

New clues on the calcium underabundance in early-type galaxies

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ABSTRACT

We use our new stellar population models, which include effects from variable element abundance ratios, to model the Ca4227 absorption-line indices of early-type galaxies, and to derive calcium element abundances. We find that calcium, although being an α -element, is depressed with respect to the other α -elements by up to a factor of 2. This confirms quantitatively earlier speculations that early-type galaxies are calcium-underabundant. We find a clear correlation between the α /Ca ratio and central velocity dispersion, which implies that more massive galaxies are more calcium-underabundant. Interestingly, this correlation extends down to the dwarf spheroidal galaxies of the Local Group for which α /Ca ratios have been measured from high-resolution spectroscopy of individual stars. The increase of calcium underabundance with galaxy mass balances the higher total metallicities of more massive galaxies, so that calcium abundance in early-type galaxies is fairly constant and, in particular, does not increase with increasing galaxy mass. This result may be the key to understanding why the Ca II triplet absorption of early-type galaxies at 8600 Å is constant to within 5 per cent over a large range of velocity dispersions. The origin of the calcium underabundance in early-type galaxies remains to be understood. We argue that formation time-scales are disfavoured to produce calcium underabundance, and that the option of metallicity-dependent supernova yields may be the most promising track to follow.

Key words: galaxies: abundances – galaxies: elliptical and lenticular, cD – galaxies: evolution – galaxies: formation – galaxies: stellar content.

1 INTRODUCTION

In the late seventies, it was first noticed that early-type galaxies show relatively weak Ca II triplet (CaT) absorption at 8600 Å (Cohen 1979; Faber & French 1980). The metallicities derived from these lines lie well below solar, which seems hard to reconcile with the super-solar metallicities obtained from the classical Lick indices Mg₂ and (Fe) (e.g. Trager et al. 2000a; Kuntschner et al. 2001; Thomas, Maraston & Bender 2002; Maraston et al. 2003). Moreover, the very tight correlation between Mg₂ and the velocity dispersion σ (e.g. Bender, Burstein & Faber 1993) has not been found for the CaT line (Cohen 1979; Faber & French 1980). Subsequent measurements of CaT absorption in early-type galaxies, providing increasingly better data quality, confirm these early results (Terlevich, Diaz & Terlevich 1990; Peletier et al. 1999). Very recently, Saglia et al. (2002) measured the CaT absorption in 94 early-type galaxies at very high signal-to-noise ratios, using an improved index definition (Cenarro et al. 2001). They again find a relatively weak CaT absorption, and even a mild anti-correlation between CaT and σ with a remarkably small scatter (see also Cenarro et al. 2003).

Saglia et al. (2002) show that neither uncertainties in the CaT modelling nor an already previously suggested steepening of the initial mass function provide compelling explanations. A very attractive way out is the conclusion that early-type galaxies are indeed underabundant in the element calcium. The main problem is that this interpretation remains speculative as long as the sensitivity of the CaT index to calcium abundance is not known and, in particular, calcium-underabundant stellar population models of CaT are not available.

Interestingly, the blue Lick index Ca4227 at $\lambda \approx 4227$ Å is also very weak in early-type galaxies, which again indicates a possible calcium underabundance (Worthey 1992; Vazdekis et al. 1997; Trager et al. 1998). A similar conclusion has been drawn on the basis of Ca4455 index measurements (Worthey 1998; Henry & Worthey 1999). We note, however, that Ca4455 – despite its name – appears to be only sensitive to the abundances of iron-peak elements (Tripicco & Bell 1995), so that the weak Ca4455 absorption reflects the well-known iron underabundance in early-type galaxies (e.g. Worthey, Faber & González 1992). Instead, Ca4227 is sensitive to calcium abundance, and – different from CaT – we now have a set of stellar population models at hand for Ca4227 (Thomas, Maraston & Bender 2003, hereafter TMB) that explicitly take

