents infimum of N, $\bar{N} = N + \Delta N$ its supremum and $\left[\underline{N},\ \bar{N}\right]$ represents the interval of N, $N \in \left[\underline{N},\ \bar{N}\right]$. This mathematical definition of interval system holds for vectors as well as for matrices, as reported in [14].

The use of Proportional-plus Integral-plus Derivative (PID) controllers for de-centralized control of industrial process plant is well known [15]. PID controllers are known for the advantages they offer in terms of tracking, robustness, disturbance rejection and noise sensitivity. Though most of the industries employ classical PID tuning method proposed by Zeigler-Nichols (Z-N), it has been seen that PID tuning method by Z-N becomes insufficient in certain cases where high performance is a basic requirement. Among the various methods proposed by researchers, the gain-phase margin-based method [16], modified Z-N method [17], dominant pole placement method [18] and Smith predictor based PID tuning method [19] have gained much priority.

Linear Quadratic Regulator (LQR) based optimal PID controller design is widely used in modern control theory due to its high robustness. In [20] He et al. has been proposed the PID controller tuning using LQR approach for low order time delay system by using specified weight matrices($\mathbf{Q}_0 = \mathbf{Q}_0^{\mathrm{T}} \ge 0, \mathbf{R}_0 = \mathbf{R}_0^{\mathrm{T}} > 0$) depends on the open loop and closed loop damping ratio (ξ_0^{ol}, ξ_0^{cl}), natural frequency ($\omega_0^{ol}, \omega_0^{cl}$) and closed loop real pole λ_0 of the nominal system. The $\mathbf{Q}_0$ and $\mathbf{R}_0$ for this controller has been designed in such a way that the controller meets the desired performance. Banerjee et al. [21] has extended this methodology for an interval system for a Pressurized heavy water reactor. However, the methodology presented in [21] only establishes uncertainty handling capability of the corresponding nominal plant PID controller.

In this paper, the robust PID controller design technique for a PWR has been proposed by selecting the weighting matrix ($\mathbf{Q}$) for specified closed loop damping ratio (ζ_0^{cl}), natural frequency (ω_0^{cl}), and real pole (λ_0). The interval algorithm is used to design the proposed controller. The disturbances are considered in the dynamics of the process as well as in the corresponding input of the system which are expressed in terms of bounded parametric uncertainty. The design of the proposed robust PID controller has been framed with LMI rather than the direct optimal PID controller [20] approach by selecting $\mathbf{Q}_0$ and $\mathbf{R}_0$ to obtain an efficacious controller as the former one can handle more parametric uncertainties.

The rest of the paper is organized as follows: Section II describes the PWR model. Section III proposes the optimal PID controller design and the LMI also which has been framed with LQR by the proposed matrices. Section IV shows the simulation result for PWR during power maneuvering. Finally, section V concludes the paper.

II. PWR Model with Sensor and Actuator

In this section, it is attempted to obtain an inexact transfer function model for a PWR defined by (3). A lumped parameter point kinetics model of a PWR has been considered with thermal hydraulic model. The point kinetics model is given by following equations:

$$\frac{dP}{dt} = \frac{\rho_t - \beta}{\Lambda} P + \lambda C \quad (4)$$

$$\frac{dC}{dt} = \beta \frac{P}{\Lambda} - \lambda C \quad (5)$$

where P and ρ_t represent neutronic power and total reactivity respectively. Λ denotes neutron generation time. λ and β are average of decay constant and delayed neutrons fraction of six groups of delayed neutrons respectively. C is delayed neutron precursor concentration.

The core thermal-hydraulics model is given by Mann's model [22] which considers two coolant lumps for every fuel lump. It is given by

$$\frac{dT_f}{dt} = H_f P - \frac{1}{\tau_f}\left(T_f - T_{c1}\right) \quad (6)$$

$$\frac{dT_{c1}}{dt} = H_c P + \frac{1}{\tau_c}\left(T_f - T_{c1}\right) - \frac{2}{\tau_r}\left(T_{c1} - T_{cin}\right) \quad (7)$$

$$\frac{dT_{c2}}{dt} = H_c P + \frac{1}{\tau_c}\left(T_f - T_{c1}\right) - \frac{2}{\tau_r}\left(T_{c2} - T_{c1}\right) \quad (8)$$

where T_f is average fuel temperature; T_{c1} and T_{c2} are average coolant temperatures in node 1 and node 2 respectively; T_{cin} is inlet temperatures of the first coolant node; H_f and H_c characterize the rate of rise of fuel and coolant temperatures respectively; τ_f and τ_c are time constants representing mean time for heat transfer from fuel to coolant and from core outlet to inlet respectively whereas τ_r represents coolant residence time in the core. The heat transfer coefficient from fuel to coolant is assumed to be constant.

