JMSE **2017**, 2(4), 252–289
doi:10.3724/SP.J.1383.204012
http://www.jmse.org.cn/
http://engine.scichina.com/publisher/CSPM/journal/JMSE

Article

Option Pricing under the Double Exponential Jump-Diffusion Model with Stochastic Volatility and Interest Rate

Rongda Chen [1,2], Zexi Li [1,3], Liyuan Zeng [4], Lean Yu [5,*], Qi Lin [1] and Jia Liu [6]

1 School of Finance, Zhejiang University of Finance and Economics, Hangzhou 310018, China; rongdachen@163.com; qlin_sf@zufe.edu.cn
2 Collaborative Innovation Center for Wealth Management and Quantitative Investment, Zhejiang University of Finance and Economics, Hangzhou 310018, China
3 School of Economics and Management, University of Chinese Academy of Sciences, Beijing 100190, China; lizexi17@mails.ucas.ac.cn
4 PricewaterhouseCoopers Zhong Tian LLP, Beijing Branch, Beijing 100020, China; anna.l.zeng@cn.pwc.com
5 School of Economics and Management, Beijing University of Chemical Technology, Beijing 100029, China
6 Salford Business School, Salford University, Salford M5 4WTU, UK; j.liu@salford.ac.uk
* Correspondence: yulean@amss.ac.cn

Received: 26 July 2017; Accepted: 3 December 2017; Published: 21 December 2017

Abstract: This paper proposes an efficient option pricing model that incorporates stochastic interest rate (SIR), stochastic volatility (SV), and double exponential jump into the jump-diffusion settings. The model comprehensively considers the leptokurtosis and heteroscedasticity of the underlying asset's returns, rare events, and an SIR. Using the model, we deduce the pricing characteristic function and pricing formula of a European option. Then, we develop the Markov chain Monte Carlo method with latent variable to solve the problem of parameter estimation under the double exponential jump-diffusion model with SIR and SV. For verification purposes, we conduct time efficiency analysis, goodness of fit analysis, and jump/drift term analysis of the proposed model. In addition, we compare the pricing accuracy of the proposed model with those of the Black–Scholes and the Kou (2002) models. The empirical results show that the proposed option pricing model has high time efficiency, and the goodness of fit and pricing accuracy are significantly higher than those of the other two models.

Keywords: Option pricing model; Stochastic interest rate; Stochastic volatility; Double exponential jump; Markov Chain Monte Carlo with Latent Variable

1. Introduction

In the last decade, many studies have examined the pricing of financial securities using the various jump-diffusion models (Kou, 2002; Leippold and Wu, 2002; Glasserman and Kou, 2003; Kou & Wang, 2004; Espinosa and Vives, 2006;

Cheng and Scaillet, 2007; Wong and Lo, 2009; Kim and Kim, 2011; Chiarell and Kang, 2013; Kim et al., 2014; Wang et al., 2016; Kiesel and Rahe, 2017; Leippold and Schärer, 2017). In addition, research departments in the financial industry have begun using jump-diffusion models as an evaluation tool (Zhang and Wang, 2013). This increase in interest is due largely to the following reasons. First, jump-diffusion models can partly explain the leptokurtic features of the distribution of the asset returns, which may have a higher peak and heavier tails than in the case of the normal distribution. Second, these models can deal effectively with other empirical phenomena, such as the volatility smiles in option markets. As a result, they are used to model both the overreaction (attributed to heavy tails) and the under-reaction (attributed to high peaks) to external good or bad news in financial markets (Barndorff-Nielsen and Shephard, 2006; Maekawa et al., 2008). Thus, the jump component of the jump-diffusion settings can be viewed as the financial market's response to signals provided by external events.

The two prominent normal jump-diffusion models (Merton, 1976) and the double exponential jump-diffusion model (Kou, 2002) can all partly interpret the leptokurtic features of a distribution and the volatility smile. As a result, many studies have employed normal jump-diffusion settings in an attempt to explain two empirical phenomena: normal jump-diffusion models utilizing a stochastic interest rate (SIR), and the stochastic volatility (SV) of Glasserman and Kou (2003), Johannes (2004), Espinosa and Vives (2006), Bo et al. (2010), Beliaeva and Nawalkha (2012), Pillay and O'Hara (2011), and Simonato (2011).

Empirical studies have indicated that the double exponential jump-diffusion model fits the asset price process better than the normal jump-diffusion model does (Ramezani and Zeng, 1998; Barndorff-Nielsen and Shephard, 2006; Maekawa et al., 2008), which has seen the double exponential jump-diffusion model gaining the wider acceptance of the two. As a result, researchers have begun developing more general models by integrating a SIR or SV into the double exponential jump-diffusion models. Here, examples include the works of Zhang et al. (2012), Zhang and Wang (2013), Huang et al. (2014), and Leippold and Vasiljevi´c (2017).

However, some models that have been constructed to determine option prices do not simultaneously incorporate an SIR, SV, and double exponential jumps and, therefore, do not fully capture the leptokurtic features of asset returns or the volatility smile in option markets. For example, these models have been unable to capture volatility clustering and the heteroscedasticity effect. Furthermore, the spot interest rate is a fundamental economic variable in the asset price process, and so cannot be treated as a constant. Zhang and Wang (2013) proposed an option pricing model that does integrate an SIR, SV, and double exponential jumps. However, their pricing formula is limited to determining the price of a European option, and so cannot be generalized to other financial derivatives.

In addition, a typical shortcoming of these previous studies is that most focus on numerical computations using hypothesized parameters. In other words, the parameters are not estimated from real data. Incorporating an SIR, SV, and jumps simultaneously increases the complexity of estimating the parameters. The maximum likelihood estimation method cannot be applied in this case, because it is very difficult to obtain the probability density function of the jump-diffusion model (Li et al., 2016). In addition, the moment estimation method cannot be used because the models' high-order moments have many parameters and are extremely complex. Another method use for parameter estimation is the Markov chain Monte Carlo (MCMC) method, which constructs a Markov chain from the parameter samples (Geman and Geman, 1984; Martino et al., 2015; Meyer et al., 2008; Gilks et al., 1995). Then, a stable distribution of the parameters is obtained from the chain. The MCMC method has strong adaptability, which can be used as the basis for estimating the parameters of jump-diffusion models.

However, when estimating the parameters of the double exponential jump-diffusion model, there are inevitably some latent variables, such as the jump time, jump amplitude, and the volatility of each time point. Eraker

(2001) proposed an effective method for dealing with latent variables in the MCMC model. Hu et al. (2006) and Zhou et al. (2013) proposed using an MCMC estimation for the double exponential jump-diffusion model, but they assume that the interest rate and volatility are constant. Then, Yu et al. (2011) proposed using the MCMC method for the Lévy jump model. Integrating the ideas of these propositions, this paper proposes a method that uses the Markov chain Monte Carlo model with latent variable (MCMC-LV) to solve the problem of the parameters in the jump-diffusion model making it difficult to estimate.

In addition, we propose several other models that can explain the leptokurtic features and the volatility smile in option pricing. For example, Chen et al. (2016) proposed American option pricing under generalized mixed fractional Brown motion (GMFBM), using numerical methods to solve the linear complementarity problem. However, the parameters of the model are difficult to estimate, and the SIR risk and credit risk cannot be included in the model. In addition to fractional Brownian motion, the Lévy process is one of the most popular topics among researchers. Kleinert and Korbel (2016) pointed out that the Lévy process can produce infinite jumps and infinite variance and, hence, is considered an ideal method to describe heteroscedasticity and rare events (jumps). Fajardo and Farias (2010) price options by applying the hyperbolic distribution function proposed by Barndorff-Nielsen (1977), and assuming that the underlying asset return obeys the Lévy process generated by a multidimensional generalized hyperbolic distribution. Gong and Zhuang (2016) price options using the underlying asset price process that contains Lévy volatility and Lévy jump processes. However, the constrained nonlinear optimization method used by these models lacks stability in the estimated parameters.

In summary, there is still a need for a pricing model that simultaneously incorporates an SIR, SV, and double exponential jumps, with an effective parameter estimation method for the asset price process. Therefore, we propose such a model under the affine jump diffusion structure (AJD). It is assumed that SV and the SIR obey the Cox–Ingersoll–Ross (CIR) process, and that the jump of the underlying asset price follows a double exponential jump. Our model can incorporate credit risk and expand along multiple dimensions.

The remainder of the paper is organized as follows. In Section 2, we derive the pricing characteristic function and obtain the pricing formula of a generalized European option, under which a pricing function suitable for the quick and accurate valuation of European options can be obtained. In Section 3, we propose the MCMC-LV method to estimate the parameters in the jump-diffusion model, taking into account SV and an SIR. In Section 4, we first verify the effectiveness of the proposed model using a time efficiency analysis, goodness of fit analysis, and jump/drift term analysis. Then, we compare the pricing accuracy of the proposed model with those of the Black–Scholes (BS) and Kou (2002) models (alternative double exponential jump-diffusion option pricing models) and the real market price (10 50ETF European options expiring in June 2016 on the Shanghai Stock Exchange (SSE), including five call options and five put options with different strike prices). Section 5 concludes the paper.

2. Option Pricing Model under Double Exponential Jump-Diffusion Settings, with SV and SIR

2.1. Double Exponential Jump-Diffusion Process, with SV and SIR

In probability space $(\Omega, \mathcal{F}, \mathbb{P})$, $\mathcal{F}$ is the filtration generated by the Brownian motion and the jump process at time t, $0 \le t \le T$, and $\mathbb{P}$ is a real probability measure. Suppose the instantaneous interest rate $r(t)$ and the SV $\sigma(t)$ of the underlying asset of returns are governed by the following CIR process. The basic state process $X(t)$ (i.e., the underlying asset price $S(t) = e^{X(t)}$) at time t is given by

$$d\begin{bmatrix} X(t) \\ r(t) \\ \sigma(t) \end{bmatrix} = d\Upsilon(t) = u\left(\Upsilon(t)\right)dt + \sigma\left(\Upsilon(t)\right)dW(t) + F_t dJ(t)$$
$$= \begin{bmatrix} u \\ \kappa_r(\bar{r} - r(t)) \\ \kappa_\sigma\left(\bar{\sigma} - \sigma(t)\right) \end{bmatrix} dt + \begin{bmatrix} \sqrt{\sigma(t)} & 0 & 0 \\ 0 & \sigma_r\sqrt{r(t)} & 0 \\ 0 & 0 & \sigma_\sigma\sqrt{\sigma(t)} \end{bmatrix} d\begin{bmatrix} W_1(t) \\ W_r(t) \\ W_\sigma(t) \end{bmatrix} + \begin{bmatrix} v_t \\ 0 \\ 0 \end{bmatrix} dJ(t), \tag{1}$$

where u is the constant drift rate; v_t follows a double exponential distribution at time t; SV $\sigma(t)$ follows a CIR process; the mean-reverting rate κ_σ, long-term volatility $\bar{\sigma}$, and volatility of the volatility σ_σ are constant, the interest rate $r(t)$ follows a CIR process; and the mean-reverting rate κ_r, long-term mean $\bar{r}$, and variation coefficient σ_r are constants. Then, $J(t)$ is a Poisson process with intensity λ, and $W_1(t), W_\sigma(t), W_r(t)$, and $J(t)$ are mutually independent Brownian motions. Moreover, v_t has an asymmetric double exponential distribution, with density

$$f(v) = p \cdot \eta_1 e^{-\eta_1 v} 1_{\{v \geq 0\}} + q \cdot \eta_2 e^{\eta_2 v} 1_{\{v < 0\}}, (\eta_1 > 1, \eta_2 > 0),$$

in other words, $v \overset{d}{=} \begin{cases} \xi^+, & with \quad probability \quad p \\ -\xi^-, & with \quad probability \quad q \end{cases}$, where ξ^+ and ξ^- are exponential random variables with means $1/\eta_1$ and $1/\eta_2$, respectively, the notation $\overset{d}{=}$ means "equal in distribution," $p \geq 0$, $q \geq 0$, $p + q = 1$, and $\xi^+, -\xi^-$ denote the probabilities of upward and downward jumps, respectively. In addition, the drift term $u\left(\Upsilon(t)\right)$, and the volatility term $\sigma\left(\Upsilon(t)\right)$ are assumed to depend affinely on $\Upsilon(t)$; that is, these terms at time t are given by

$$u\left(\Upsilon(t)\right) = \begin{bmatrix} u \\ \kappa_r \bar{r} \\ \kappa_\sigma \bar{\sigma} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\kappa_r & 0 \\ 0 & 0 & -\kappa_\sigma \end{bmatrix} \Upsilon(t) = U_0 + U_1 \Upsilon(t)$$
$$\sigma\left(\Upsilon(t)\right)^T \sigma\left(\Upsilon(t)\right) = \begin{bmatrix} \sigma(t) & 0 & 0 \\ 0 & \sigma_r^2 r(t) & 0 \\ 0 & 0 & \sigma_\sigma^2 \sigma(t) \end{bmatrix} = \Sigma_0 + \Sigma_M \Upsilon(t).$$
$$\Sigma_0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \Sigma_M = \begin{bmatrix} 0 & 0 & 1 \\ 0 & \sigma_r^2 & 0 \\ 0 & 0 & \sigma_\sigma^2 \end{bmatrix}, r(t) = \rho_0 + \rho_1^T \Upsilon(t), \rho_0 = 0, \rho_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

At this point, we depict three processes: the basic state process, volatility process, and interest rate process. Let the price of the underlying asset $S(t) = e^{X(t)}$ be as mentioned above. Then, we have

$$d\left(e^{X(t)}\right) = e^{X(t-)} dX^c(t) + \frac{1}{2} e^{X(t-)} dX^c(t) dX^c(t) + (e^{X(t-)+v_t} - e^{X(t-)}) dJ(t)$$
$$dS(t) = S(t-)(u + \frac{1}{2}\sigma(t))dt + S(t-)\sqrt{\sigma(t)} dW_1(t) + S(t-)(e^{v_t} - 1) dJ(t).$$

Let $V_t = e^{v_t}, Y_t = V_t - 1$. Then, we obtain

$$\begin{aligned} &dS(t)=S(t-)(u+\frac{1}{2}\sigma(t))dt+S(t-)\sqrt{\sigma(t)}dW_1(t)+S(t-)Y_t dJ(t)\\ &d\sigma(t)=\kappa_\sigma\left(\bar{\sigma}-\sigma(t)\right)dt+\sigma_\sigma\sqrt{\sigma(t)}dW_\sigma(t)\\ &dr(t)=\kappa_r(\bar{r}-r(t))dt+\sigma_r\sqrt{r(t)}dW_r(t)\end{aligned} \quad . \tag{2}$$

In order to demonstrate these processes from a clear perspective, the number of underlying assets is assumed to be one, and the default risk and the correlations between $W_1(t), W_\sigma(t), W_r(t), J(t), \nu_t$ are ignored. In the following sections, we begin by using Equations (1) and (2) to obtain the pricing characteristic function $\psi^\chi(\mu, \Upsilon(t), t, T)$ of financial derivatives.

2.2. Double Exponential Jump-Diffusion Process with SV and SIR under the Risk Neutral Probability Measure

To get the pricing formula for financial securities, it is necessary to carry out a measure transformation for the double exponential jump process with SV and SIR under the risk neutral probability measure. In this section, Equations (1) and (2) are transformed under the risk neutral probability measure $\tilde{\mathbb{P}}$. Equations (1) and (2) show that a double exponential jump process $Y_t dJ(t)$ cannot be ignored in the process of transforming the measure.

Let $W(t)=\left(W_1(t), W_\sigma(t), W_r(t)\right)^T$ be a three-dimensional Brownian motion under real probability measure $\mathbb{P}$, and $Q(t)=\sum_{i=1}^{J(t)} Y_i$ be a compound Poisson process with intensity λ ($\lambda \in \mathbb{R}^+$) and jump amplitude $Y_1, Y_2, \cdots$ that follows the probability density function $f(y)$ under the real probability measure $\mathbb{P}$. In addition, $\theta(t)=\left(\theta_1(t), \theta_2(t), \theta_3(t)\right)^T$ is a three-dimensional adapted process under the real probability measure $\mathbb{P}$, where $\|\theta(t)\|=\left(\sum_{i=1}^{3}\theta_i^2(t)\right)^{\frac{1}{2}}$.