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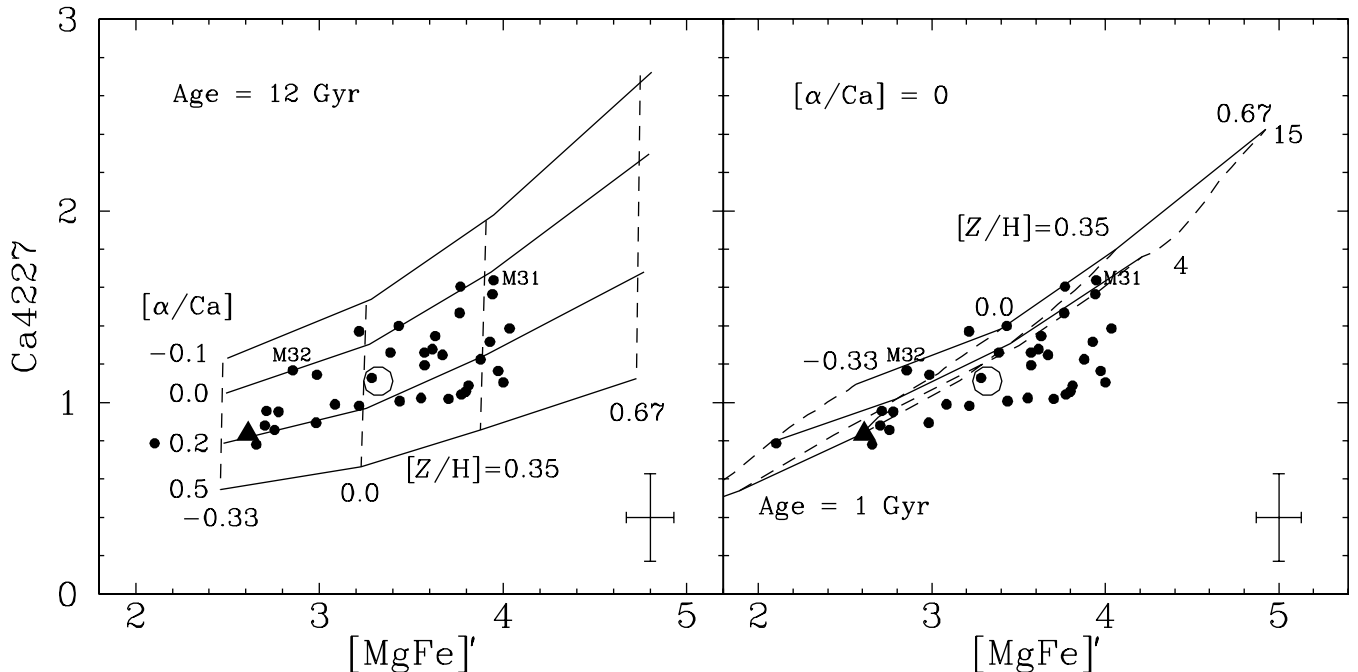


Figure 1. Early-type galaxies in the $[MgFe]'$ -Ca4227 plane. Galaxy data (filled circles) are from Trager et al. (1998). The open circle is their median. The filled triangle is the integrated Galactic bulge light from Puzia et al. (2002). Lines are stellar population models from Thomas et al. (2003). Left-hand panel: models for constant α/Ca ratios $[\alpha/\text{Ca}] = -0.1, 0.0, 0.2$ and 0.5 (solid lines) and constant metallicities $[Z/H] = -0.33, 0.0, 0.35$ and 0.67 (dashed lines) at a fixed age of $t = 12$ Gyr. Right-hand panel: models for constant ages of 1, 4, and 15 Gyr (solid lines), and the constant metallicities $[Z/H] = -0.33, 0.0, 0.35$ and 0.67 (dashed lines) at a fixed solar α/Ca element ratio ($[\alpha/\text{Ca}] = 0$).

abundance effects of individual elements into account and, in particular, are computed for different calcium abundances. Here, we use these models to analyse the data of Trager et al. (1998) and – for the first time – to quantitatively derive calcium element abundances of early-type galaxies.

In Section 2, we briefly introduce the stellar population model. The results are presented in Section 3 and discussed in Section 4.