In this paper it is assumed that the reactor is running on alternate mode, i.e., there is no constraint on average coolant temperature [23]. The change in total reactivity is considered due to control rod movement and due to reactivity feedbacks from fuel and coolant temperatures. Here, control rod acts as an actuator and this actuator movement can be represented by following sets of equations:

$$\frac{d\rho_{ex}}{dt} = Gz \tag{9}$$

$$\frac{dz}{dt} = \alpha_{rod} i_{rod} \tag{10}$$

where ρ_{ex} is the external reactivity injected into the reactor core due to the control rod movement, G is the reactivity worth of control rod, z is speed of the control rod, α_{rod} is the scalar multiplier and i_{rod} is the applied control signal to regulating the control movement.

The total reactivity can be obtained by

$$\rho_t = \rho_{ex} + \alpha_f T_f + \alpha_c T_{c1} + \alpha_c T_{c2} \tag{11}$$

where, α_f and α_c represent the temperature coefficients of reactivity due to fuel and coolant respectively.

A self-powered neutron detector (SPND) is used to measure the reactor core power which produces current corresponding to the power and can be represented by (12)

$$\frac{di_{lin}}{dt} = -\frac{1}{\tau_{lin}} i_{lin} + \frac{K_{lin}}{\tau_{lin}} \frac{P}{P_0} + \frac{4}{\tau_{lin}} \tag{12}$$

where, i_{lin} is the output current of SPND amplifier, k_{lin} and τ_{lin} are the gain and time constant of the amplifier respectively.

Equations (4)-(12) are used to develop a state-vector model for the PWR of the form

$$\dot{\hat{\mathbf{x}}}(t) = \hat{\mathbf{A}}\hat{\mathbf{x}}(t) + \hat{\mathbf{B}}\hat{\mathbf{u}}(t) \text{ and } \hat{y}(t) = \hat{\mathbf{C}}\hat{\mathbf{x}}(t) \tag{13}$$

where $\hat{\mathbf{x}} = \begin{bmatrix} P & C & T_f & T_{c1} & T_{c2} & i_{lin} & \rho_{ex} & z \end{bmatrix}$ denotes the set of state variables around an equilibrium state and $\hat{\mathbf{u}}$ is the change in current to drive the control rod. $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$ can be computed by (14) and (15).

$$\hat{\mathbf{A}} = \begin{bmatrix} -\frac{\beta}{\Lambda} & \lambda & \frac{P_0\alpha_f}{\Lambda} & \frac{P_0\alpha_c}{\Lambda} & \frac{P_0\alpha_c}{\Lambda} & 0 & \frac{P_0\alpha_f}{\Lambda} & 0 \\ \frac{\beta}{\Lambda} & -\lambda & 0 & 0 & 0 & 0 & 0 & 0 \\ H_f & 0 & -\frac{1}{\tau_f} & \frac{1}{\tau_f} & 0 & 0 & 0 & 0 \\ H_c & 0 & \frac{1}{\tau_c} & -\left(\frac{1}{\tau_c}+\frac{2}{\tau_r}\right) & 0 & 0 & 0 & 0 \\ H_c & 0 & \frac{1}{\tau_c} & -\left(\frac{1}{\tau_c}-\frac{2}{\tau_r}\right) & -\frac{2}{\tau_r} & 0 & 0 & 0 \\ \frac{K_{lin}}{\tau_{lin}} & 0 & 0 & 0 & 0 & -\frac{1}{\tau_{lin}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & G \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{14}$$

$$\hat{\mathbf{B}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_{rod} \end{bmatrix}^{\mathrm{T}} \tag{15}$$

A MATLAB simulation model has been constructed using (14) and (15) to identify the model as an interval system form of (3). For model identification, the current for control rod movement is taken as input and the sensor current is taken as output. The identification is carried out in MATLAB using the System Identification Toolbox and the data for identification were obtained from [24] and [25]. For 60% and 100% power levels, the identification data has been collected from simulation model run for 25s with 0.01 sampling time. The transfer function models for various power levels have been identified using the Process Model estimation of the System Identification Toolbox and only the models with best-fit (more than 99%) are selected for study. The identification process is carried out in accordance with the technique mentioned in [26].