According to theorems on measure transformations of compound Poisson processes and Brownian motions, the Radon–Nikodym derivative can be obtained as follows:

$$\begin{aligned} &Z(t)=Z\left(Z_1(t), Z_2(t)\right)=\frac{d\tilde{\mathbb{P}}}{d\mathbb{P}}=Z_1(t)Z_2(t)\\ &Z_1(t)=e^{-\int_0^t \theta(u)^T dW(u)-\frac{1}{2}\int_0^t\|\theta(u)\|^2 du}, Z_2(t)=e^{(\lambda-\tilde{\lambda})t}\prod_{i=1}^{J(t)}\frac{\tilde{\lambda}\tilde{f}(Y_i)}{\lambda f(Y_i)}\end{aligned}'$$

where $Z_1(t)$ is the Radon–Nikodym derivative under the affine process, $Z_2(t)$ is the Radon–Nikodym derivative under the jump process, $Z(t)$ is the Radon–Nikodym derivative of the joint measure transformation under both the affine and the jump process.

Then, under the risk-neutral probability measure $\tilde{\mathbb{P}}$, the basic state process $X(t)$, interest rate process $r(t)$, and volatility process $\sigma(t)$ at time t are given by:

$$d\begin{bmatrix} X(t)\\ r(t)\\ \sigma(t)\end{bmatrix}=d\Upsilon(t)=u\left(\Upsilon(t)\right)dt+\sigma\left(\Upsilon(t)\right)d\tilde{W}(t)+F_t dJ(t),$$

$$u\left(\Upsilon(t)\right)=\begin{bmatrix}-\lambda\bar{Y}\\ \kappa_r\bar{r}\\ \kappa_\sigma\bar{\sigma}\end{bmatrix}+\begin{bmatrix}0 & 1 & -\frac{1}{2}\\ 0 & -\kappa_r & 0\\ 0 & 0 & -\kappa_\sigma\end{bmatrix}\Upsilon(t)=U_0+U_1\Upsilon(t)$$

where
$$\sigma\left(\Upsilon(t)\right)^T\sigma\left(\Upsilon(t)\right)=\begin{bmatrix}\sigma(t) & 0 & 0\\ 0 & \sigma_r{}^2 r(t) & 0\\ 0 & 0 & \sigma_\sigma{}^2\sigma(t)\end{bmatrix}=\Sigma_0+\Sigma_M\Upsilon(t)\quad. \tag{3}$$

$$\Sigma_0=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix},\Sigma_M=\begin{bmatrix}0&0&1\\0&\sigma_r^2&0\\0&0&\sigma_\sigma^2\end{bmatrix},r(t)=\rho_1{}^T\Upsilon(t),\rho_1=\begin{bmatrix}0\\1\\0\end{bmatrix}$$

Equation (3) is the no-arbitrage formula of Equations (1) and (2) under the risk neutral probability measure $\widetilde{\mathbb{P}}$, which is the formula of the double exponential jump-diffusion process with SV and SIR for the underlying asset price process under the risk-neutral probability measure. Then, $\tilde{W}(t)=\left(\tilde{W}_1(t),\tilde{W}_\sigma(t),\tilde{W}_r(t)\right)^T$ is a three-dimensional Brownian motion under the risk-neutral probability measure $\widetilde{\mathbb{P}}$.

2.3. Characteristic Function of Underlying Asset and its Option Pricing Formula

In order to apply the double exponential jump-diffusion process models with SV and SIR to broaden the range of financial derivative pricing problems, a generalized pricing formula is required for most of the financial derivatives in the market.

From Equation (3), the characteristic coefficient of the double exponential jump-diffusion process model with SV and SIR is $\chi=\left(U_0,U_1,\Sigma_0,\Sigma_M,\rho_1\right)$. Because $\Upsilon(t)$ is Markovian, χ decides the following characteristic function of the underlying asset (financial derivatives):

$$\psi^\chi(\mu,\Upsilon(t),t,T)=\widetilde{\mathbb{E}}\left(e^{-\int_t^T r(s)ds}e^{\mu^T\Upsilon(T)}\,|\mathcal{F}(t)\right),\text{ where }\mu=\begin{bmatrix}\mu_1\\ \mu_2\\ \mu_3\end{bmatrix}\in\mathbb{R}^3.$$

The characteristic function $\psi^\chi(\mu,\Upsilon(t),t,T)$ denotes the value of an underlying asset at time t that pays $e^{\mu^T\Upsilon(T)}$ at future time T, based on all information up to t. The characteristic function $\psi^\chi(\mu,\Upsilon(t),t,T)$ is an effective tool for pricing options and a wide class of financial derivatives. To solve the characteristic function, the Feynman–Kac Theorem needs to be used with the double exponential jump model with SV and SIR.

In order to solve the characteristic function, we assume that the characteristic function $\psi^\chi(\mu,\Upsilon(t),t,T)$ has a solution of the following form, as per Shreve (2004):

$$\psi^\chi(\mu,\Upsilon(t),t,T)=\exp\left(\alpha(\mu,t,T)+\beta(\mu,t,T)^T\Upsilon(t)\right),$$

where $\beta(\mu,t,T)=\begin{bmatrix}\beta^1(\mu,t,T)\\ \beta^2(\mu,t,T)\\ \beta^3(\mu,t,T)\end{bmatrix}$.

The solutions are as follows (See proof in Appendix A):

$$\beta^1(\mu,t,T)=\mu_1$$

$$\beta^3(\mu,t,T)=\frac{\tan\left(\frac{\sqrt{\sigma_\sigma^2\mu_1(\mu_1-1)-\kappa_\sigma^2}(T-t)}{2}+\arctan\left(\frac{\sigma_\sigma^2\mu_3-\kappa_\sigma}{\sqrt{\sigma_\sigma^2\mu_1(\mu_1-1)-\kappa_\sigma^2}}\right)\right)\sqrt{\sigma_\sigma^2\mu_1(\mu_1-1)-\kappa_\sigma^2}+\kappa_\sigma}{\sigma_\sigma^2}$$

$$\beta^2(\mu,t,T)=\frac{\tan\left(\frac{\sqrt{2\sigma_r^2(\mu_1-1)-\kappa_r^2}(T-t)}{2}+\arctan\left(\frac{\sigma_r^2\mu_2-\kappa_r}{\sqrt{2\sigma_r^2(\mu_1-1)-\kappa_r^2}}\right)\right)\sqrt{2\sigma_r^2(\mu_1-1)-\kappa_r^2}+\kappa_r}{\sigma_r^2} \quad (5)$$

$$\alpha(\mu,t,T)=\lambda\bar{Y}\mu_1(t-T)-\lambda\left(\frac{q\eta_2}{\eta_2+\mu_1}-\frac{p\eta_1}{\mu_1-\eta_1}\right)(t-T)-\left(\frac{\kappa_r^2\bar{r}}{\sigma_r^2}+\frac{\kappa_\sigma^2\bar{\sigma}}{\sigma_\sigma^2}\right)(t-T)-$$

$$\frac{\kappa_r\bar{r}}{\sigma_r^2}2\ln\left|\cos\left(\frac{\sqrt{2\sigma_r^2(\mu_1-1)-\kappa_r^2}(T-t)}{2}\right)-\sin\left(\frac{\sqrt{2\sigma_r^2(\mu_1-1)-\kappa_r^2}(T-t)}{2}\right)\frac{\sigma_r^2\mu_r-\kappa_r}{\sqrt{2\sigma_r^2(\mu_1-1)-\kappa_r^2}}\right|-$$

$$\frac{\kappa_\sigma\bar{\sigma}}{\sigma_\sigma^2}2\ln\left|\cos\left(\frac{\sqrt{\sigma_\sigma^2\mu_1(\mu_1-1)-\kappa_\sigma^2}(T-t)}{2}\right)-\sin\left(\frac{\sqrt{\sigma_\sigma^2\mu_1(\mu_1-1)-\kappa_\sigma^2}(T-t)}{2}\right)\frac{\sigma_\sigma^2\mu_3-\kappa_\sigma}{\sqrt{\sigma_\sigma^2\mu_1(\mu_1-1)-\kappa_\sigma^2}}\right|$$

Equation (5) determines an analytic expression for $\psi^\chi(\mu,\Upsilon(t),t,T)$. For some simple underlying assets with due payments, $\psi^\chi(\mu,\Upsilon(t),t,T)$ can be used directly. However, for derivative securities such as European options, the inverse Fourier transform should be applied to the characteristic function $\psi^\chi(\mu,\Upsilon(t),t,T)$.

Next, we derive a European option pricing formula under the double exponential jump model with SV and SIR, based on the characteristic function $\psi^\chi(\mu,\Upsilon(t),t,T)$.

Let the price of a generalized European option be

$$C^\chi(\delta,\varepsilon,c,\Upsilon(t),t,T)=\widetilde{\mathbb{E}}\left(e^{-\int_t^T r(s)ds}\left(e^{\delta^T\bullet\Upsilon(T)}-c\right)\bullet 1_{\varepsilon^T\bullet\Upsilon(T)\geq\ln(c)}\,|\mathcal{F}(t)\right),$$

where $\delta=\begin{bmatrix}\delta_1\\\delta_2\\\delta_3\end{bmatrix}\in\mathbb{R}^3$, $\varepsilon=\begin{bmatrix}\varepsilon_1\\\varepsilon_2\\\varepsilon_3\end{bmatrix}\in\mathbb{R}^3$ (i.e., when $\varepsilon^T\bullet\Upsilon(T)\geq\ln(c)$ is satisfied), and the option pays $e^{\delta^T\bullet\Upsilon(T)}-c$ at time T, and zero otherwise. Then, c is a constant, and $\ln(c)$ is a threshold, similar to the strike price of a call option.

Let $G^\chi(\delta,\varepsilon,y,\Upsilon(t),t,T)=\widetilde{\mathbb{E}}\left(e^{-\int_t^T r(s)ds}e^{\delta^T\bullet\Upsilon(T)}\bullet 1_{\varepsilon^T\bullet\Upsilon(T)\leq y}\,|\mathcal{F}(t)\right)$ be the price of a derivative security at time t. Then, this derivative security pays $e^{\delta^T\bullet\Upsilon(T)}$ at time T when $\varepsilon^T\bullet\Upsilon(T)\leq y$, and zero otherwise. Thus, the pricing formula of the generalized European option can be written as: $C^\chi(\delta,\varepsilon,c,\Upsilon(t),t,T)=G^\chi(\delta,-\varepsilon,-\ln(c),\Upsilon(t),t,T)-c\bullet G^\chi(0,-\varepsilon,-\ln(c),\Upsilon(t),t,T)$.

From Fubini's theorem, we define the probability measure $\widetilde{\mathbb{P}}$ on filtration $\mathcal{F}(t)$, and define the Lebesgue measure on the real number axis using Borel σ-algebra as σ-addable and complete. Therefore, the order of the derivation and expectation can be exchanged; that is:

$$G^{\chi}{}_{y}(\delta,\varepsilon,y,\Upsilon(t),t,T)=\frac{\partial G^{\chi}(\delta,\varepsilon,y,\Upsilon(t),t,T)}{\partial y}=$$

$$\widetilde{\mathbb{E}}\left(e^{-\int_t^T r(s)ds}e^{\delta^T\bullet\Upsilon(T)}\bullet\delta\left(y-\varepsilon^T\bullet\Upsilon(T)\right)|\mathcal{F}(t)\right),$$

where $G^{\chi}{}_{y}(\delta,\varepsilon,y,\Upsilon(t),t,T)$ is the partial derivative of $G^{\chi}(\delta,\varepsilon,y,\Upsilon(t),t,T)$ on y. According to the definition, $G^{\chi}{}_{y}(\delta,\varepsilon,y,\Upsilon(t),t,T)$ is absolutely integrable on real number $y\in\mathbb{R}$; thus, $G^{\chi}{}_{y}(\delta,\varepsilon,y,\Upsilon(t),t,T)\in L_1[-\infty,\infty]$. Therefore, the Fourier transform and inverse Fourier transform of $G^{\chi}{}_{y}(\delta,\varepsilon,y,\Upsilon(t),t,T)$ exist. Similarly, the Fourier–Stieltjes transform and inverse Fourier–Stieltjes transform of $G^{\chi}(\delta,\varepsilon,y,\Upsilon(t),t,T)$ exist. The Fourier–Stieltjes transform of $G^{\chi}(\delta,\varepsilon,y,\Upsilon(t),t,T)$ is defined as follows:

$$\hat{G}^{\chi}(\delta,\varepsilon,\omega,\Upsilon(t),t,T)=\int_{y\in\mathbb{R}}e^{i\omega y}dG^{\chi}(\delta,\varepsilon,y,\Upsilon(t),t,T).$$

Following Fubini's theorem, we exchange the order of the expectation and the integration:

$$\hat{G}^{\chi}(\delta,\varepsilon,\omega,\Upsilon(t),t,T)=\widetilde{\mathbb{E}}\left(e^{-\int_t^T r(s)ds}e^{\delta^T\bullet\Upsilon(T)}\bullet\int_{y\in\mathbb{R}}e^{i\omega y}\delta\left(y-\varepsilon^T\bullet\Upsilon(T)\right)dy\,|\mathcal{F}(t)\right)$$
$$=\psi^{\chi}(\delta^T+i\omega\varepsilon^T,\Upsilon(t),t,T).$$

Then, from Lévy's inversion theorem and Gil-Pelaez (1951), we have:

$$G^{\chi}(\delta,\varepsilon,y,\Upsilon(t),t,T)=\frac{\psi^{\chi}(\delta,\Upsilon(t),t,T)}{2}-\frac{1}{\pi}\int_0^{\infty}\mathrm{Im}\left(\frac{e^{-i\omega y}\psi^{\chi}(\delta^T+i\omega\varepsilon^T,\Upsilon(t),t,T)}{\omega}\right)d\omega. \tag{6}$$

From Equation (6), we obtain the price of the generalized call option, as follows:

$$\begin{aligned}&C^{\chi}(\delta,\varepsilon,c,\Upsilon(t),t,T)=G^{\chi}(\delta,-\varepsilon,-\ln(c),\Upsilon(t),t,T)-c\bullet G^{\chi}(0,-\varepsilon,-\ln(c),\Upsilon(t),t,T)\\&=\frac{\psi^{\chi}(\delta,\Upsilon(t),t,T)-c\psi^{\chi}(0,\Upsilon(t),t,T)}{2}+\\&\frac{1}{\pi}\int_0^{\infty}\mathrm{Im}\left(\frac{e^{i\omega\ln(c)}\left(c\psi^{\chi}(-i\omega\varepsilon^T,\Upsilon(t),t,T)-\psi^{\chi}(\eta^T-i\omega\varepsilon^T,\Upsilon(t),t,T)\right)}{\omega}\right)d\omega\end{aligned} \tag{7}$$

$$\begin{aligned}&C^{\chi}(\delta,\varepsilon,c,\Upsilon(t),t,T)=c\bullet G^{\chi}(0,\varepsilon,\ln(c),\Upsilon(t),t,T)-G^{\chi}(\delta,\varepsilon,\ln(c),\Upsilon(t),t,T)\\&=\frac{c\psi^{\chi}(0,\Upsilon(t),t,T)-\psi^{\chi}(\delta,\Upsilon(t),t,T)}{2}+\\&\frac{1}{\pi}\int_0^{\infty}\mathrm{Im}\left(\frac{e^{-i\omega\ln(c)}\left(\psi^{\chi}(\eta^T+i\omega\varepsilon^T,\Upsilon(t),t,T)-c\psi^{\chi}(i\omega\varepsilon^T,\Upsilon(t),t,T)\right)}{\omega}\right)d\omega\end{aligned}. \tag{8}$$

The price of the option based on the double exponential jump model with SV and SIR, based on Equations (7) and (8), can be computed quickly and accurately using the inverse Fourier transform method.

3. Parameter Estimation Based on the MCMC-LV Method in the Jump-Diffusion Model

In this section, we analyze estimates of the parameters for the double exponential jump-diffusion model with SV and SIR. That is, a Markov chain Monte Carlo with latent variable (MCMC-LV) method is used to estimate the parameters of the jump-diffusion model. The MCMC-LV method has strong extensibility. If we add more latent variables such as credit risks, correlation coefficients, or more jumps, the MCMC-LV method can still be extended and used.