2 MODELS

In TMB, we present stellar population models for the first time with different chemical mixtures and element abundance ratios. We computed all optical Lick indices from CN_1 to TiO_2 in the wavelength range $4000 \lesssim \lambda \lesssim 6500 \text{ \AA}$ of simple stellar population (SSP) models, varying the abundance ratios of α -elements (i.e. O, Mg, Ca, Na, Si, Ti) to iron peak elements (Fe, Cr) and to the individual elements carbon, nitrogen and calcium. The impact from the element abundance changes on the Lick absorption-line indices is taken from Tripicco & Bell (1995). The models comprise ages between 1 and 15 Gyr and metallicities between 1/200 and 5 solar. We refer to TMB for more details.

Particular care was taken to calibrate the SSP models on globular cluster data, which, most importantly, include objects with relatively high – namely solar – metallicities (Puzia et al. 2002). We match their Ca4227 indices very well with models in which calcium is enhanced like the other α -elements with respect to the iron-peak elements, in agreement with results from high-resolution spectroscopy of individual stars (see TMB). Note that calcium itself is an α -element. Having the issue of weak calcium absorption in early-type galaxies in mind, we developed additional models in which calcium is detached from the group of α -elements, allowing for variations of the α/Ca element ratio. In TMB, models of the index Ca4227 for the

element ratios $[\alpha/\text{Ca}] = -0.1, 0.0, 0.2$ and 0.5 are provided, with $[\alpha/\text{Ca}] \equiv \log(X_\alpha/X_{\text{Ca}}) - \log(X_\alpha/X_{\text{Ca}})_\odot$.

We will use these models to derive α/Ca ratios of early-type galaxies. Note that the above notation relates the element ratio to the solar value. We will speak of calcium underabundance if α/Ca is larger than solar, hence $[\alpha/s\text{Ca}] > 0$.

3 RESULTS

We analyse the early-type galaxy data of Trager et al. (1998). This turned out to be the only data set in the literature with Ca4227 indices for ‘normal’ early-type galaxies. We do not use the sample of Longhetti et al. (2000), because it consists of peculiar, disturbed and shell galaxies. Still, it should be kept in mind that the Trager et al. (1998) sample contains a rather inhomogeneous morphological mix, with many S0 and later-type galaxies included. From this sample, we selected those objects, for which the error in the Ca4227 measurement is smaller than 0.25 \AA , which yields 39 galaxies with a median error of 0.2 \AA .

3.1 $[MgFe]'$ versus Ca4227

In Fig. 1, we show Ca4227 as a function of the index $[MgFe]'$, with $[MgFe]' \equiv \sqrt{\text{Mgb}(0.72\text{Fe}5270 + 0.28\text{Fe}5335)}$ as defined by TMB. $[MgFe]'$ is independent of α/Fe ratio variations, and therefore a good tracer of total metallicity. SSP models for the α/Ca ratios $[\alpha/\text{Ca}] = -0.1, 0.0, 0.2$ and 0.5 and the metallicities $[Z/H] = -0.33, 0.0, 0.35$ and 0.67 at the fixed age $t = 12$ Gyr are plotted in the left-hand panel (see labels in the figure). Models of constant metallicity and varying α/Ca ratio (dashed lines) are almost perfectly vertical, indicating that $[MgFe]'$ is not sensitive to calcium abundance. Solid lines in the left-hand panel of Fig. 1 are models with constant α/Ca

ratios. Note that the models in the $[\text{MgFe}']$ –Ca4227 plane are highly degenerate in age, as both indices have similar age dependences. This is shown with the right-hand panel of Fig. 1, in which models with ages 1, 4, and 15 Gyr and metallicities $[Z/H] = -0.33, 0.0, 0.35$ and 0.67 are plotted (see labels in the figure) – the α/Ca ratio being fixed at the solar value. The left-hand panel of Fig. 1 therefore is well suited for isolating α/Ca ratio effects.

The galaxy data are shown as filled circles; the open circle is their median. The filled triangle is the integrated light of the Galactic bulge (Puzia et al. 2002). The data lie below the model with solar α/Ca ratio; no combination of age and metallicity can be found to match the data. α/Ca ratios from solar to roughly 3 solar ($0 \lesssim [\alpha/\text{Ca}] \lesssim 0.5$) are obtained. The maximal calcium underabundance relative to the other α -elements is therefore about a factor of 2–3. We are confident that this detection of calcium underabundance is significant in spite of the relatively large observational errors for two reasons. (1) The Ca4227 range covered by the data plotted here is consistent with the data of Longhetti et al. (2000), which have – although being disturbed and shell galaxies – smaller observational errors ($< 0.09 \text{ \AA}$). (2) The median of the sample (open circle) is at $\text{Ca4227} = 1.1 \pm 0.03 \text{ \AA}$, significantly below the model with solar α/Ca . From the location of the median, we infer that calcium is depleted in early-type galaxies by typically a factor of 1.4.