III. Robust PID Controller Design using LMI

In this section the design methodology for a robust PID controller has been established. For designing a PID controller as reported in [20], the nominal plant can be considered as

$$G_0(s) = \frac{r_0(s)}{q_0(s)} = \frac{L_0}{s^2 + a_0 s + b_0} \tag{16}$$

If $y(s)$ is the output of the plant $G_0(s)$, then the PID controller is given by

$$G_c(s) = k_p + \frac{k_i}{s} + k_d s \tag{17}$$

as an optimal Full State Feedback Controller (FSFC) $\mathbf{K} = \left[k_i, k_p, k_d\right]$ by considering the augmented state vector

$$\left[\int y(t), y(t), \dot{y}(t)\right]^{\mathrm{T}} \tag{18}$$

It can be shown that the controller gains obtained by this method follows from a unique solution $\mathbf{P} = \mathbf{P}^{\mathrm{T}} > \mathbf{0}$ of an Algebraic Riccati Equation (ARE) which is formulated with the constraints on λ_0, ζ_0^{cl}, and ω_0^{cl}.

The PID controller $G_c(s)$ (17) is then designed for $G_0(s)$ to yield a nominal-closed loop system of the form

$$G_{cl}(s)=\frac{L_o\left(k_p+\frac{k_i}{s}+k_d s\right)}{\left(s+\lambda_0\right)\left(s^2+2\zeta_0^{cl}\omega_0^{cl}s+\left(\omega_0^{cl}\right)^2\right)}:\quad \lambda_0\in\mathbb{R}^+ \tag{19}$$

with specified ζ_0^{cl}, ω_0^{cl} and λ_0. It is clear from (19) that $G_{cl}(s)$ shall have three roots, of which the root at $s=-\lambda_0$ is always real and other two roots are complex conjugate. The values λ_0 are usually user specified and a large value of λ_0 over $\zeta_0^{cl},\omega_0^{cl}$ ensures that the time response of $G_{cl}(s)$ is governed by the complex roots of $G_{cl}(s)$. The resultant PID controller is thus an optimal controller which guarantees a specified closed-loop response for the nominal plant. However, it is clear from above that the controller $G_c(s)$ designed for $G_0(s)$ does not guarantee the desired closed-loop response for $G(s,\mathbf{v})$ with bounded parametric uncertainty. Thus, in this section a LMI based controller design criterion is proposed, which ensures that if $G_c(s)$ is used to control the interval plant $G(s,\mathbf{v})$ (3), the controller will still remain a robust one for $G(s,\mathbf{v})$ and produce a closed-loop interval system of the form

$$\left.\begin{aligned}&G_{cl}(s,\mathbf{v}')=\frac{L\left(k_p+\frac{k_i}{s}+k_d s\right)}{\left(s+\lambda\zeta^{cl}\omega^{cl}\right)\left(s^2+2\zeta^{cl}\omega^{cl}s+\left(\omega^{cl}\right)^2\right)};\\&\lambda\in\left[\underline{\lambda},\ \bar{\lambda}\right];\ \zeta^{cl}\in\left[\underline{\zeta}^{cl},\ \bar{\zeta}^{cl}\right];\ \omega^{cl}\in\left[\underline{\omega}^{cl},\ \bar{\omega}^{cl}\right];\end{aligned}\right\} \tag{20}$$

where the parameter vector $\mathbf{v}'=\left[\lambda,\zeta^{cl},\omega^{cl}\right]$ (21)

It is, therefore, ensured that the controller $G_c(s)$ designed for a nominal plant $G_0(s)$ produces a closed loop time response with bounded parametric uncertainties $\mathbf{v}'$ in the presence of bounded parametric uncertainties $\mathbf{v}$ present in the nominal plant.