3.1. Sketch of MCMC-LV Method

The MCMC-LV method that we use to estimate parameters for the double exponential jump-diffusion model with SV and the SIR can be divided into two parts. First, we estimate the underlying asset price process:

$$\begin{aligned} dS(t) &= S(t-)(\mu + \frac{1}{2}\sigma(t))dt + S(t-)\sqrt{\sigma(t)}dW_1(t) + S(t-)(e^{v_t}-1)dJ(t) \\ d\sigma(t) &= \kappa_\sigma\left(\bar{\sigma}-\sigma(t)\right)dt + \sigma_\sigma\sqrt{\sigma(t)}dW_\sigma(t) \end{aligned} . \quad (9)$$

Then, we estimate the interest rate process:

$$dr(t) = \kappa_r(\bar{r} - r(t))dt + \sigma_r\sqrt{r(t)}dW_r(t) . \quad (10)$$

Note that the parameter estimations of Equations (9) and (10) are independent of each other.

Because $J(t)$, $\sigma(t)$, and v_t cannot be observed directly, the parameter estimation cannot be achieved directly using the Gibbs algorithm and sample data of the underlying assets. Thus, the time series of $J(t)$, $\sigma(t)$, and v_t are regarded as the latent variables being estimated.

For a CIR interest rate, there are no latent variables. Therefore, we use Metropolis–Hastings sampling for the MCMC (MCMC-MH).

3.2. MCMC-LV Method for the Underlying Asset Price Process

According to Hu et al. (2006), the differential expression of $S(t)$ and $\sigma(t)$ in Equation (9) are changed into discrete form, as follows:

$$\begin{aligned} & S(t+\Delta t) - S(t) = \Delta S(t) = S(t-)\left(\mu + \frac{1}{2}\sigma(t)\right)\Delta t + S(t-)\sqrt{\sigma(t)}W(\Delta t) + S(t-)\left(e^{v_t}-1\right)B_t \\ & \Delta\sigma(t) = \sigma(t) - \sigma(t-\Delta t) = \kappa_\sigma\left(\bar{\sigma} - \sigma(t-\Delta t)\right)\Delta t + \sigma_\sigma\sqrt{\sigma(t-\Delta t)}W_\sigma(\Delta t) \\ & \begin{cases} y(t) = \dfrac{\Delta S(t)}{S(t-)} = \left(\mu + \dfrac{1}{2}\sigma(t)\right)\Delta t + \sqrt{\sigma(t)}W(\Delta t) + Y_t B_t \\ \sigma(t) = \sigma(t-\Delta t) + \kappa_\sigma\left(\bar{\sigma} - \sigma(t-\Delta t)\right)\Delta t + \sigma_\sigma\sqrt{\sigma(t-\Delta t)}W_\sigma(\Delta t) \end{cases} \quad \left(t = 0,1,2,\ldots,M-1\right) \end{aligned} . \quad (11)$$

Here, B_t follows a 0–1 distribution, where the probability of 1 is $\lambda\Delta t$. Then, $Y_t = V_t - 1, V_t = e^{v_t}$, v_t has an asymmetric double exponential distribution, with density $f(v) = p\cdot\eta_1 e^{-\eta_1 v}1_{\{v\geq 0\}} + q\cdot\eta_2 e^{\eta_2 v}1_{\{v<0\}}, (\eta_1 > 1, \eta_2 > 0)$.

If we let $t = 0$ in Equation (13), we get $f\left(\sigma(0)\,|\,\Im_2, \sigma(-1)\right)$. It is inevitable that we will encounter $\sigma(-1)$.

Therefore, we define $\sigma(-1)$ as an additional parameter that needs to be estimated. Thus, we define the following parameter sets:

$$\begin{aligned} &\Im_1 = \{\mu, \sigma, \lambda, \eta_1, \eta_2, p\} \\ &\Im_2 = \{\kappa_\sigma, \bar{\sigma}, \sigma_\sigma\} \\ &\Im = \{\mu, \sigma, \lambda, \eta_1, \eta_2, p, \kappa_\sigma, \bar{\sigma}, \sigma_\sigma, \sigma(-1)\} \end{aligned}.$$

It is assumed that the parameters in $\Im$ have mutually independent prior distributions. Thus, the probability density function (PDF) of $\Im$ is $\pi(\Im) = \pi(\Im_1)\pi(\Im_2)\pi(\sigma(-1))$, where the PDFs are equal to:

$$\begin{aligned} &\pi(\Im_1) = \pi(\mu)\pi(\sigma)\pi(\lambda)\pi(\eta_1)\pi(\eta_2)\pi(p) \\ &\pi(\Im_2) = \pi(\kappa_\sigma)\pi(\bar{\sigma})\pi(\sigma_\sigma) \end{aligned}. \quad (12)$$

Based on other relevant studies (Yu et al., 2011; Hu et al., 2006; Lu and Hua, 2010; Han et al., 2010; Yang et al., 2010), we set the prior distribution of the parameters as follows:

$$\begin{aligned} &\mu \sim N(\hat{\mu}_1, \hat{\sigma}_1^2), \sigma \sim \ln N(\hat{\mu}_2, \hat{\sigma}_2^2), \lambda \sim \mathrm{B}(\hat{\alpha}_1, \hat{\beta}_1) \\ &\eta_1 \sim \Gamma(\hat{k}_1, \hat{\theta}_1), \eta_2 \sim \Gamma(\hat{k}_2, \hat{\theta}_2), p \sim \mathrm{B}(\hat{\alpha}_2, \hat{\beta}_2) \\ &\kappa_\sigma \sim \mathrm{B}(\tilde{\alpha}_1, \tilde{\beta}_1), \bar{\sigma} \sim \ln N(\tilde{\mu}_1, \tilde{\sigma}_1^2), \sigma_\sigma \sim \ln N(\tilde{\mu}_2, \tilde{\sigma}_2^2)' \\ &\pi(\sigma(-1)) \sim \ln N(\hat{\mu}_3, \hat{\sigma}_3^2) \end{aligned} \quad (13)$$

where $\hat{\mu}_1, \hat{\sigma}_1, \hat{\mu}_2, \hat{\sigma}_2, \hat{\alpha}_1, \hat{\beta}_1, \hat{k}_1, \hat{\theta}_1, \hat{k}_2, \hat{\theta}_2, \hat{\alpha}_2, \hat{\beta}_2, \tilde{\alpha}_1, \tilde{\beta}_1, \tilde{\mu}_1, \tilde{\sigma}_1, \tilde{\mu}_2, \tilde{\sigma}_2, \hat{\mu}_3, \hat{\sigma}_3$ are parameters of prior distributions, which are set artificially based on experience and the statistical results from the experiment. The types of the prior distributions are shown in Table 1.

Table 1. The Description of the Prior Distributions

Symbol	Description	PDF
$N(\mu,\sigma^2)$	Normal Distribution	$\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, x \in (-\infty, +\infty)$
$\ln N(\mu,\sigma^2)$	Log-Normal Distribution	$\frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{(\ln x-\mu)^2}{2\sigma^2}}, x \in (0, +\infty)$
$\Gamma(k,\theta)$	Gamma Distribution	$\frac{1}{\Gamma(k)\theta^k} x^{k-1} e^{-\frac{x}{\theta}}, \Gamma(z) = \int_0^\infty x^{z-1} e^{-x} dx, x \in (0, +\infty)$
$Beta(\alpha,\beta)$	Beta Distribution	$\frac{x^{\alpha-1}(1-x)^{\beta-1}\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}, \Gamma(z) = \int_0^\infty x^{z-1} e^{-x} dx, x \in [0,1]$

In Equation (11), it is assumed that $t = 0,1,\ldots,M-1,(\Delta t = 1)$ denote M days (on the assumption that we have already collected M days of asset prices), $\vec{y} = (y(0), y(1), \ldots, y(M-1))$ is a vector of observable daily returns, $\vec{Y} = (Y_0, Y_1, \ldots, Y_{M-1})$ is a vector of unobservable daily jump amplitudes, $\vec{B} = (B_0, B_1, \ldots, B_{M-1})$ is a vector of unobservable daily jump indicators, and $\vec{\sigma} = (\sigma(0), \sigma(1), \ldots, \sigma(M-1))$ is a vector of unobservable daily volatility. In addition, $\vec{y}$ is the only sample that can be observed directly from the M days of asset prices. Therefore, we define $\vec{y}$ as a manifest variable. Then, $\vec{Y}, \vec{B}, \vec{\sigma}$ are samples that exist in the model settings, but that cannot be observed from the M days of asset prices. Therefore, we define $\vec{Y},\vec{B},\vec{\sigma}$ as latent variables. We undertake the following steps to estimate the values of the parameters $\Im_1,\Im_2$, based on the manifest variable $\vec{y}$ and the latent variables $\vec{Y},\vec{B},\vec{\sigma}$.

If the M days of asset prices have not yet been collected, the samples $\vec{y},\vec{Y},\vec{B},\vec{\sigma}$ will be random variables. Define the conditional probability density function (CPDF) of the samples of $\vec{y},\vec{Y},\vec{B}$ at day $t\left(t=0,1,...,M-1\right)$ as $f\left(y(t),Y_t,B_t\mid\Im_1,\Im_2,\sigma(t)\right)$. Define the CPDF of the samples of $\vec{\sigma}$ at day t as $f\left(\sigma(t)\mid\Im_2,\sigma(t-\Delta t)\right)$. According to Appendix B.1, expressions for these formulae are derived as follows:

$$f\left(y(t),Y_t,B_t\mid\Im_1,\Im_2,\sigma(t)\right)=\frac{1}{\sqrt{2\pi}\sqrt{\sigma(t)}\sqrt{\Delta t}}e^{-\frac{\left[y(t)-\left(\mu+\frac{1}{2}\sigma(t)\right)\Delta t-Y_tB_t\right]^2}{2\sigma(t)\Delta t}}\bullet$$
$$\left[\left(1-\lambda\Delta t\right)\delta(B_t)+\lambda\Delta t\delta(B_t-1)\right]\bullet\left[I_{\{-1<Y_t<0\}}\left(1-p\right)\frac{\eta_2}{Y_t+1}e^{\eta_2\ln\left(Y_t+1\right)}+I_{\{Y_t\geq 0\}}p\frac{\eta_1}{Y_t+1}e^{-\eta_1\ln\left(Y_t+1\right)}\right] \tag{14}$$

$$f\left(\sigma(t)\mid\Im_2,\sigma(t-\Delta t)\right)=\frac{1}{\sqrt{2\pi}\sigma_\sigma\sqrt{\sigma(t-\Delta t)}\sqrt{\Delta t}}e^{-\frac{\left\{\sigma(t)-\left[\sigma(t-\Delta t)+\kappa_\sigma\left(\bar{\sigma}-\sigma(t-\Delta t)\right)\Delta t\right]\right\}^2}{2\sigma_\sigma{}^2\sigma(t-\Delta t)\Delta t}}. \tag{15}$$

From Equations (12)–(15), the joint distribution of $\vec{Y},\vec{B},\vec{\sigma},\Im,\vec{y}$ (including all samples and parameters) is

$$\begin{aligned}
&f\left(\vec{Y},\vec{B},\vec{\sigma},\Im,\vec{y}\right)\\
&=\pi(\Im_1)\pi(\Im_2)\pi(\sigma(-1))f\left(\vec{Y},\vec{B},\vec{y},\vec{\sigma}\mid\Im_1,\Im_2,\sigma(-1)\right)\\
&=\pi(\Im)\prod_{t=0}^{M-1}\left[f\left(Y_t,B_t,y(t)\mid\Im_1,\Im_2,\sigma(t)\right)f\left(\sigma(t)\mid\Im_2,\sigma(t-1)\right)\right]\\
&=\pi(\Im)\prod_{t=0}^{M-1}\left\{\begin{array}{l}\frac{1}{\sqrt{2\pi}\sqrt{\sigma(t)}\sqrt{\Delta t}}e^{-\frac{\left[y(t)-\left(\mu+\frac{1}{2}\sigma(t)\right)\Delta t-Y_tB_t\right]^2}{2\sigma(t)\Delta t}}\bullet\left[I_{\{-1<Y_t<0\}}\left(1-p\right)\frac{\eta_2}{Y_t+1}e^{\eta_2\ln\left(Y_t+1\right)}+I_{\{Y_t\geq 0\}}p\frac{\eta_1}{Y_t+1}e^{-\eta_1\ln\left(Y_t+1\right)}\right]\bullet\\
\left[\left(1-\lambda\Delta t\right)\delta(B_t)+\lambda\Delta t\delta(B_t-1)\right]\bullet\frac{1}{\sqrt{2\pi}\sigma_\sigma\sqrt{\sigma(t-\Delta t)}\sqrt{\Delta t}}e^{-\frac{\left\{\sigma(t)-\left[\sigma(t-\Delta t)+\kappa_\sigma\left(\bar{\sigma}-\sigma(t-\Delta t)\right)\Delta t\right]\right\}^2}{2\sigma_\sigma{}^2\sigma(t-\Delta t)\Delta t}}\end{array}\right\}.\\
&\left(t=0,1,2,...,M-1\right)
\end{aligned} \tag{16}$$

Equation (16) is the key PDF needed for the MCMC simulation, and will be discussed and used later.

After the derivation of Equation (16), MCMC Gibbs sampling (Geman and Geman, 1984) is needed to obtain the sample for the calculation of the model parameters $\Im$. The Gibbs sampling repeatedly draws samples of $\Im$ from a Markov process with discrete time and continuous state. Discrete time means that each round of sampling is a time node of the Markov process. Continuous state means that the samples are drawn from a continuous probability distribution in each round of sampling. After the samples drawn from the Markov chain reach a steady-state distribution, after many rounds of sampling, the model parameters $\Im$ can be estimated by the mean (median or mode) of the samples.

Before beginning the MCMC Gibbs sampling, the only known information is the M days of asset prices, which can be transformed into daily return samples $\vec{y}$. Because $\Im$ is an unknown parameter set and $\vec{Y},\vec{B},\vec{\sigma}$ are latent variables, the first step of the Gibbs sampling is to artificially set the initial values of $\vec{Y},\vec{B},\vec{\sigma},\Im$ as $\vec{Y}_0,\vec{B}_0,\vec{\sigma}_0,\Im_0$, based on reasonable statistics, where

$$\vec{Y}_0 = \left(Y_{0,0}, Y_{0,1}, \dots, Y_{0,M-1}\right), \vec{B}_0 = \left(B_{0,0}, B_{0,1}, \dots, B_{0,M-1}\right)$$
$$\vec{\sigma}_0 = \left(\sigma_{0,0}, \sigma_{0,1}, \dots, \sigma_{0,M-1}\right), \Im_0 = \left(u_0, \sigma_0, \lambda_0, \eta_{0,1}, \eta_{0,2}, p_0, \kappa_{\sigma 0}, \bar{\sigma}_0, \sigma_{\sigma 0}, \sigma_0(-1)\right).$$

As demonstrated in the previous steps, $\vec{Y}_0, \vec{B}_0, \vec{\sigma}_0$ can be regarded as the unobservable daily jump amplitude, unobservable daily jump indicators, and unobservable daily volatility, respectively. Then, $\Im_0$ is the initial value of the parameters waiting to be estimated. Let $\vec{Y}_m, \vec{B}_m, \vec{\sigma}_m, \Im_m$ be the samples drawn in the $m-th$ round of Gibbs sampling $(m = 1, 2, 3, \dots, k, k+1, \dots, N)$, where

$$\vec{Y}_m = \left(Y_{m,0}, Y_{m,1}, \dots, Y_{m,M-1}\right), \vec{B}_m = \left(B_{m,0}, B_{m,1}, \dots, B_{m,M-1}\right)$$
$$\vec{\sigma}_m = \left(\sigma_{m,0}, \sigma_{m,1}, \dots, \sigma_{m,M-1}\right), \Im_m = \left(u_{m,s}, \sigma_m, \lambda_m, \eta_{m,1}, \eta_{m,2}, p_m, \kappa_{\sigma m}, \bar{\sigma}_m, \sigma_{\sigma m}, \sigma_m(-1)\right).$$

Suppose that the underlying Markov chain of the Gibbs sampling is $\wp$ (with discrete time and continuous state), which has a stationary distribution with CPDF $f\left(\vec{Y}, \vec{B}, \vec{\sigma}, \Im \mid \vec{y}\right)$ (Equation (16), conditioned on $\vec{y}$). Here, $\vec{Y}_m, \vec{B}_m, \vec{\sigma}_m, \Im_m$ represents the samples drawn from state m of $\wp$, and $k(k \gg 1)$ is a sufficiently large number, beyond which the samples $\vec{Y}_m, \vec{B}_m, \vec{\sigma}_m, \Im_m, (m \geq k)$ can be regarded as samples drawn from a stationary distribution with a CPDF $f\left(\vec{Y}, \vec{B}, \vec{\sigma}, \Im \mid \vec{y}\right)$ of the Markov chain $\wp$.