The large triangle in Fig. 1 is the average of 15 Galactic bulge fields measured by Puzia et al. (2002). Obviously, the Galactic bulge is also calcium-underabundant. Its α/Ca ratio is consistent with the median of the early-type galaxy sample, at somewhat lower total metallicity. This result is in good agreement with the detailed abundance analysis of individual field bulge stars from high-resolution spectra by McWilliam & Rich (1994). These authors measure super-solar α/Ca ratios ($[\alpha/\text{Ca}] \sim 0.2$) – and hence calcium underabundance – for the metal-rich stars ($[\text{Fe}/\text{H}] > -0.5$) of their sample.

The galaxies M31 and M32 are labelled in Fig. 1. M31 has stronger Ca4227 and $[\text{MgFe}']$ indices than M32, so that both objects have very similar α/Ca ratios ($[\alpha/\text{Ca}] \sim 0$). Curiously, M32 shows stronger CaT absorption than M31 (Kormendy & Bender 1999), which seems to indicate higher calcium abundance in M32. Note however, that the CaT index is severely contaminated by Paschen absorption lines (Cenarro et al. 2001). As M32 is younger than M31 (Trager et al. 2000b), the strong CaT absorption index of M32 is most likely caused by an increase of Paschen line absorption.

3.2 Correlations with galaxy velocity dispersion

In Fig. 2, the index Ca4227 (top panel) and the α/Ca ratios derived from Fig. 1 (bottom panel) are plotted as functions of central velocity dispersion (circles). More precisely, using the TMB models, we determine ages, metallicities and α/Ca ratios from the indices $H\beta$, $[\text{MgFe}']$ and Ca4227. It should be emphasized that uncertainties in the stellar population ages have only a minor effect on the determination of the α/Ca ratio as demonstrated by Fig. 1. Velocity dispersions are taken from Whitmore, McElroy & Tonry (1985), Faber et al. (1989), Bender, Burstein & Faber (1992), and González (1993). The triangles in Fig. 2 are the Local Group dwarf spheroidal galaxies Carina, Draco, Fornax, Leo I, Sextans and Ursa Minor. Their mean α/Ca ratios are the average of the element abundances of individual stars (~ 5 per object) determined from high-resolution spectroscopy (Shetrone, Côté & Sargent 2001; Shetrone et al. 2003). These authors derive α -element abundances by averaging the abundances of the elements Mg, Ca and Ti. Central velocity dispersions are taken from the compilation of Mateo (1998).

Ca4227 correlates with velocity dispersion within the measurement errors. The increase of Ca4227 with increasing σ_0 , however, is balanced by the simultaneous increase of $[\text{MgFe}']$, so that α/Ca also increases with velocity dispersion. A linear least-squares fit (solid line) to the giant galaxy data yields

$$[\alpha/\text{Ca}] = -0.03 + 0.0008 * \sigma_0. \quad (1)$$

In other words, more massive galaxies are more metal-rich and more calcium-underabundant. The admittedly large scatter of the relation between α/Ca and σ_0 is fully consistent with the errors in $[\alpha/\text{Ca}]$ (~ 0.1 dex), which are caused mainly by the large observational errors of Ca4227. Ca4227 is a rather weak absorption line and requires high signal-to-noise ratios to be measured accurately. Given these large errors, it seems even surprising that such a clear correlation could be found. Most interestingly, it is very consistent with the α/Ca ratios of the Local Group dwarf spheroidals. We note that the large spread in Fig. 2 may also come from the inhomogeneous morphological mix in the Trager et al. (1998) sample. Ca4227 index measurements for a similarly large sample of elliptical galaxies at significantly higher signal-to-noise ratios than the Trager et al. (1998) data would be very useful to check the present result.