As reported in He et al. [20] with the augmented state vector (18), the optimal PID controller can be represented as

$$\mathbf{K}=\mathbf{R}_0^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P}=\left[k_i\quad k_p\quad k_d\right] \tag{22}$$

where $\mathbf{P}=\mathbf{P}^{\mathrm{T}}>0$ and $\mathbf{R}_0=\mathbf{R}_0^{\mathrm{T}}>0$ which can be obtained from ARE

$$\mathbf{A}_0^{\mathrm{T}}\mathbf{P}+\mathbf{P}\mathbf{A}_0-\mathbf{P}\mathbf{B}_0\mathbf{R}_0^{-1}\mathbf{B}_0^{\mathrm{T}}\mathbf{P}+\mathbf{Q}_0=\mathbf{0} \tag{23}$$

with $\mathbf{Q}_0=\mathbf{Q}_0^{\mathrm{T}}\geq 0$ that minimizes the cost function

$$J=\frac{1}{2}\int_0^\infty\left[\mathbf{x}^{\mathrm{T}}(t)\mathbf{Q}_0\mathbf{x}(t)+\mathbf{u}^{\mathrm{T}}(t)\mathbf{R}_0\mathbf{u}(t)\right]\mathrm{d}t \tag{24}$$

Using the augmented state vector (18), the nominal plant (16) may be represented in the form of state space model as

$$\left.\begin{aligned}\dot{\mathbf{x}}(t)&=\mathbf{A}_0\mathbf{x}(t)+\mathbf{B}_0\mathbf{u}(t)\\y(t)&=\mathbf{C}\mathbf{x}(t)\end{aligned}\right\} \tag{25}$$

where

$$\mathbf{A}_0=\begin{bmatrix}0&1&0\\0&0&1\\0&-b_0&-a_0\end{bmatrix};\mathbf{B}_0=\begin{bmatrix}0\\0\\L_0\end{bmatrix}\text{ and }\mathbf{C}=\begin{bmatrix}0&1&0\end{bmatrix} \tag{26}$$

Considering (19) and the closed loop system poles with optimal PID controller $\left(-\lambda_0,-\xi_0^{cl}\omega_0^{cl}\pm\omega_0^{cl}\sqrt{1-\left(\xi_0^{cl}\right)^2}\right)$ the weighting matrix $\mathbf{Q}$ can be designed from ARE (23) with $\mathbf{R}_0=[r]$ and

$$\mathbf{Q}_0=\begin{bmatrix}q_0^{11}&0&0\\0&q_0^{22}&0\\0&0&q_0^{33}\end{bmatrix} \tag{27}$$

where

$$\left.\begin{aligned}q_o^{11}&=\frac{r}{L_0^{\ 2}}\left[\lambda_0^2\left(\omega_0^{ci}\right)^4\right]\\q_o^{22}&=\frac{r}{L_0^{\ 2}}\left[\left(\omega_0^{ci}\right)^4+\left\{4\left(\xi_0^{cl}\right)^2-2\right\}\lambda_0^2\left(\omega_0^{ci}\right)^2-b_0^{\ 2}\right]\\q_o^{33}&=\frac{r}{L_0^{\ 2}}\left[\left\{\lambda_0^2+\left(4\left(\xi_0^{cl}\right)^2-2\right)\left(\omega_0^{ci}\right)^2\right\}-a_0^{\ 2}+2b_0\right]\end{aligned}\right\} \tag{28}$$

The corresponding state space model of (3) for a system with interval has been represented as

$$\dot{\mathbf{x}}(t)=\mathbf{A}\mathbf{x}(t)+\mathbf{B}\mathbf{u}(t);\quad \mathbf{A}\in\left[\underline{\mathbf{A}},\bar{\mathbf{A}}\right]\text{ and }\mathbf{B}\in\left[\underline{\mathbf{B}},\ \bar{\mathbf{B}}\right] \tag{29}$$

where

$$\mathbf{A}=\begin{bmatrix}0&1&0\\0&0&1\\0&-b&-a\end{bmatrix};\mathbf{B}=\begin{bmatrix}0\\0\\L\end{bmatrix};a\in\left[\underline{a},\ \bar{a}\right],b\in\left[\underline{b},\ \bar{b}\right],L\in\left[\underline{L},\bar{L}\right] \tag{30}$$