Here, $N(N \gg k)$ is the maximum number of Gibbs sampling rounds, as set by the researcher. The greater the value of N, the larger the computing capacity that is required. After N rounds of Gibbs sampling, the estimated model parameters are:

$$\Im = \frac{\sum_{m=k}^{N} \Im_m}{N-k}. \tag{17}$$

From the above discussion, the key to performing Gibbs sampling is to determine the transition kernel that leads to Markov chain $\wp$ with stationary distribution CPDF $f\left(\vec{Y}, \vec{B}, \vec{\sigma}, \Im \mid \vec{y}\right)$.

Define $f\left(\vec{Y}_m, \vec{B}_m, \vec{\sigma}_m, \Im_m \mid \vec{y}\right)$ as the CPDF of the Markov chain $\wp$ at stationary state $m(m \gg k)$. Assume the transition kernel is:

$$\begin{aligned} & p\left(\vec{Y}_m, \vec{B}_m, \vec{\sigma}_m, \Im_m, \vec{Y}_{m+1}, \vec{B}_{m+1}, \vec{\sigma}_{m+1}, \Im_{m+1}\right) \\ & = f\left(\vec{Y}_{m+1} \mid \vec{B}_m, \vec{\sigma}_m, \Im_m, \vec{y}\right) \bullet f\left(\vec{B}_{m+1} \mid \vec{Y}_{m+1}, \vec{\sigma}_m, \Im_m, \vec{y}\right) \bullet \\ & f\left(\vec{\sigma}_{m+1} \mid \vec{Y}_{m+1}, \vec{B}_{m+1}, \Im_m, \vec{y}\right) \bullet f\left(\Im_{m+1} \mid \vec{Y}_{m+1}, \vec{B}_{m+1}, \vec{\sigma}_{m+1}, \vec{y}\right) \end{aligned}. \tag{18}$$

Then the Markov chain $\wp$ has a unique stationary distribution, with PDF $f\left(\vec{Y}, \vec{B}, \vec{\sigma}, \Im \mid \vec{y}\right)$ (Equation (16), conditioned on $\vec{y}$) that satisfies the Chapman–Kolmogorov Equation (See proof in Appendix B.2):

$$\begin{aligned} & \int\int\int\int f\left(\vec{Y}_m, \vec{B}_m, \vec{\sigma}_m, \Im_m \mid \vec{y}\right) \bullet p\left(\vec{Y}_m, \vec{B}_m, \vec{\sigma}_m, \Im_m, \vec{Y}_{m+1}, \vec{B}_{m+1}, \vec{\sigma}_{m+1}, \Im_{m+1}\right) d\vec{Y}_m d\vec{B}_m d\vec{\sigma}_m d\Im_m \\ & = f\left(\vec{Y}_{m+1}, \vec{B}_{m+1}, \vec{\sigma}_{m+1}, \Im_{m+1} \mid \vec{y}\right) \end{aligned}.$$

Suppose we have already obtained the $m-th$ round Gibbs samples $\vec{Y}_m, \vec{B}_m, \vec{\sigma}_m, \Im_m \ (m = 1,2,3,...)$. Define:

$f_{\vec{B},\vec{\sigma},\Im,\vec{y}}\left(\vec{Y}\right) = f\left(\vec{Y} \mid \vec{B},\vec{\sigma},\Im,\vec{y}\right)$ as the CPDF of $f\left(\vec{Y},\vec{B},\vec{\sigma},\Im,\vec{y}\right)$ (Equation (16)),

$f_{\vec{Y},\vec{\sigma},\Im,\vec{y}}\left(\vec{B}\right) = f\left(\vec{B} \mid \vec{Y},\vec{\sigma},\Im,\vec{y}\right)$ as the CPDF of $f\left(\vec{Y},\vec{B},\vec{\sigma},\Im,\vec{y}\right)$,

$f_{\vec{Y},\vec{B},\Im,\vec{y}}\left(\vec{\sigma}\right) = f\left(\vec{\sigma} \mid \vec{Y},\vec{B},\Im,\vec{y}\right)$ as the CPDF of $f\left(\vec{Y},\vec{B},\vec{\sigma},\Im,\vec{y}\right)$, and

$f_{\vec{Y},B,\vec{\sigma},\vec{y}}\left(\Im\right) = f\left(\Im \mid \vec{Y},\vec{B},\vec{\sigma},\vec{y}\right)$ as the CPDF of $f\left(\vec{Y},\vec{B},\vec{\sigma},\Im,\vec{y}\right)$.

When taking the $m+1-th$ round of sampling, the following procedures are necessary, according to the transition kernel (18):

(1) Draw a sample $\vec{Y}_{m+1}$ from the CPDF $f_{\vec{B}_m,\vec{\sigma}_m,\Im_m,\vec{y}}\left(\vec{Y}_{m+1}\right)$.

(2) Draw a sample $\vec{B}_{m+1}$ from the CPDF $f_{\vec{Y}_{m+1},\vec{\sigma}_m,\Im_m,\vec{y}}\left(\vec{B}_{m+1}\right)$.

(3) Draw a sample $\vec{\sigma}_{m+1}$ from the CPDF $f_{\vec{Y}_{m+1},\vec{B}_{m+1},\Im_m,\vec{y}}\left(\vec{\sigma}_{m+1}\right)$.

(4) Draw a sample $\Im_{m+1}$ from the CPDF $f_{\vec{Y}_{m+1},\vec{B}_{m+1},\vec{\sigma}_{m+1},\vec{y}}\left(\Im_{m+1}\right)$.

However, the CPDFs of the above four steps are difficult to calculate. In order to find alternative ways to so, we determine the following relationships:

$$f_{\vec{B}_m,\vec{\sigma}_m,\Im_m,\vec{y}}\left(\vec{Y}_{m+1}\right) = \frac{f\left(\vec{Y}_{m+1},\vec{B}_m,\vec{\sigma}_m,\Im_m,\vec{y}\right)}{\int f\left(\vec{Y}_{m+1},\vec{B}_m,\vec{\sigma}_m,\Im_m,\vec{y}\right) d\vec{Y}_{m+1}} = c_{Ym} \bullet f\left(\vec{Y}_{m+1},\vec{B}_m,\vec{\sigma}_m,\Im_m,\vec{y}\right), c_{Ym} \in \mathbb{R}^+$$

$$f_{\vec{Y}_{m+1},\vec{\sigma}_m,\Im_m,\vec{y}}\left(\vec{B}_{m+1}\right) = \frac{f\left(\vec{Y}_{m+1},\vec{B}_{m+1},\vec{\sigma}_m,\Im_m,\vec{y}\right)}{\int f\left(\vec{Y}_{m+1},\vec{B}_{m+1},\vec{\sigma}_m,\Im_m,\vec{y}\right) d\vec{B}_{m+1}} = c_{Bm} \bullet f\left(\vec{Y}_{m+1},\vec{B}_{m+1},\vec{\sigma}_m,\Im_m,\vec{y}\right), c_{Bm} \in \mathbb{R}^+$$

$$f_{\vec{Y}_{m+1},\vec{B}_{m+1},\Im_m,\vec{y}}\left(\vec{\sigma}_{m+1}\right) = \frac{f\left(\vec{Y}_{m+1},\vec{B}_{m+1},\vec{\sigma}_{m+1},\Im_m,\vec{y}\right)}{\int f\left(\vec{Y}_{m+1},\vec{B}_{m+1},\vec{\sigma}_{m+1},\Im_m,\vec{y}\right) d\vec{\sigma}_{m+1}} = c_{\sigma m} \bullet f\left(\vec{Y}_{m+1},\vec{B}_{m+1},\vec{\sigma}_{m+1},\Im_m,\vec{y}\right), c_{\sigma m} \in \mathbb{R}^+$$

$$f_{\vec{Y}_{m+1},\vec{B}_{m+1},\vec{\sigma}_{m+1},\vec{y}}\left(\Im_{m+1}\right) = \frac{f\left(\vec{Y}_{m+1},\vec{B}_{m+1},\vec{\sigma}_{m+1},\Im_{m+1},\vec{y}\right)}{\int f\left(\vec{Y}_{m+1},\vec{B}_{m+1},\vec{\sigma}_{m+1},\Im_{m+1},\vec{y}\right) d\Im_{m+1}} = c_{\Im m} \bullet f\left(\vec{Y}_{m+1},\vec{B}_{m+1},\vec{\sigma}_{m+1},\Im_{m+1},\vec{y}\right), c_{\Im m} \in \mathbb{R}^+,$$

where $f(\ )$ denotes Equation (16). Based on the above four relationships, we can perform the following equivalent process:

(5) Extract the sample $\vec{Y}_{m+1}$ from PDF $f_1\left(\vec{Y}_{m+1}\right) = c_{Ym} \bullet f\left(\vec{Y}_{m+1},\vec{B}_m,\vec{\sigma}_m,\Im_m,\vec{y}\right)$ ($f(\ )$ here is Equation (16)).

(6) Extract the sample $\vec{B}_{m+1}$ from PDF $f_2\left(\vec{B}_{m+1}\right) = c_{Bm} \bullet f\left(\vec{Y}_{m+1},\vec{B}_{m+1},\vec{\sigma}_m,\Im_m,\vec{y}\right)$.

(7) Extract the sample $\vec{\sigma}_{m+1}$ from PDF $f_3\left(\vec{\sigma}_{m+1}\right) = c_{\sigma m} \bullet f\left(\vec{Y}_{m+1},\vec{B}_{m+1},\vec{\sigma}_{m+1},\Im_m,\vec{y}\right)$.

(8) Extract the sample $\Im_{m+1}$ from PDF $f_4\left(\Im_{m+1}\right) = c_{\Im m} \bullet f\left(\vec{Y}_{m+1},\vec{B}_{m+1},\vec{\sigma}_{m+1},\Im_{m+1},\vec{y}\right)$.

Let $\vec{Y}_0, \vec{B}_0, \vec{\sigma}_0, \Im_0$ be the initial sample, set artificially using reasonable statistics. Sampling along the process (5) → (6) → (7) → (8) → (5) → (6) ⋯ stated above, the samples gradually converge to the stationary distribution $f\left(\vec{Y}, \vec{B}, \vec{\sigma}, \Im \mid \vec{y}\right)$. After extensive rounds of cycling, Equation (17) can be used to estimate $\Im$. Because the CPDFs in the above four procedures are complex, the best way to solve the problem is to use the accept–rejection method (see Glasserman, 2004) to produce the Monte Carlo sampling.

3.3. MCMC-MH Method for Interest Rate Process

The differential expression of $r(t)$ in Equation (10) is changed into discrete form, as follows:

$$\begin{aligned} &dr(t) = \kappa_r(\overline{r} - r(t))dt + \sigma_r \sqrt{r(t)} dW_r(t) \\ &y(t) = r(t+\Delta t) - r(t) = \Delta r(t) = \kappa_r(\overline{r} - r(t))\Delta t + \sigma_r \sqrt{r(t)} W_r(\Delta t). \\ &\left(t = 0,1,2,\ldots,M-1\right) \end{aligned} \tag{19}$$

In Equation (19), we assume that $t = 0,1,\ldots,M-1, (\Delta t = 1)$ denotes M days (assume that we have already collected M days of interest rates). Then, $\vec{r} = \left(r(0), r(1), \ldots, r(M-1)\right)$ is a vector of daily interest rates, $\vec{y} = \left(y(0), y(1), \ldots, y(M-1)\right)$ is an observable vector of daily $\Delta r(t)$, and $\Im = \left\{\kappa_r, \overline{r}, \sigma_r\right\}$ is the parameter set. We further assume that the parameters in $\Im$ have mutually independent prior distributions. Thus, the PDF of $\Im$ is $\pi(\Im) = \pi(\kappa_r)\pi(\overline{r})\pi(\sigma_r)$. Based on other relevant studies (Yu et al., 2011; Hu et al., 2006; Lu and Hua, 2010; Han et al., 2010; Yang et al., 2010), the prior distributions of the parameters are set as

$$\kappa_r \sim Beta(\hat{\alpha}_1, \hat{\beta}_1), \overline{r} \sim N(\hat{u}_1, \hat{\sigma}_1^2), \sigma_r \sim \ln N(\hat{u}_2, \hat{\sigma}_2^2), \tag{20}$$

where $\hat{\alpha}_1, \hat{\beta}_1, \hat{\mu}_1, \hat{\sigma}_1, \hat{\mu}_2, \hat{\sigma}_2$ are the parameters of the prior distributions, which are set artificially based on the statistical results and experience. The prior distributions are shown in Table 1.

Similarly to the proof in Appendix B.1, we can obtain the CPDF of a single sample as follows:

$$f\left(y(t) \mid r(t), \Im\right) = \frac{1}{\sqrt{2\pi}\sigma_r \sqrt{r(t)\Delta t}} e^{-\frac{\left(y(t) - \kappa_r(\overline{r} - r(t))\Delta t\right)^2}{2\sigma_r^2 r(t)\Delta t}}. \tag{21}$$

Because the joint random variables $\left(\vec{y}, \Im\right), t = 0,1,\ldots,M-1$ are independent of each other, conditioned on $\vec{r}$, from Equations (20) and (21), we obtain the CPDF of the joint random variables $\left(\vec{y}, \Im\right)$, as follows:

$$\begin{aligned} f\left(\Im, \vec{y} \mid \vec{r}\right) &= \pi(\Im) f\left(\vec{y} \mid \vec{r}, \Im\right) = \pi(\Im) \prod_{t=0}^{M-1} f\left(y(t) \mid r(t), \Im\right) \\ &= \pi(\Im) \frac{1}{\left(2\pi \prod_{t=0}^{M-1} r(t)\right)^{\frac{M}{2}} \sigma_r^{M}} e^{-\sum_{t=0}^{M-1} \frac{\left(y(t) - \kappa_r(\overline{r} - r(t))\right)^2}{2\sigma_r^2 r(t)}} \end{aligned} \tag{22}$$

$$f\left(\Im \mid \vec{y}, \vec{r}\right) = \frac{f\left(\Im, \vec{y} \mid \vec{r}\right)}{\int f\left(\Im, \vec{y} \mid \vec{r}\right) d\Im} \propto f\left(\Im, \vec{y} \mid \vec{r}\right). \tag{23}$$

After the derivation of the important Equations (22) and (23), we need to use MCMC Metropolis–Hastings sampling (Martino et al., 2015; Meyer et al., 2008; Gilks et al., 1995) to obtain the sample needed for the calculation of the model parameters $\Im$.

The sampling process is a Markov chain $\aleph$ with discrete time and continuous state. Then, $\Im_m = \left(\kappa_{m,r}, \overline{r}_m, \sigma_{m,r}\right)$ is the sample from the $m-th, m=1,2,3...$ round of sampling, and $\Im_{m+1} = \left(\kappa_{m+1,r}, \overline{r}_{m+1}, \sigma_{m+1,r}\right)$ is the sample from the $m+1-th$ round of sampling. Choose $p(\Im_{m+1} \mid \Im_m, \vec{y}, \vec{r}) = \pi(\Im_{m+1})\alpha(\Im_{m+1} \mid \Im_m, \vec{y}, \vec{r})$ as the transition kernel, where:

$$\alpha(\Im_{m+1} \mid \Im_m, \vec{y}, \vec{r}) = \min\left\{1, \frac{f\left(\Im_{m+1} \mid \vec{y}, \vec{r}\right)\pi(\Im_m)}{f\left(\Im_m \mid \vec{y}, \vec{r}\right)\pi(\Im_{m+1})}\right\}, m = 1,2,3\ldots.$$

Here, $f\left(\Im_m \mid \vec{y}, \vec{r}\right)$ is Equation (23). The Markov chain $\aleph$ formed by the transition kernel has a unique stationary distribution, and its PDF is $f\left(\Im \mid \vec{y}, \vec{r}\right)$ (see Appendix B.3 for the proof).