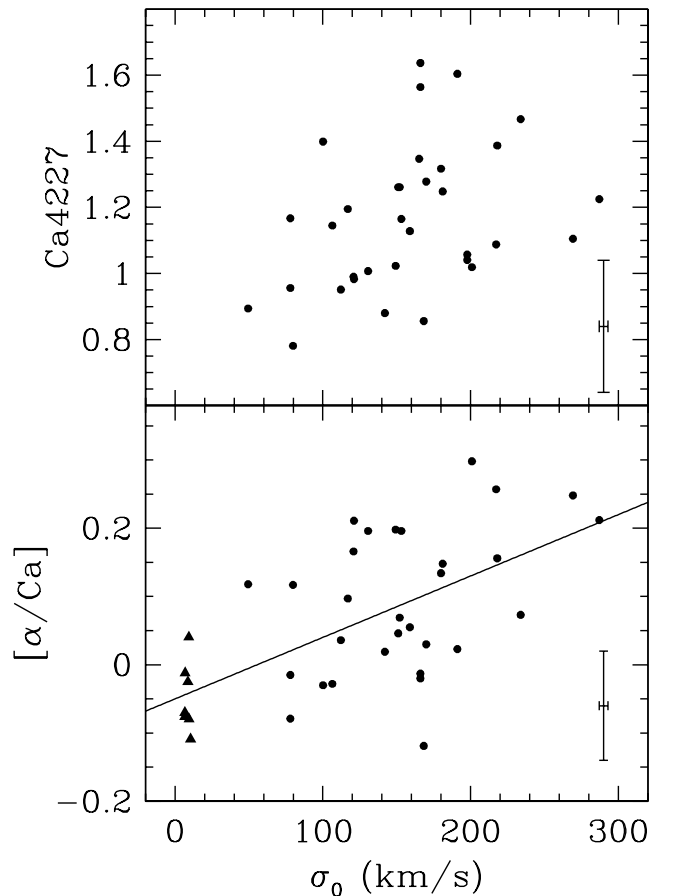


Figure 2. Ca4227 index (top panel) and α/Ca ratio (bottom panel) of early-type galaxies as functions of central velocity dispersion. Ca4227 data (circles) are from Trager et al. (1998). α/Ca ratios are derived from Fig. 1. Triangles are Local Group dwarf spheroidal galaxies for which α/Ca ratios are measured in individual stars (Shetrone et al. 2001, 2003). Central velocity dispersions are from Whitmore et al. (1985), Faber et al. (1989), Bender et al. (1992), González (1993) and Mateo (1998). The solid line is a linear least-squares fit to the giant galaxy data (circles).

The increase of $[\alpha/\text{Ca}]$ by ~ 0.2 dex between $\sigma_0 = 100$ and 300 km s^{-1} is comparable to the typical metallicity increase (Trager et al. (2000b); Kuntschner et al. (2001); D. Thomas et al., in preparation) in that mass range. Hence calcium abundance stays approximately constant or may even decrease (the present data quality does not allow us to discriminate these possibilities). The Ca4227 index increases, mainly because of its sensitivity not only to calcium abundance but also to total metallicity (Tripicco & Bell 1995). The Paschen line corrected Ca II triplet index (CaT*) at 8600 \AA (Cenarro et al. 2001), instead, is found to be constant to within 5 per cent over the same range of velocity dispersions (Saglia et al. 2002; Cenarro et al. 2003). This result can be understood now, provided that the index CaT* is more sensitive to calcium abundance than Ca4227.

4 DISCUSSION AND CONCLUSIONS

In the previous section, we showed that early-type galaxies are calcium-underabundant. The reason for this chemical peculiarity is not yet understood. In this section, we will briefly discuss possible mechanisms that can lead to the formation of calcium-underabundant stellar populations.