The ARE for the interval plant (3) may be re-written as

$$\mathbf{A}^{\mathrm{T}}\mathbf{P}+\mathbf{P}\mathbf{A}-\mathbf{P}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P}+\mathbf{Q}=0 \tag{31}$$

$$\text{with }\ \mathbf{Q}\in\left[\underline{\mathbf{Q}},\ \bar{\mathbf{Q}}\right]\text{and }\mathbf{R}=[\gamma r];\quad \gamma\in\left[\underline{\gamma},\bar{\gamma}\right] \tag{32}$$

By (30) and (31) the interval weighting matrix $\mathbf{Q}$ can be represented as

$$\left.\begin{aligned}&\mathbf{Q}=\begin{bmatrix}q^{11}&0&0\\0&q^{22}&0\\0&0&q^{33}\end{bmatrix};\\&q^{11}\in\left[\underline{q}^{11},\ \bar{q}^{11}\right],q^{22}\in\left[\underline{q}^{22},\ \bar{q}^{22}\right],q^{33}\in\left[\underline{q}^{33},\ \bar{q}^{33}\right]\end{aligned}\right\} \tag{33}$$

where

$$\left.\begin{aligned}q^{11}&=\frac{\gamma r}{L^2}\left[\lambda^2\left(\omega^{cl}\right)^4\right]\\q^{22}&=\frac{\gamma r}{L^2}\left[\left(\omega_0^{ci}\right)^4+\left\{4\left(\xi^{cl}\right)^2-2\right\}\lambda^2\left(\omega^{cl}\right)^2-b^2\right]\\q^{33}&=\frac{\gamma r}{L^2}\left[\left\{\lambda^2+\left(4\left(\xi^{cl}\right)^2-2\right)\left(\omega^{cl}\right)^2\right\}-a^2+2b\right]\end{aligned}\right\}$$

(34)

It can be seen from (33) and (34) that the weighting matrices $\mathbf{Q}_0$ and $\mathbf{Q}$ are the function of a_0, b_0, L_0, λ_0, ζ_0^{cl},

ω_0^{cl} and a , b , L , λ , ζ^{cl} , ω^{cl} respectively with the variation of input weighting matrix $\mathbf{R}$.

To design an identical PID controller which remains optimal for both the system (nominal as well as interval), the following lemma must hold:

Lemma 1

If a continuous time ARE have a common unique solution $\mathbf{P}=\mathbf{P}^{\mathrm{T}}>0$ for both the nominal system (16) as well as interval system (3) with $\mathbf{Q}_0$ and $\mathbf{Q}$ to obtain identical optimal PID controller gain (22), then for the interval system, the input weight matrix $\mathbf{R}$ must be directly proportional to the variation of the input matrix $\mathbf{B}$ within the prescribed interval.

Proof:

To obtain an identical optimal PID controller gain with common $\mathbf{P}$ for the nominal system (16) as well as interval system (3),the following condition must be satisfied

$$\begin{aligned}&\mathbf{K}=\mathbf{R}_0^{-1}\mathbf{B}_0^{\mathrm{T}}\mathbf{P}=\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P}\\ &\Rightarrow \mathbf{R}_0^{-1}\mathbf{B}_0^{\mathrm{T}}=\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\\ &\Rightarrow r^{-1}L=(\gamma r)^{-1}(\gamma L)\end{aligned} \quad (35)$$

Hence, the Lemma 1 holds true.

It has been established in (35) that to obtain a common optimal PID controller gain there must be a unique common solution of $\mathbf{P}$ for the nominal system (16) as well as the interval system (3). Thus, to obtain a common $\mathbf{P}$ an LMI technique has been adopted in terms of LMI based LQR with $\mathbf{x}(0)$ being the initial state, the performance index can be characterized as $J_{\min}=\mathbf{x}^{\mathrm{T}}(0)\mathbf{P}\mathbf{x}(0)$.