Let $\Im_0 = \left(\kappa_{0,r}, \overline{r}_0, \sigma_{0,r}\right)$ be the initial sample, set artificially using reasonable statistics. Based on the transition kernel, we employ the following sampling process:

(1) Draw a sample $\Im_m = \left(\kappa_{m,r}, \overline{r}_m, \sigma_{m,r}\right), m = 1,2,3,...,k,k+1,...,N$ from $\pi(\Im_m)$.

(2) Accept $\Im_m$, with probability $\alpha(\Im_m \mid \Im_{m-1}, \vec{y}, \vec{r})$.

(3) If $\Im_m$ is rejected, repeat (1)→(2) until $\Im_m$ is accepted.

(4) If $\Im_m$ is accepted and m does not reach N, repeat the above steps.

Note that $k\left(k \gg 1\right)$ is a sufficiently large number, beyond which the samples $\Im_m\left(m \geq k\right)$ can be regarded as being drawn from a stationary distribution, with CPDF $f\left(\Im \mid \vec{y}, \vec{r}\right)$, of the Markov chain $\aleph$.

In addition, $N\left(N \gg k\right)$ is the maximum number of sampling rounds, as set by the researcher. The greater the value of N, the larger the computing capacity that is required. After N rounds of sampling, the estimated parameters are equal to:

$$\Im = \frac{\sum_{m=k}^{N} \Im_m}{N-k}.$$

4. Empirical Analysis

In this section, we present the empirical results from the proposed option pricing model (the option pricing model under the double exponential jump-diffusion settings with SV and SIR). First we conduct time efficiency analysis, goodness of fit analysis, and jump/drift term analysis of the proposed model. Then, we compare the pricing accuracy of the proposed model to those of the BS and Kou (2002) models (alternative double exponential jump-diffusion option pricing models). In order to evaluate the option pricing accuracy, we select 10 50ETF European options from the Shanghai Stock Exchange (SSE), expiring in June 2016, including five call options and five put options with different strike prices. In addition, we collect samples of option prices from 63 trading days between 1 February 2016 and 6 May 2016.

To determine the theoretical option price of the proposed model on each trading day between 1 February 2016, and 6 May 2016, we need to use the MCMC-LV method to estimate the parameters of the double exponential jump-diffusion model with SV and the SIR. We collect 572 days (before 6 May 2016) of 50ETF closing prices and O/N SHIBOR rates, which are used as the training samples of the MCMC-LV method. On each trading day, the parameters of the proposed model are set equal to the average values of 2000 parameter samples from the MCMC-LV method, the training samples of which are the 50ETF prices and the O/N SHIBOR rates for the previous 500 trading days. Table C1 (see Appendix C) shows the results of the estimated parameters for the 63-day period using the MCMC-LV method.

As shown in Table C1, each trading day has a unique set of parameters. The training samples for the parameters on each day are the 50ETF closing prices and the O/N SHIBOR rates over the previous 500 trading days. Therefore, the parameters for the different trading days are similar, but not the same. With regard to the parameters of the 50ETF closing prices, the drift rate μ denotes the ability to achieve long-term stable returns. Because the training samples are all taken from a bear market (although the 500-day sampling period is not long enough to extend beyond the bear market), the drift parameters on the 63 trading days are barely positive. Here, λ denotes the intensity of the double exponential jumps, and η_1, and η_2 denote the amplitudes of the positive jump and negative jump, respectively. The greater the values of the parameters, the larger the jump amplitude will be. Then, p is the probability of positive jumps, κ_σ denotes the mean-reverting rates rate, $\bar{\sigma}$ is the long term volatility, σ_σ is the volatility of volatility, and $\sigma(1)$ is the initial volatility (a latent variable of MCMC-LV method). With regard to the O/N SHIBOR rates, κ_r is the mean-reverting rate of O/N SHIBOR rates, $\bar{r}$ is the long-term mean, and σ_r is the variation coefficient.

Table 2. Average Time Required for a Single Sample of Parameters under MCMC-LV

Length of Learning Sample	Time * Required for a Single Sample of Parameters of 50ETF Model (s)	Time * Required for a Single Sample of Parameters of O/N SHIBOR Model (s)
500	5.741	3.81E-04
450	4.982	3.86E-04
400	4.791	3.59E-04
350	4.374	4.43E-04
300	4.272	4.57E-04
250	3.914	5.79E-04
200	3.307	3.97E-04
150	2.814	4.05E-04
100	2.589	3.61E-04
50	2.2143	4.15E-04

Note: * 100 times average.

MCMC-LV sampling is conducted using Matlab 2015b on a personal computer configured as follows: Intel(R) Core(TM) i7-4500U CPU @ 1.80GHz 2.40GHz, 8.00 GB RAM, and 64 bit Windows 10 operating system. Table 2 and Figure 1 show that far more time is required to determine a single sample of parameters for the underlying asset model than is the case in the interest rate model. For example, it takes approximately 5.741 seconds, on average, to draw a sample of parameters for the 50ETF model using 500 training samples (500 days of 50EFT returns), but only takes about 3.81E-04 seconds to draw a sample of parameters for the interest rate model. This is because the 50ETF model combines the double-exponential jump, CIR volatility, and the CIR interest rate, and has nine parameters that need to be estimated. In comparison, the interest rate model is related to only one CIR process, and has just three parameters that need to be estimated. This is why we do not adopt the mixed-exponential jump (Cai and Kou, 2011) and the hyper-exponential jump (Cai and Kou, 2012), because too many parameters can lead to a heavy computational burden.

In Figure 1, the time required for a single sample of parameters of the 50ETF model decreases linearly with the number of training samples (days of 50EFT returns). However, the requirement for a single sample of parameters for the interest rate model displays quite a volatile relationship with the number of training samples, in an order of magnitude of 10^{-4}.

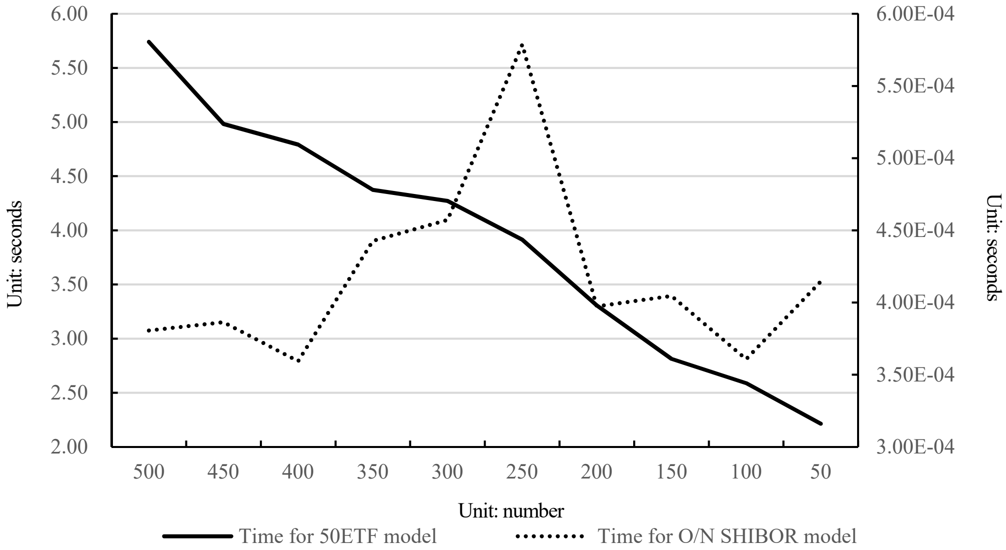

Figure 1. Average Time Required for a Single Sample of Parameters under MCMC-LV

The option pricing is calculated using Matlab 2015b on a personal computer configured as follows: Intel(R) Core(TM) i7-4500U CPU @ 1.80GHz 2.40GHz, 8.00 GB RAM, and a 64 bit Windows 10 operating system. Table C2 (See Appendix C) shows the estimated European option prices using the double exponential jump-diffusion option pricing model with SV and SIR (i.e., Equations (9) and (10)) using the estimated parameters in Table C1.

To verify the effectiveness of the proposed model, the BS and Kou models are employed for a price comparison. Table C3 (See Appendix C) shows the estimated option prices from the BS model, using a moment estimation to obtain the parameters. Table C4 (See Appendix C) shows the estimated option prices from the Kou model, applying the MCMC-LV model for the parameter estimation.

In addition, Table 3 and Figure 2 show the average time required to determine the call and put option prices under the proposed model. Fast results can be achieved by reducing the number of sampling points in the numerical integration. The pricing of a put option is marginally faster than that of a call option. In addition, as the number of sampling points increases (below 10240 points), the computational time increases slowly. However, when the number of sampling points rises above 10240 points, the computational time increases rapidly.

Table 3. Average Computational Time of Call/Put Option Price

Number of Sampling Points * in Numerical Integration	The Natural Logarithm of the Number of Sampling Points	Average ** Computational Time of Call Option	Average ** Computational Time of Put Option
20	3.00	6.56E-03	6.13E-03
40	3.69	7.40E-03	7.00E-03
80	4.38	7.67E-03	7.51E-03
160	5.08	8.53E-03	8.34E-03
320	5.77	8.27E-03	8.04E-03
640	6.46	1.36E-02	1.37E-02
1280	7.15	2.02E-02	1.89E-02
2560	7.85	2.63E-02	2.59E-02
5120	8.54	4.28E-02	4.10E-02
10240	9.23	4.59E-02	4.34E-02
20480	9.93	1.81E-01	1.69E-01
40960	10.62	2.41E-01	2.28E-01

Notes: * The pricing error tolerance is below 10E-14 in all cases. ** 400 times average.

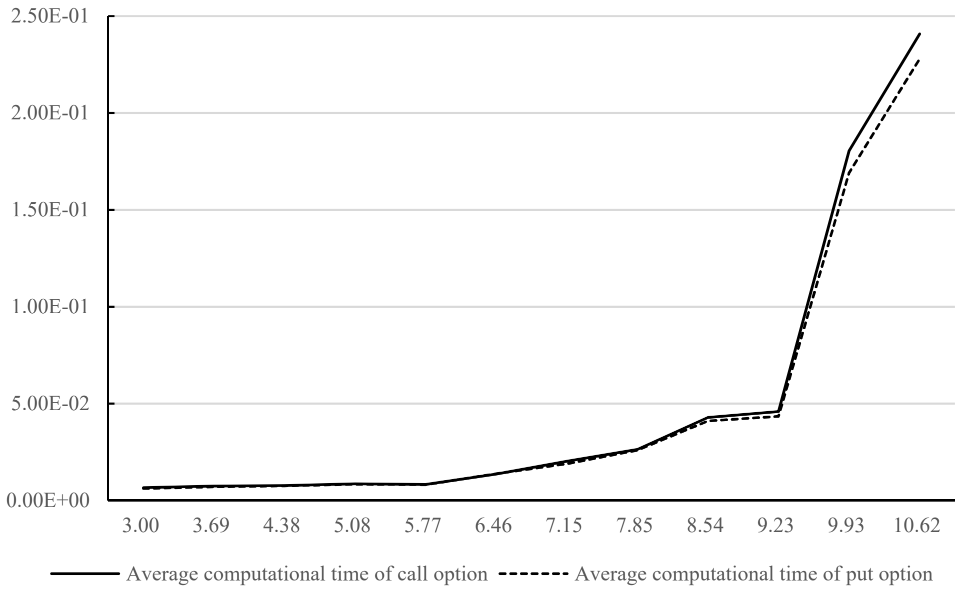

Figure 2. Average Computational Time of Call/Put Option Price

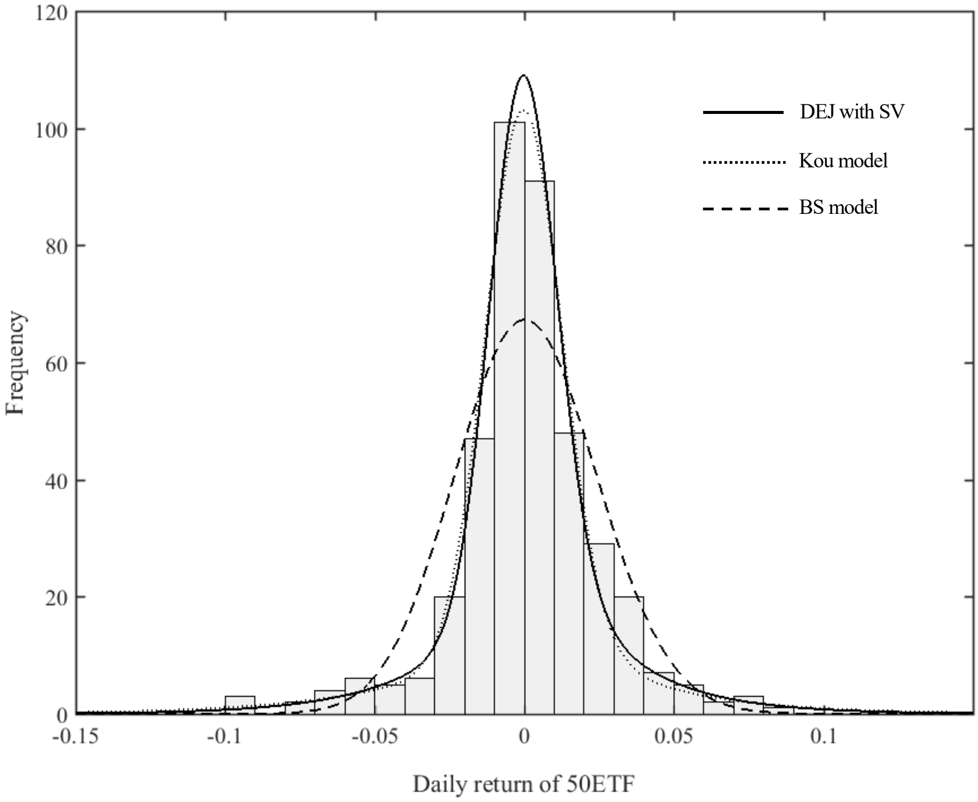

Figure 3. The Goodness of Fit of the Proposed Double Exponential Jump Model with Stochastic Volatility, Kou Model and BS Model to 50ETF Daily Return

Figure 3 compares the goodness of fit of the proposed model, the Kou model and the BS model for 50ETF daily returns. The proposed model clearly fits the 50ETF daily returns best. The only difference between the proposed model and the Kou model in terms of 50ETF daily returns is the SV. The SV clearly contributes to a higher peak and a fatter tail. Moreover, the only difference between the Kou model and the BS model is the double-exponential jump, which clearly contributes to the higher peak and the fatter tail of the Kou model.

In order to compare the goodness of fit quantitatively, a K–S test is carried out. Let $F_n(x)$ be the empirical function of the sample of 50ETF daily returns. Let $F(x)$, $F_{Kou}(x)$, and $F_{BS}(x)$ be the cumulative probability distribution functions of the proposed model, BS model, and Kou model, respectively. Then, the null hypotheses of the three models are as follows:

The proposed model: $H_0 = \{F_n(x) \text{ follows } F(x)\}$

The Kou model: $H_0^{Kou} = \{F_n(x) \text{ follows } F_{Kou}(x)\}$

The BS model: $H_0^{BS} = \{F_n(x) \text{ follows } F_{BS}(x)\}$

The confidence level is $\alpha = 0.05$. The K–S test results of the three models are shown in Table 4.