4.1 Formation time-scales and the initial mass function

More massive early-type galaxies have higher α/Fe ratios and higher average ages (e.g. Trager et al. 2000b; Thomas et al. 2002; Terlevich & Forbes 2002; Caldwell, Rose & Concannon 2003), which indicate faster formation histories of their stellar populations (Matteucci 1994; Thomas, Greggio & Bender 1999). Note that high α/Fe ratios are actually caused by an underabundance of the element iron, rather than by an overabundance of the α -elements (Buzzoni et al. (1994); Trager et al. 2000a; TMB). They are a good measure for star formation time-scales, because a substantial fraction of iron comes from the delayed enrichment by Type Ia supernovae (Greggio & Renzini 1983; Matteucci & Greggio 1986; Thomas, Greggio & Bender 1998). Note that the old, metal-poor stars in the halo of the Milky Way, which formed at an early stage of the evolution of the Galaxy, are also underabundant in iron (McWilliam 1997, and references therein). It may, therefore, seem straightforward to also link the underabundance of calcium with formation time-scales.

The situation for the α/Ca ratio, however, is entirely different. The old, metal-poor halo stars are not underabundant in calcium (McWilliam 1997), hence the solar α/Ca ratio is already at place in the very beginning of the chemical evolution of the Milky Way. In other words, calcium, which is actually an α -element, is enriched in lockstep with the other α -elements. Moreover, Type Ia supernovae produce mainly iron and do not contribute significantly to the enrichment of calcium (Nomoto, Thielemann & Yokoi 1984). The calcium underabundance found in early-type galaxies can therefore not be explained through the link of formation time-scales with delayed Type Ia supernova enrichment.

Within Type II supernova nucleosynthesis, calcium comes mainly from supernovae of the less massive progenitors with $m \lesssim 20 M_{\odot}$ (Woosley & Weaver 1995; Thielemann, Nomoto & Hashimoto 1996). If the chemical enrichment is dominated by stars more massive than $\sim 20 M_{\odot}$, calcium underabundance can be achieved (Mollá & García-Vargas 2000). Because of the short lifetimes of these stars, however, extremely short formation time-scales ($\sim 10^7 \text{ yr}$) would be required, implying unreasonably high star formation rates of up to $10^5 M_{\odot}$ per year.

Alternatively, a flattening of the initial mass function (IMF) enhances the contribution from high-mass stars to the chemical enrichment. Integrating the supernova yields of Woosley & Weaver (1995) with variable IMF slopes, we find that an abundance ratio of $[\alpha/\text{Ca}] \sim 0.2$ requires extremely shallow IMF slopes of the order of $x \sim 0.2$ (Salpeter being $x = 1.35$). A flattening of the IMF to produce calcium underabundance can therefore be ruled out, as such shallow IMFs violate other observational constraints: in particular, they predict stellar mass-to-light ratios (Maraston 1998) well above the dynamical determinations (Gerhard et al. 2001).

4.2 Metallicity-dependent yields

A more appealing solution may be connected to a metallicity dependence of supernova yields, in the sense that in metal-rich supernovae, the production (or at least the ejection) of calcium is depressed (Worthey 1998).

From the theoretical side of Type II supernova modelling, there are no clear indications for the existence of metallicity-dependent yields. The calculations of Woosley & Weaver (1995, model B) predict only a very mild increase of the α/Ca ratio with increasing metallicity in the supernova ejecta for the metallicity range $1/10$ to solar. Extrapolating these yields to double-solar metallicity leads to $[\alpha/\text{Ca}]$ ratios that are ~ 0.2 dex below the values derived here for massive early-type galaxies. If metallicity effects are at work, they must set in at super-solar metallicities, a range which has not yet been explored in Type II supernova nucleosynthesis calculations.

This option gets support from the fact that only the stars with metallicities above solar in the McWilliam & Rich (1994) data are calcium-underabundant. It would also be very consistent with the $\alpha/\text{Ca}-\sigma$ correlation of early-type galaxies found in this paper. The most significant increase of the α/Ca ratio (by ~ 0.2 dex) with galaxy mass occurs among giant galaxies in a regime of super-solar metallicity, and goes along with a relatively mild increase of metallicity by ~ 0.3 dex. At sub-solar metallicities, instead, in the transition from giant to dwarf galaxies, the α/Ca ratio decreases by only 0.1 dex, in spite of the huge drop in metallicity by more than 1.5 dex. To conclude, the possibility of metallicity-dependent yields at super-solar metallicities remains a promising track to follow in order to find the origin of the calcium underabundance in early-type galaxies.

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