Using the LMI method, the ARE (23) can be written with the constraints $\hat{\mathbf{P}}$, $\mathbf{M}$ as reported in [27] by

$$\min_{\hat{\mathbf{P}},\mathbf{M}} \mathbf{x}^{\mathrm{T}}(0)\hat{\mathbf{P}}^{-1}\mathbf{x}(0) \quad (36)$$

where the objective (36) has an upper bound δ , represented in an LMI form as

$$\begin{bmatrix}\delta & \mathbf{x}^{\mathrm{T}}(0)\\ \mathbf{x}(0) & \hat{\mathbf{P}}^{-1}\end{bmatrix}\geq 0 \quad (37)$$

subject to

$$\begin{bmatrix}\mathbf{A}_0\hat{\mathbf{P}}+\hat{\mathbf{P}}\mathbf{A}_0^{\mathrm{T}}+\mathbf{B}_0\mathbf{M}+\mathbf{M}^{\mathrm{T}}\mathbf{B}_0^{\mathrm{T}} & \hat{\mathbf{P}} & \mathbf{M}^{\mathrm{T}}\\ \hat{\mathbf{P}} & -\mathbf{Q}_0^{-1} & \mathbf{0}\\ \mathbf{M} & \mathbf{0} & -\mathbf{R}_0^{-1}\end{bmatrix}\leq 0;\quad \hat{\mathbf{P}}>0 \quad (38)$$

for the nominal system and,

$$\begin{bmatrix}\mathbf{A}\hat{\mathbf{P}}+\hat{\mathbf{P}}\mathbf{A}^{\mathrm{T}}+\mathbf{B}\mathbf{M}+\mathbf{M}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}} & \hat{\mathbf{P}} & \mathbf{M}^{\mathrm{T}}\\ \hat{\mathbf{P}} & -\mathbf{Q}^{-1} & \mathbf{0}\\ \mathbf{M} & \mathbf{0} & -\mathbf{R}^{-1}\end{bmatrix}\leq 0;\quad \hat{\mathbf{P}}>0 \quad (39)$$

for the interval system, where $\mathbf{M}=\mathbf{K}\hat{\mathbf{P}}$ and $\hat{\mathbf{P}}=\mathbf{P}^{-1}$.

It can be seen from [27] that the design of a controller using the LMI technique is much more efficient as it incorporates several control parameters required for a controller design and can be used to solve convex optimization problems [27].

The matrix $\mathbf{Q}$ is a positive semi-definite and it can be tested over the entire convex box of $\left[L,a,b,\lambda,\zeta^{cl},\omega^{cl}\right]$ using the results presented in [28]. It has been established in [28] that for a symmetric interval matrix such as $\mathbf{Q}$, that if

$$\mathbf{Q_c}=\frac{1}{2}\left(\underline{\mathbf{Q}}+\bar{\mathbf{Q}}\right) \quad (40)$$

and

$$\mathbf{Q}^{\Delta}=\frac{1}{2}\left(\bar{\mathbf{Q}}-\underline{\mathbf{Q}}\right) \quad (41)$$

Then $\mathbf{Q}$ is positive semi-definite for the entire interval $\left[\underline{\mathbf{Q}},\bar{\mathbf{Q}}\right]$ if,

$$\lambda_{\min}(\mathbf{Q}_c)\geq\rho(\mathbf{Q}^{\Delta}) \quad (42)$$

IV. Simulation and Results

In this section, the effectiveness of proposed robust PID controller has been demonstrated. Fig.1 shows the schematic diagram of the control scheme of PWR. An interval transfer function for PWR representing the 60%FP-100%FP with 10% bounded parametric uncertainty has been considered on thermal hydraulic data presented in [24]. The interval identified transfer function and the value of corresponding PID controller obtained by proposed methodology in section III and the methodology proposed in [20] have been depicted in Table I. The values of λ_0 , ζ_0^{cl} , and ω_0^{cl} have been chosen as 3, 0.8, and 0.02 respectively to obtain the gain of the PID controller. The nominal system of the corresponding interval transfer function has been computed by interval algorithm presented in [14]. In this paper all interval arithmetic have been calculated by INTLAB [29] toolbox and all LMIs are design by YALMIP [30] toolbox.

The transfer function presented in Table 1 is controlled by both the controller proposed in section III and the PID controller as depicted in He et al. in [20]. Figs. 2 and 3 show the closed-loop response of the nominal as well as interval system by the PID controller using the proposed method and followed by the method used by He et al. [20]. It can be depicted clearly from the above figures that the PID controller gain obtained from the method by He et al. [20] can control the interval system but the time domain performance parameters like peak overshoot and settling time are larger than that obtained from the proposed PID controller.