Table 4. K-S Test Results of the Three Models

Empirical Function	Distribution	K Statistic	K_α (Critical Value)	$P-Value$
	$F(x)$	0.8487	1.3581	0.4673
SSE 50ETF return Empirical function $F_n(x)$	$F_{Kou}(x)$	1.0229	1.3581	0.2462
	$F_{BS}(x)$	2.0916	1.3581	3.1713E-04

It is obvious that the K -statistics of $F(x)$ and $F_{Kou}(x)$ both stay below the critical value, and that the p-values of $F(x)$ and $F_{Kou}(x)$ are above the confidence interval. Thus, $H_0^{SVI-AJD}$ and H_0^{Kou} cannot be rejected. While the K statistic of $F(x)$ is smaller than that of $F_{Kou}(x)$, the p-value of $F(x)$ is bigger than that of $F_{Kou}(x)$, which indicates that the goodness of fit of $F(x)$ is much better than that of $F_{Kou}(x)$.

On the other hand, the K -statistic of $F_{BS}(x)$ is much bigger than the critical value, with a p-value that is much smaller than 0.05. Therefore, we reject the null hypothesis H_0^{BS}, where $F_n(x)$ does not follow $F_{BS}(x)$. The above experimental results show that the proposed model has the highest goodness of fit, while the next best is the Kou model, followed by the BS model.

In order to explore the contribution of the jump term and the drift term to asset price fluctuations, it is crucial to calculate the proportion of the drift/jump term accounts for the 50ETF daily return. From Equation (11), the daily return of 50ETF can be written in discrete form:

$$y(t) = \left(\mu + \frac{1}{2}\sigma(t)\right)\Delta t + \sqrt{\sigma(t)}W(\Delta t) + Y_t B_t \,,$$

where $Y_t B_t$ is the jump term, and the rest is the drift term. There are 500 points in each of Figures 4 and 5. Each point in the two figures denotes a specific day in the 500-day period before 6 May 2016. The vertical axis of Figure 4 is $\left(\mu + \frac{1}{2}\sigma(t)\right)\Delta t + \sqrt{\sigma(t)}W(\Delta t) \Big/ y(t)$, and the vertical axis of Figure 5 is $Y_t B_t \big/ y(t)$. According to Figures 4 and 5, despite some anomalies around zero, a greater absolute value for the return implies a smaller proportion of the drift term and a larger proportion of the jump term. Thus, the jump term is good at explaining significant fluctuations in 50ETF prices, while the drift term is good at explaining small fluctuations in 50ETF prices.

In order to compare the accuracy of the pricing model, we propose using $\frac{MSE}{VAR}$, MRE, and $\frac{ME}{STD}$ to measure the discrepancies between the estimated option price and the real closing price. The definitions of the three indices are shown in Table 5.

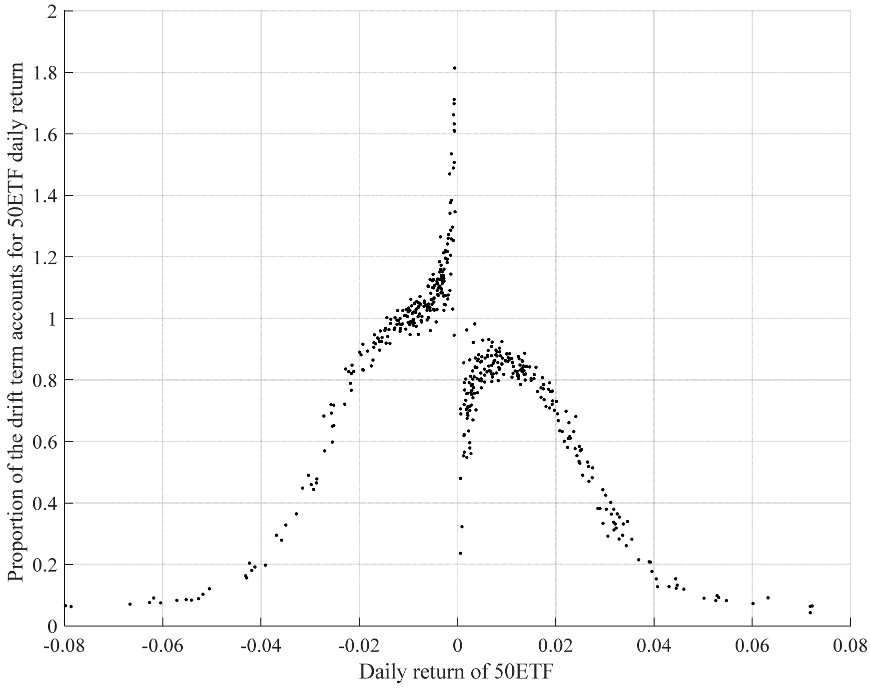

Figure 4. Proportion of the Drift Term Accounts for 50ETF Daily Return

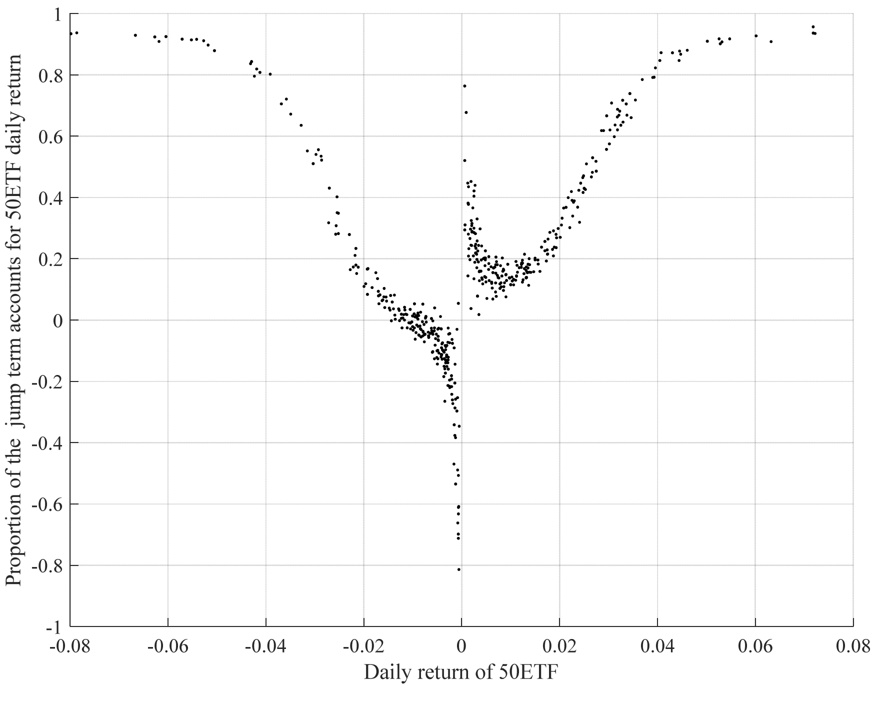

Figure 5. Proportion of the Jump Term Accounts for 50ETF Daily Return

Table 5. The Definition of $\frac{MSE}{VAR}$, MRE and $\frac{ME}{STD}$

Index	Definition *	Explanation
$\frac{MSE}{VAR}$	$\frac{\frac{1}{n}\sum_{i=1}^{n}\left(real\ price_i - estimated\ price_i\right)^2}{var\ of\ real\ prices}, i = 1,2,\ldots,n$	The ratio of the mean square error between the price estimated by the option pricing model and the real price to the variance of the actual price sample.
MRE	$\frac{1}{n}\sum_{i=1}^{n}\frac{\left\|real\ price_i - estimated\ price_i\right\|}{real\ price_i}, i = 1,2,\ldots,n$	The mean relative error between the estimated price from the option pricing model and the real price.
$\frac{ME}{STD}$	$\frac{\frac{1}{n}\sum_{i=1}^{n}\left\|real\ price_i - estimated\ price_i\right\|}{std\ of\ real\ prices}, i = 1,2,\ldots,n$	The ratio of the absolute error between the estimated price by option pricing model and the real price and the standard deviation of the actual price.

Notes: * $real\ price_i$ is the real option price on the i th trading day, $estimated\ price_i$ is the estimated option price by option pricing model on trading day i, n is the number of trading days (In this section $n = 63$). $var\ of\ real\ prices$ is the variance of real option price, $std\ of\ real\ prices$ is the standard deviation of real option price.

Each of these three indicators can be compared horizontally and vertically. In a horizontal comparison, we compare the pricing accuracy of different option contracts under the same model, while in a vertical comparison, we compare the pricing accuracy of different models under the same option contract. The comparison results are shown in Table 6.

Table 6. The Empirical Test of $\frac{MSE}{VAR}$, MRE, and $\frac{ME}{STD}$

Model Type	Error Type	50ETF/ Call/June/ 1.8 *	50ETF/ Call/June/ 2.0	50ETF/ Call/June/ 2.2	50ETF/ Call/June/ 2.4	50ETF/ Call/June/ 2.6	50ETF/Put /June/ 1.8 **	50ETF/ Put/June/ 2.0	50ETF/ Put/June/ 2.2	50ETF/ Put/June/ 2.4	50ETF/ Put/June/ 2.6	Mean
The proposed model	MSE/VAR	48.99%	108.94%	161.83%	124.83%	101.10%	35.11%	27.03%	27.55%	33.25%	40.47%	**70.91%**
	MRE	15.41%	24.12%	42.83%	73.92%	63.77%	42.61%	34. 41%	16.44%	10.60%	9.07%	**33.32%**
	MAE/STD	61.71%	93.18%	112.65%	95.48%	76.43%	43.10%	42.21%	42.92%	45.36%	52.91%	**66.60%**
BS	MSE/VAR	74.27%	201.50%	363.75%	358.89%	284.85%	13.46%	15.78%	18.13%	24.08%	34.05%	**138.88%**
	MRE	19.64%	34.99%	66.20%	125.58%	142.08%	52.74%	38.98%	15.32%	8.78%	8.00%	**51.23%**
	MAE/STD	78.37%	135.48%	181.02%	177.12%	154.66%	26.90%	33.21%	35.77%	37.78%	47.19%	**90.75%**
Kou	MSE/VAR	359.88%	1231.63%	2895.27%	4740.74%	8075.94%	89.90%	45.64%	23.82%	12.59%	19.26%	**1749.47%**
	MRE	44.71%	89.02%	186.49%	418.04%	708.90%	262.11%	100.29%	26.09%	9.51%	5.46%	**185.06%**
	MAE/STD	179.65%	344.83%	533.59%	680.24%	876.28%	93.08%	61.40%	39.81%	33.14%	32.84%	**287.50%**

Notes: * For the SSE 50ETF call option expiring in June 2016, the expiration date is the fourth Wednesday in June, and the strike price is 1.8. ** For the 50ETF put option expiring in June 2016, the expiration date is the fourth Wednesday in June, and the strike price is 1.8. The dashed boxes show the minimum values of each type of error of corresponding to an option contract.

The mean $\frac{MSE}{VAR}$, mean MRE, and mean $\frac{ME}{STD}$ under the proposed option pricing model are 70.91%, 33.32%, and 66.60%, respectively, each of which is the smallest of the three models. Therefore, the proposed option pricing model has the highest average accuracy in terms of pricing all SSE 50ETF options expiring in June 2016. For all SSE

50ETF call options expiring in June 2016, the proposed option pricing model (i.e., the option pricing model under the double exponential jump-diffusion settings with SV and SIR) has the smallest pricing error for each error type.

Compared with the Kou model, it seems that the more the put option is out-of-the-money, the higher is the accuracy of the proposed option pricing model. The pricing accuracy of "50ETF/Put/June/1.8" and "50ETF/Put/June/2.0" under the proposed option pricing model is higher than that of the Kou model, while the pricing accuracy of "50ETF/Put/June/2.4" and "50ETF/Put/June/2.6" under the Kou model is higher than that of proposed model and the BS model.

The mean $\frac{MSE}{VAR}$, mean MRE, and mean $\frac{ME}{STD}$ of the Kou model are the largest among the three models, increasing to 1749.47%, 185.06%, and 287.50% respectively, partly owing to large errors in the pricing of call options.

5. Conclusions

We propose an efficient options pricing model that incorporates the SIR, SV, and the double exponential jump into the jump-diffusion settings. The model comprehensively considers the leptokurtosis and heteroscedasticity of the return distribution, rare events, and the SIR. We determine the pricing characteristic function and pricing formula of a European option. We develop the MCMC-LV method to estimate the parameters of the double exponential jump diffusion model with SIR and SV. From our time efficiency analysis, the MCMC-LV for 50ETF constitutes the main time consumption, while the MCMC-LV method for interest rates is very fast. The reason is that the 50ETF model combines a double-exponential jump, CIR volatility, and the CIR interest rate, and has nine parameters to be estimated. In comparison, the interest rate model is related to only one CIR process, and has only three parameters that are needed to be estimated.

The goodness of fit test shows that the proposed model best fits the 50ETF daily return. It is clear that the double-exponential jump and SV contribute to a higher peak and fatter tails. The jump/drift term analysis shows that the jump term is good at explaining great fluctuations in 50ETF prices, and that the drift term is good at explaining small fluctuations in 50ETF prices.

We also compared the pricing accuracy between the proposed model and the BS and the Kou (2002) models using real market data. We selected 10 50ETF European options from the Shanghai Stock Exchange (SSE), expiring in June 2016, as a sample, including five call options and five put options with different strike prices. Then, we calculated the theoretical prices of all 50ETF call and put options expiring in June 2016. The empirical results show that the average pricing accuracy of our proposed model is much higher than that of the BS and Kou models, especially for call options. With regard to our model, fast pricing results can be achieved by reducing the number of sampling points in the numerical integration. Our model suggests that it is more efficient to keep the number of sampling points below 10240 during option pricing, because when the number of sampling points increases beyond 10240 points, the computational time increases rapidly.

The proposed option pricing model has several limitations in dealing with path-dependent derivatives and American options. One suggested way is to treat T in $\psi^{\chi}(\mu,\nu,\Upsilon(t),t,T)$ as the stopping time. This method requires further investigation.

Acknowledgments: This work is partially supported by the grants from the National Natural Science Foundation of China (NSFC No. 71471161), the Key Programs of the National Natural Science Foundation of China (NSFC Nos. 71631005 and 71433001), the National Natural Science Foundation of China (NSFC No. 71703142), and Zhejiang College Students' Science Innovation Project (Xin Miao Project) on "Research on Integrated Risk Measurement of Structured Financial Products Based on Affine Jump Diffusion Process" (No. 2016R414069).

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A. Derivation of the Characteristic Function

From Equation (3):

$\chi = (U_0, U_1, \Sigma_0, \Sigma_M, \rho_1)$.

$\psi^{\chi}(\mu, \Upsilon(t), t, T) = \widetilde{\mathbb{E}}\left(e^{-\int_t^T r(s)ds} e^{\mu^T \Upsilon(T)} \,|\mathcal{F}(t) \right)$, where $\mu = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{bmatrix} \in \mathbb{R}^3$

Let $\psi^{\chi}(\mu, \Upsilon(t), t, T) = \exp\left(\alpha(\mu,t,T) + \beta(\mu,t,T)^T \Upsilon(t)\right)$, where $\beta(\mu,t,T) = \begin{bmatrix} \beta^1(\mu,t,T) \\ \beta^2(\mu,t,T) \\ \beta^3(\mu,t,T) \end{bmatrix}$.

Then, applying the Feynman–Kac Theorem to the characteristic function $\psi^{\chi}(\mu, \Upsilon(t), t, T)$, the differential of the discounted characteristic function is

$$
\begin{aligned}
& d\left(D(t)\psi^{\chi}(\mu, \Upsilon(t), t, T)\right) \\
& = \left(\alpha_t(\mu,t,T) + \beta_t(\mu,t,T)^T \Upsilon(t-)\right) D(t)\psi^{\chi}(\mu, \Upsilon(t-), t, T)dt - r(t)D(t)\psi^{\chi}(\mu, \Upsilon(t-), t-, T)dt + \\
& \beta(\mu,t,T)^T D(t)\psi^{\chi}(\mu, \Upsilon(t-), t, T)d\Upsilon(t-) + \frac{1}{2}\beta(\mu,t,T)^T \beta(\mu,t,T)^T D(t)\psi^{\chi}(\mu, \Upsilon(t-), t-, T)d\Upsilon(t-)d\Upsilon(t-) + . \\
& \left[e^{\beta(\mu,t,T)^T \Delta\Upsilon(t)} - 1\right] D(t)\psi^{\chi}(\mu, \Upsilon(t-), t, T)dJ(t) - \lambda E\left[e^{\beta(\mu,t,T)^T \Delta\Upsilon(t)} - 1\right] D(t)\psi^{\chi}(\mu, \Upsilon(t-), t, T)dt + \\
& \lambda E\left[e^{\beta(\mu,t,T)^T \Delta\Upsilon(t)} - 1\right] D(t)\psi^{\chi}(\mu, \Upsilon(t-), t, T)dt
\end{aligned}
$$

Under the risk-neutral probability measure $\widetilde{\mathbb{P}}$, the discounted characteristic function $D(t)\psi^{\chi}(\mu, \Upsilon(t), t, T) = \widetilde{\mathbb{E}}\left(D(T)\psi^{\chi}(\mu, \Upsilon(t), T, T) \,|\mathcal{F}(t) \right)$ is martingale, thus, the coefficient of dt is 0.

Then, we have

$$
\begin{aligned}
& \alpha_t(\mu,t,T) - \lambda\overline{Y}\beta^1(\mu,t,T) + \kappa_r \overline{r}\beta^2(\mu,t,T) + \kappa_\sigma \overline{\sigma}\beta^3(\mu,t,T) + \lambda E\left[e^{\beta^1(\mu,t,T)\nu_t} - 1\right] + \\
& \left[\frac{1}{2}\left(\beta^2(\mu,t,T)\right)^2 \sigma_r{}^2 + \beta_t^2(\mu,t,T) - 1 + \beta^1(\mu,t,T) - \kappa_r\beta^2(\mu,t,T)\right] r(t) + \\
& \beta_t^1(\mu,t,T)X(t-) + \left[\frac{1}{2}\beta^3(\mu,t,T)^2\sigma_\sigma{}^2 + \beta_t^3(\mu,t,T) - \frac{1}{2}\beta^1(\mu,t,T) - \kappa_\sigma\beta^3(\mu,t,T) + \frac{1}{2}\beta^1(\mu,t,T)^2\right]\sigma(t) = 0
\end{aligned}
.
$$

Determining the probability density function of ν, we need to calculate $\lambda E\left[e^{\beta^1(\mu,t,T)\nu_t} - 1\right]$:

$$
f(\nu) = p \cdot \eta_1 e^{-\eta_1\nu} 1_{\{\nu \geq 0\}} + q \cdot \eta_2 e^{\eta_2\nu} 1_{\{\nu < 0\}}, (\eta_1 > 1, \eta_2 > 0)
$$

$$
\lambda E\left[e^{\beta^1(\mu,t,T)\nu_t} - 1\right] = \mathbb{E}\,\lambda\left(e^{\mu_1\nu_t} - 1\right) = \lambda\int_{-\infty}^{0} q\eta_2 e^{(\eta_2+\mu_1)\nu}d\nu + \lambda\int_{0}^{\infty} p\eta_1 e^{(\mu_1-\eta_1)\nu}d\nu - \lambda
$$

$$= \lambda \frac{q\eta_2}{\eta_2+\mu_1}\int_{-\infty}^{0} e^{(\eta_2+\mu_1)v} d(\eta_2+\mu_1)v + \lambda \frac{p\eta_1}{\mu_1-\eta_1}\int_{0}^{\infty} e^{(\mu_1-\eta_1)v} d(\mu_1-\eta_1)v - \lambda$$
$$= \lambda\left(\frac{q\eta_2}{\eta_2+\mu_1} - \frac{p\eta_1}{\mu_1-\eta_1} - 1\right)(\mu_1 < \eta_1).$$

The ODEs can be written as:

$$\alpha_t(\mu,t,T) - \lambda\bar{Y}\beta^1(\mu,t,T) + \kappa_r\bar{r}\beta^2(\mu,t,T) + \kappa_\sigma\bar{\sigma}\beta^3(\mu,t,T) + \lambda\left(\frac{q\eta_2}{\eta_2+\mu_1} - \frac{p\eta_1}{\mu_1-\eta_1} - 1\right) = 0$$
$$\frac{1}{2}\left(\beta^2(\mu,t,T)\right)^2 {\sigma_r}^2 + \beta_t^2(\mu,t,T) - 1 + \beta^1(\mu,t,T) - \kappa_r\beta^2(\mu,t,T) = 0 ,$$
$$\frac{1}{2}\beta^3(\mu,t,T)^2 {\sigma_\sigma}^2 + \beta_t^3(\mu,t,T) - \frac{1}{2}\beta^1(\mu,t,T) - \kappa_\sigma\beta^3(\mu,t,T) + \frac{1}{2}\beta^1(\mu,t,T)^2 = 0$$

with boundary conditions:

$$\begin{aligned} \beta^1(\mu,t,T) &= \mu_1 \\ \beta^2(\mu,T,T) &= \mu_2 \\ \beta^3(\mu,T,T) &= \mu_3 \\ \alpha(\mu,T,T) &= 0 \end{aligned}.$$

The solutions are:

$$\beta^1(\mu,t,T) = \mu_1$$
$$\beta^3(\mu,t,T) = \frac{\tan\left(\frac{\sqrt{{\sigma_\sigma}^2\mu_1(\mu_1-1)-{\kappa_\sigma}^2}(T-t)}{2} + \arctan\left(\frac{{\sigma_\sigma}^2\mu_3-\kappa_\sigma}{\sqrt{{\sigma_\sigma}^2\mu_1(\mu_1-1)-{\kappa_\sigma}^2}}\right)\right)\sqrt{{\sigma_\sigma}^2\mu_1(\mu_1-1)-{\kappa_\sigma}^2} + \kappa_\sigma}{{\sigma_\sigma}^2}$$
$$\beta^2(\mu,t,T) = \frac{\tan\left(\frac{\sqrt{2{\sigma_r}^2(\mu_1-1)-{\kappa_r}^2}(T-t)}{2} + \arctan\left(\frac{{\sigma_r}^2\mu_2-\kappa_r}{\sqrt{2{\sigma_r}^2(\mu_1-1)-{\kappa_r}^2}}\right)\right)\sqrt{2{\sigma_r}^2(\mu_1-1)-{\kappa_r}^2} + \kappa_r}{{\sigma_r}^2}.$$
$$\alpha(\mu,t,T) = \lambda\bar{Y}\mu_1(t-T) - \lambda\left(\frac{q\eta_2}{\eta_2+\mu_1} - \frac{p\eta_1}{\mu_1-\eta_1}\right)(t-T) - \left(\frac{{\kappa_r}^2\bar{r}}{{\sigma_r}^2} + \frac{{\kappa_\sigma}^2\bar{\sigma}}{{\sigma_\sigma}^2}\right)(t-T) -$$
$$\frac{\kappa_r\bar{r}}{{\sigma_r}^2} 2\ln\left|\cos\left(\frac{\sqrt{2{\sigma_r}^2(\mu_1-1)-{\kappa_r}^2}(T-t)}{2}\right) - \sin\left(\frac{\sqrt{2{\sigma_r}^2(\mu_1-1)-{\kappa_r}^2}(T-t)}{2}\right)\frac{{\sigma_r}^2\mu_r-\kappa_r}{\sqrt{2{\sigma_r}^2(\mu_1-1)-{\kappa_r}^2}}\right| -$$
$$\frac{\kappa_\sigma\bar{\sigma}}{{\sigma_\sigma}^2} 2\ln\left|\cos\left(\frac{\sqrt{{\sigma_\sigma}^2\mu_1(\mu_1-1)-{\kappa_\sigma}^2}(T-t)}{2}\right) - \sin\left(\frac{\sqrt{{\sigma_\sigma}^2\mu_1(\mu_1-1)-{\kappa_\sigma}^2}(T-t)}{2}\right)\frac{{\sigma_\sigma}^2\mu_3-\kappa_\sigma}{\sqrt{{\sigma_\sigma}^2\mu_1(\mu_1-1)-{\kappa_\sigma}^2}}\right|$$

Appendix B. MCMC-LV (MCMC-MH) Method for Assets Price (Interest Rate)

B.1. Derivation of the Conditional Probability Density Function

$$
f\left(y(t),Y_t,B_t \mid \Im_1,\Im_2,\sigma(t)\right)=
$$

$$
\frac{1}{\sqrt{2\pi}\sqrt{\sigma(t)}\sqrt{\Delta t}}e^{-\frac{\left[y(t)-\left(\mu+\frac{1}{2}\sigma(t)\right)\Delta t-Y_tB_t\right]^2}{2\sigma(t)\Delta t}}\bullet\left[\left(1-\lambda\Delta t\right)\delta(B_t)+\lambda\Delta t\delta(B_t-1)\right]\bullet
$$

$$
\left[I_{\{-1<Y_t<0\}}\left(1-p\right)\frac{\eta_2}{Y_t+1}e^{\eta_2\ln(Y_t+1)}+I_{\{Y_t\geq 0\}}p\frac{\eta_1}{Y_t+1}e^{-\eta_1\ln(Y_t+1)}\right]
$$

$$
f\left(\sigma(t)\mid \Im_2,\sigma(t-\Delta t)\right)=\frac{1}{\sqrt{2\pi}\sigma_\sigma\sqrt{\sigma(t-\Delta t)}\sqrt{\Delta t}}e^{-\frac{\left\{\sigma(t)-\left[\sigma(t-\Delta t)+\kappa_\sigma\left(\bar{\sigma}-\sigma(t-\Delta t)\right)\Delta t\right]\right\}^2}{2\sigma_\sigma{}^2\sigma(t-\Delta t)\Delta t}}
$$

Proof:

$$
f\left(y(t)\mid u_s,\sigma(t),Y_t,B_t\right)=\frac{1}{\sqrt{2\pi}\sqrt{\sigma(t)}\sqrt{\Delta t}}e^{-\frac{\left(y(t)-\left(\mu+\frac{1}{2}\sigma(t)\right)\Delta t-Y_tB_t\right)^2}{2\sigma(t)\Delta t}}
$$

$$
f\left(B_t\mid\lambda\right)=\left(1-\lambda\Delta t\right)\delta(B_t)+\lambda\Delta t\delta(B_t-1),\delta(\bullet)\sim Dirac-delta-function
$$

$$
f\left(Y_t\mid\eta_1,\eta_2,p\right)=F_{Y_t}\left(Y_t\mid\eta_1,\eta_2,p\right)=\frac{\partial P\left(y\leq Y_t\right)}{Y_t}=\frac{\partial P\left(\nu_t\leq\ln\left(Y_t+1\right)\right)}{Y_t}
$$

$$
=\frac{\partial}{Y_t}\left[I_{\{-1<Y_t<0\}}\int_{-\infty}^{\ln(Y_t+1)}\left(1-p\right)\eta_2e^{\eta_2y}dy+I_{\{Y_t\geq 0\}}\left(\int_0^{\ln(Y_t+1)}p\eta_1e^{-\eta_1y}dy+\left(1-p\right)\right)\right]
$$

$$
=\frac{\partial}{Y_t}\left[I_{\{-1<Y_t<0\}}\left(1-p\right)e^{\eta_2\ln(Y_t+1)}+I_{\{Y_t\geq 0\}}\left(1-pe^{-\eta_1\ln(Y_t+1)}\right)\right]
$$

$$
=I_{\{-1<Y_t<0\}}\left(1-p\right)\frac{\eta_2}{Y_t+1}e^{\eta_2\ln(Y_t+1)}+I_{\{Y_t\geq 0\}}p\frac{\eta_1}{Y_t+1}e^{-\eta_1\ln(Y_t+1)}
$$

According to Yu et al. (2011), we have

$$
\begin{aligned}
&f\left(y(t),Y_t,B_t\mid\mu,\sigma,\lambda,\eta_1,\eta_2,p,\sigma(t)\right)\\
&=f\left(y(t),Y_t,B_t\mid\Im_1,\Im_2,\sigma(t)\right)\\
&=f\left(y(t)\mid\mu,\sigma(t),Y_t,B_t\right)f\left(B_t\mid\lambda\right)f\left(Y_t\mid\eta_1,\eta_2,p\right)\\
&=\frac{1}{\sqrt{2\pi}\sqrt{\sigma(t)}\sqrt{\Delta t}}e^{-\frac{\left(y(t)-\left(\mu+\frac{1}{2}\sigma(t)\right)\Delta t-Y_tB_t\right)^2}{2\sigma(t)\Delta t}}\bullet\left[\left(1-\lambda\Delta t\right)\delta(B_t)+\lambda\Delta t\delta(B_t-1)\right]\bullet\\
&\left[I_{\{-1<Y_t<0\}}\left(1-p\right)\frac{\eta_2}{Y_t+1}e^{\eta_2\ln(Y_t+1)}+I_{\{Y_t\geq 0\}}p\frac{\eta_1}{Y_t+1}e^{-\eta_1\ln(Y_t+1)}\right]
\end{aligned}.
$$

B.2. Proof of the Transition Kernel of the Markov Chain $\wp$

Define $f\left(\vec{Y}_m,\vec{B}_m,\vec{\sigma}_m,\Im_m\mid\vec{y}\right)$ as the conditional probability density function (CPDF) of the Markov chain $\wp$ at stationary state $m\left(m \gg k\right)$. Assume the transition kernel is:

$$\begin{aligned} & p\left(\vec{Y}_m, \vec{B}_m, \vec{\sigma}_m, \Im_m, \vec{Y}_{m+1}, \vec{B}_{m+1}, \vec{\sigma}_{m+1}, \Im_{m+1}\right) \\ & = f\left(\vec{Y}_{m+1} \mid \vec{B}_m, \vec{\sigma}_m, \Im_m, \vec{y}\right) \bullet f\left(\vec{B}_{m+1} \mid \vec{Y}_{m+1}, \vec{\sigma}_m, \Im_m, \vec{y}\right) \bullet \\ & f\left(\vec{\sigma}_{m+1} \mid \vec{Y}_{m+1}, \vec{B}_{m+1}, \Im_m, \vec{y}\right) \bullet f\left(\Im_{m+1} \mid \vec{Y}_{m+1}, \vec{B}_{m+1}, \vec{\sigma}_{m+1}, \vec{y}\right) \end{aligned}.$$

Then the Markov chain $\wp$ has a unique stationary distribution with probability density function $f\left(\vec{Y}, \vec{B}, \vec{\sigma}, \Im \mid \vec{y}\right)$ (Equation (16), conditioned on $\vec{y}$), which satisfies the Chapman-Kolmogorov equation:

$$\begin{aligned} & \int\int\int\int f\left(\vec{Y}_m, \vec{B}_m, \vec{\sigma}_m, \Im_m \mid \vec{y}\right) \bullet p\left(\vec{Y}_m, \vec{B}_m, \vec{\sigma}_m, \Im_m, \vec{Y}_{m+1}, \vec{B}_{m+1}, \vec{\sigma}_{m+1}, \Im_{m+1}\right) d\vec{Y}_m d\vec{B}_m d\vec{\sigma}_m d\Im_m \\ & = f\left(\vec{Y}_{m+1}, \vec{B}_{m+1}, \vec{\sigma}_{m+1}, \Im_{m+1} \mid \vec{y}\right) \end{aligned}.$$