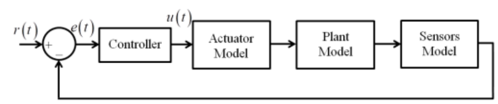

Figure 1. Schematic diagram of control for PWR.

Next, the PID controller proposed in this paper has been used in the actual nonlinear PWR plant as shown in Fig. 1. During a power maneuvering operation, first it is considered that the reactor is running at 100%FP. After 20 s demand power is changed from 100%FP to 60%FP at a ramp rate of 5% FP/s. At 300 s the demand power is changed from 60%FP to 100%FP with a rate of 5% FP/s. This power maneuvering has been depicted in Fig. 4. From Fig. 4, it can be clearly seen that the robust PID controller proposed in this paper is capable enough to handle the fast variation from 100%FP to 60%FP and vice versa. In Fig. 4, it is also depicted that both the proposed controller and the controller narrated by He et al. are capable of handling this scenario but the proposed one gives better results in terms of rise time, settling time, and maximum peak overshoot.

TABLE I. TRANSFER FUNCTION AND PID CONTROLLER GAIN FOR PWR USING BOTH THE METHODS

Transfer Function	***PID Controller Gain from the method by He. et. al. [20]***			***PID Controller Gain by proposed method***		
	k_i	k_p	k_d	k_i	k_p	k_d
$\frac{[0.0072, 0.0113]}{s^2+[0.2244, 0.3502]s+[0.0080,0.0125]}$	0.16	8.2	359.8	0.2166	18.2476	514.2625

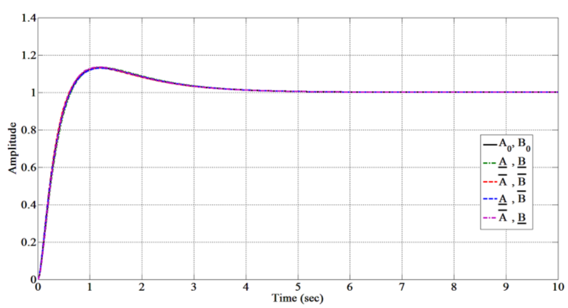

Figure 2. Interval system with propose optimal PID controller

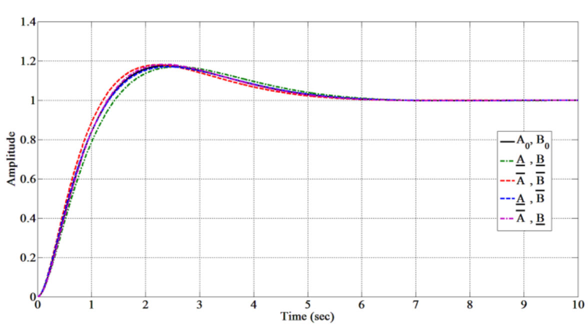

Figure 3. Interval system with optimal PID controller by He.et.al [20].

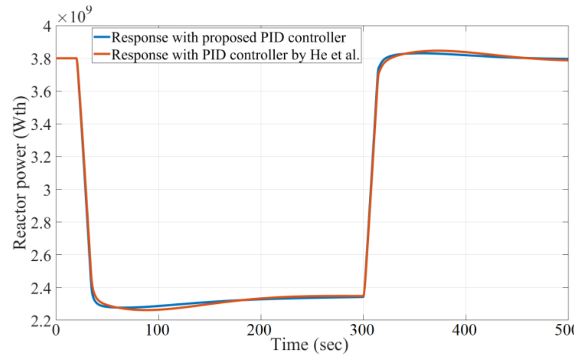

Figure 4. Power maneuvering using proposed PID controller.

V. CONCLUSION

In this paper a novel robust PID controller design technique for uncertain interval system has been proposed. The proposed controller is able to control the uncertain as well as nominal system. The effectiveness of the proposed controller design for actual nonlinear PWR plant has been put forward in the results. However, in this method, there are limitations as well. The variation of parametric uncertainties should be polytopic and the system should be identified as a second order system. Although identified system has been computed without time-delay, further work would entail the same for holistic representation of the work.

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