Proof:

$$\begin{aligned} & \int f\left(\vec{Y}_m, \vec{B}_m, \vec{\sigma}_m, \Im_m \mid \vec{y}\right) d\vec{Y}_m \bullet f\left(\vec{Y}_{m+1} \mid \vec{B}_m, \vec{\sigma}_m, \Im_m, \vec{y}\right) \\ & = \int f\left(\vec{Y}_m, \vec{B}_m, \vec{\sigma}_m, \Im_m \mid \vec{y}\right) d\vec{Y}_m \bullet \frac{f\left(\vec{Y}_{m+1}, \vec{B}_m, \vec{\sigma}_m, \Im_m \mid \vec{y}\right)}{\int f\left(\vec{Y}_m, \vec{B}_m, \vec{\sigma}_m, \Im_m \mid \vec{y}\right) d\vec{Y}_m} \\ & = f\left(\vec{Y}_{m+1}, \vec{B}_m, \vec{\sigma}_m, \Im_m \mid \vec{y}\right) \end{aligned}$$

$$\begin{aligned} & \int f\left(\vec{Y}_{m+1}, \vec{B}_m, \vec{\sigma}_m, \Im_m \mid \vec{y}\right) d\vec{B}_m \bullet f\left(\vec{B}_{m+1} \mid \vec{Y}_{m+1}, \vec{\sigma}_m, \Im_m, \vec{y}\right) \\ & = \int f\left(\vec{Y}_{m+1}, \vec{B}_m, \vec{\sigma}_m, \Im_m \mid \vec{y}\right) d\vec{B}_m \bullet \frac{f\left(\vec{Y}_{m+1}, \vec{B}_{m+1}, \vec{\sigma}_m, \Im_m \mid \vec{y}\right)}{\int f\left(\vec{Y}_{m+1}, \vec{B}_m, \Im_m \mid \vec{y}\right) d\vec{B}_m} \\ & = f\left(\vec{Y}_{m+1}, \vec{B}_{m+1}, \vec{\sigma}_m, \Im_m \mid \vec{y}\right) \end{aligned}$$

$$\begin{aligned} & \int f\left(\vec{Y}_{m+1}, \vec{B}_{m+1}, \vec{\sigma}_m, \Im_m \mid \vec{y}\right) d\vec{\sigma}_m \bullet f\left(\vec{\sigma}_{m+1} \mid \vec{Y}_{m+1}, \vec{B}_{m+1}, \Im_m, \vec{y}\right) \\ & = \int f\left(\vec{Y}_{m+1}, \vec{B}_{m+1}, \vec{\sigma}_m, \Im_m \mid \vec{y}\right) d\vec{\sigma}_m \bullet \frac{f\left(\vec{Y}_{m+1}, \vec{B}_{m+1}, \vec{\sigma}_{m+1}, \Im_m \mid \vec{y}\right)}{\int f\left(\vec{Y}_{m+1}, \vec{B}_{m+1}, \vec{\sigma}_m, \Im_m \mid \vec{y}\right) d\vec{\sigma}_m} \\ & = f\left(\vec{Y}_{m+1}, \vec{B}_{m+1}, \vec{\sigma}_{m+1}, \Im_m \mid \vec{y}\right) \end{aligned}$$

$$\begin{aligned} & \int f\left(\vec{Y}_{m+1}, \vec{B}_{m+1}, \vec{\sigma}_{m+1}, \Im_m \mid \vec{y}\right) d\Im_m \bullet f\left(\Im_{m+1} \mid \vec{Y}_{m+1}, \vec{B}_{m+1}, \vec{\sigma}_{m+1}, \vec{y}\right) \\ & = \int f\left(\vec{Y}_{m+1}, \vec{B}_{m+1}, \vec{\sigma}_{m+1}, \Im_m \mid \vec{y}\right) d\Im_m \bullet \frac{f\left(\vec{Y}_{m+1}, \vec{B}_{m+1}, \vec{\sigma}_{m+1}, \Im_{m+1} \mid \vec{y}\right)}{\int f\left(\vec{Y}_{m+1}, \vec{B}_{m+1}, \vec{\sigma}_{m+1}, \Im_m \mid \vec{y}\right) d\Im_m} \\ & = f\left(\vec{Y}_{m+1}, \vec{B}_{m+1}, \vec{\sigma}_{m+1}, \Im_{m+1} \mid \vec{y}\right) \end{aligned}$$

It's obvious that the chain is irreducible and has positive recurrence. Thus, the stationary distribution exists and is unique. This completes the proof.

B.3. Proof of the Transition Kernel of the Markov Chain $\aleph$

We define the sampling process as a Markov chain $\aleph$ with discrete time and continuous state. Here, $\Im_m = \left(\kappa_{m,r}, \overline{r}_m, \sigma_{m,r}\right)$ is the sample from the $m-th, m = 1,2,3...$ round of sampling, and $\Im_{m+1} = \left(\kappa_{m+1,r}, \overline{r}_{m+1}, \sigma_{m+1,r}\right)$ is the sample from the $m+1-th$ round of sampling. Choose $p(\Im_{m+1} \mid \Im_m, \vec{y}, \vec{r}) = \pi(\Im_{m+1})\alpha(\Im_{m+1} \mid \Im_m, \vec{y}, \vec{r})$ as the transition kernel, where:

$$\alpha(\Im_{m+1} \mid \Im_m, \vec{y}, \vec{r}) = \min\left\{1, \frac{f\left(\Im_{m+1} \mid \vec{y}, \vec{r}\right)\pi(\Im_m)}{f\left(\Im_m \mid \vec{y}, \vec{r}\right)\pi(\Im_{m+1})}\right\}, m = 1,2,3...$$

where $f\left(\Im_m \mid \vec{y}, \vec{r}\right)$ is Equation (23). The Markov chain $\aleph$ formed by the transition kernel has a unique stationary distribution, and its PDF is $f\left(\Im \mid \vec{y}, \vec{r}\right)$.

Proof:

(1) If $f\left(\Im_{m+1} \mid \vec{y}, \vec{r}\right)\pi(\Im_m) = f\left(\Im_m \mid \vec{y}, \vec{r}\right)\pi(\Im_{m+1})$, then $\alpha(\Im_{m+1} \mid \Im_m, \vec{y}, \vec{r}) = 1$, $\alpha(\Im_m \mid \Im_{m+1}, \vec{y}) = 1$. Thus, we have:

$$p(\Im_{m+1} \mid \Im_m, \vec{y}, \vec{r}) f\left(\Im_m \mid \vec{y}, \vec{r}\right) = \pi(\Im_{m+1}) f\left(\Im_m \mid \vec{y}, \vec{r}\right)$$

$$p(\Im_m \mid \Im_{m+1}, \vec{y}, \vec{r}) f\left(\Im_{m+1} \mid \vec{y}, \vec{r}\right) = \pi(\Im_m) f\left(\Im_{m+1} \mid \vec{y}, \vec{r}\right).$$

From Equation (23), we obtain the following balance conditions:

$$p(\Im_{m+1} \mid \Im_m, \vec{y}, \vec{r}) f\left(\Im_m \mid \vec{y}, \vec{r}\right) = p(\Im_m \mid \Im_{m+1}, \vec{y}, \vec{r}) f\left(\Im_{m+1} \mid \vec{y}, \vec{r}\right).$$

According to the detailed balance conditions proposed by Yu et al. (2011), the Markov chain $\aleph$ formed by the transition kernel has a unique stationary distribution, and its PDF is $f\left(\Im \mid \vec{y}, \vec{r}\right)$:

(2) If $f\left(\Im_{m+1} \mid \vec{y}, \vec{r}\right)\pi(\Im_m) > f\left(\Im_m \mid \vec{y}, \vec{r}\right)\pi(\Im_{m+1})$, then $\alpha(\Im_{m+1} \mid \Im_m, \vec{y}, \vec{r}) = 1$, $\alpha(\Im_m \mid \Im_{m+1}, \vec{y}) = \dfrac{f\left(\Im_m \mid \vec{y}, \vec{r}\right)\pi(\Im_{m+1})}{f\left(\Im_{m+1} \mid \vec{y}, \vec{r}\right)\pi(\Im_m)}$. So, we have:

$$p(\Im_{m+1} \mid \Im_m, \vec{y}, \vec{r}) f\left(\Im_m \mid \vec{y}, \vec{r}\right) = \pi(\Im_{m+1}) f\left(\Im_m \mid \vec{y}, \vec{r}\right)$$

$$\begin{aligned} &p(\Im_m \mid \Im_{m+1}, \vec{y}, \vec{r}) f\left(\Im_{m+1} \mid \vec{y}, \vec{r}\right) = f\left(\Im_{m+1} \mid \vec{y}, \vec{r}\right)\pi(\Im_m)\alpha(\Im_m \mid \Im_{m+1}, \vec{y}, \vec{r}) \\ &= f\left(\Im_m \mid \vec{y}, \vec{r}\right)\pi(\Im_{m+1}) \end{aligned}.$$

It's obvious that hat the detailed balance conditions are established. The Markov chain $\aleph$ formed by the transition kernel has unique stationary distribution and its PDF is $f\left(\Im \mid \vec{y}, \vec{r}\right)$.

(3) If $f\left(\Im_{m+1}, \vec{y}\right)\pi(\Im_m) < f\left(\Im_m, \vec{y}\right)\pi(\Im_{m+1})$, then $\alpha(\Im_{m+1} \mid \Im_m, \vec{y}, \vec{r}) = \dfrac{f\left(\Im_{m+1} \mid \vec{y}, \vec{r}\right)\pi(\Im_m)}{f\left(\Im_m \mid \vec{y}, \vec{r}\right)\pi(\Im_{m+1})}$,

$\alpha(\Im_m \mid \Im_{m+1}, \vec{y}) = 1$. So, we have:

$$\begin{aligned} &p(\Im_{m+1} \mid \Im_m, \vec{y}, \vec{r}) f\left(\Im_m \mid \vec{y}, \vec{r}\right) = \pi(\Im_{m+1}) f\left(\Im_m \mid \vec{y}, \vec{r}\right)\alpha(\Im_{m+1} \mid \Im_m, \vec{y}, \vec{r}) \\ &= f\left(\Im_{m+1} \mid \vec{y}, \vec{r}\right)\pi(\Im_m) \\ &p(\Im_m \mid \Im_{m+1}, \vec{y}, \vec{r}) f\left(\Im_{m+1} \mid \vec{y}, \vec{r}\right) = f\left(\Im_{m+1} \mid \vec{y}, \vec{r}\right)\pi(\Im_m)\alpha(\Im_m \mid \Im_{m+1}, \vec{y}, \vec{r}) \\ &= f\left(\Im_{m+1} \mid \vec{y}, \vec{r}\right)\pi(\Im_m) \end{aligned}$$

It's obvious that if detailed balance conditions are established, the Markov chain $\aleph$ formed by the transition kernel has a unique stationary distribution and its PDF is $f\left(\Im \mid \vec{y}, \vec{r}\right)$.

Appendix C. Empirical Results

Table C1. Estimate Parameters of Each Trading Day with MCMC-LV Method[1]

Date	μ	λ	η_1	η_2	p	κ_σ	$\bar{\sigma}$	σ_σ	$\sigma(1)$	κ_r	$\bar{r}$	σ_r
2016-02-01	4.16E-04	4.13E-01	4.70E+01	3.93E+01	5.90E-01	7.29E-01	1.05E-04	2.77E-03	1.68E-04	7.24E-04	3.55E-05	3.92E-04
2016-02-02	5.94E-04	4.50E-01	4.81E+01	4.02E+01	6.04E-01	6.55E-01	9.84E-05	1.79E-03	2.23E-04	2.52E-03	3.67E-05	4.00E-04
2016-02-03	4.50E-04	4.19E-01	4.77E+01	3.91E+01	6.08E-01	6.80E-01	1.09E-04	2.49E-03	1.59E-04	3.08E-03	4.55E-05	3.93E-04
2016-02-04	4.58E-04	4.06E-01	4.73E+01	3.86E+01	6.11E-01	6.68E-01	1.13E-04	2.53E-03	2.47E-04	2.14E-03	5.52E-05	3.65E-04
2016-02-05	9.61E-04	5.20E-01	5.02E+01	4.15E+01	6.18E-01	6.84E-01	7.46E-05	2.16E-03	3.09E-04	1.68E-03	5.34E-05	3.61E-04
2016-02-15	3.34E-04	3.69E-01	4.66E+01	3.77E+01	6.01E-01	6.87E-01	1.29E-04	3.17E-03	2.02E-04	6.54E-04	3.86E-05	3.65E-04
2016-02-16	5.01E-04	4.47E-01	4.81E+01	3.98E+01	6.00E-01	6.74E-01	9.34E-05	1.83E-03	1.32E-04	1.05E-03	6.34E-05	3.68E-04
2016-02-17	5.29E-04	4.49E-01	4.84E+01	4.01E+01	6.03E-01	6.53E-01	1.00E-04	2.59E-03	2.13E-04	1.17E-03	5.68E-05	3.59E-04
2016-02-18	5.50E-04	4.42E-01	4.84E+01	3.95E+01	6.11E-01	6.96E-01	1.07E-04	2.61E-03	1.76E-04	2.01E-03	5.67E-05	3.44E-04
2016-02-19	2.45E-04	3.52E-01	4.56E+01	3.69E+01	6.00E-01	6.87E-01	1.34E-04	3.31E-03	1.71E-04	1.97E-03	6.02E-05	3.31E-04
2016-02-22	4.68E-04	4.34E-01	4.77E+01	3.92E+01	6.09E-01	7.13E-01	9.88E-05	2.41E-03	1.77E-04	6.30E-03	6.00E-05	3.40E-04
2016-02-23	3.26E-04	4.16E-01	4.74E+01	3.95E+01	5.93E-01	6.53E-01	1.11E-04	1.40E-03	1.85E-04	2.95E-03	3.26E-05	3.37E-04
2016-02-24	6.93E-04	4.66E-01	4.86E+01	4.01E+01	6.13E-01	7.15E-01	9.08E-05	2.50E-03	2.25E-04	2.47E-03	9.29E-05	3.36E-04
2016-02-25	6.19E-04	4.70E-01	4.87E+01	4.01E+01	6.03E-01	6.85E-01	9.08E-05	2.44E-03	2.13E-04	6.39E-03	6.92E-05	3.35E-04
2016-02-26	4.00E-04	4.15E-01	4.70E+01	3.86E+01	5.95E-01	7.03E-01	1.08E-04	3.24E-03	1.58E-04	2.60E-03	4.63E-05	3.38E-04
2016-02-29	5.50E-04	4.47E-01	4.77E+01	3.93E+01	5.95E-01	7.21E-01	9.61E-05	2.76E-03	1.75E-04	5.89E-04	6.33E-05	3.26E-04
2016-03-01	7.22E-04	4.91E-01	4.93E+01	4.09E+01	6.08E-01	6.97E-01	8.84E-05	2.55E-03	1.62E-04	4.54E-03	5.01E-05	3.35E-04
2016-03-02	3.40E-04	3.90E-01	4.65E+01	3.82E+01	5.96E-01	7.10E-01	1.27E-04	3.41E-03	1.46E-04	2.00E-03	1.52E-05	3.25E-04
2016-03-03	5.34E-04	4.56E-01	4.82E+01	3.97E+01	5.99E-01	6.78E-01	9.77E-05	2.76E-03	2.22E-04	6.62E-04	8.17E-05	3.21E-04
2016-03-04	7.77E-04	5.11E-01	4.94E+01	4.10E+01	6.08E-01	7.10E-01	8.21E-05	2.14E-03	1.91E-04	1.98E-03	5.76E-05	3.18E-04
2016-03-07	3.57E-04	4.04E-01	4.70E+01	3.81E+01	6.05E-01	6.77E-01	1.19E-04	2.88E-03	2.59E-04	1.72E-03	6.11E-05	3.14E-04
2016-03-08	4.88E-05	3.54E-01	4.56E+01	3.73E+01	5.86E-01	6.93E-01	1.43E-04	2.81E-03	1.78E-04	1.43E-03	8.89E-05	2.98E-04
2016-03-09	6.36E-04	4.71E-01	4.84E+01	4.00E+01	6.13E-01	6.61E-01	9.27E-05	2.36E-03	3.00E-04	1.44E-03	6.35E-05	2.94E-04
2016-03-10	4.26E-04	4.27E-01	4.75E+01	3.92E+01	6.01E-01	6.66E-01	1.11E-04	2.62E-03	2.70E-04	1.02E-03	1.73E-05	2.98E-04
2016-03-11	4.76E-04	4.37E-01	4.77E+01	3.88E+01	6.11E-01	6.43E-01	1.04E-04	2.34E-03	2.02E-04	4.03E-04	6.18E-05	3.02E-04
2016-03-14	8.17E-04	5.07E-01	4.97E+01	4.07E+01	6.19E-01	7.18E-01	8.09E-05	2.47E-03	1.66E-04	2.23E-03	5.02E-05	2.98E-04
2016-03-15	6.24E-04	4.65E-01	4.79E+01	3.91E+01	6.11E-01	7.03E-01	9.00E-05	2.83E-03	1.69E-04	5.75E-03	8.60E-05	3.00E-04
2016-03-16	9.49E-04	5.32E-01	5.02E+01	4.10E+01	6.24E-01	6.75E-01	7.38E-05	2.27E-03	1.50E-04	2.41E-03	5.99E-05	2.98E-04
2016-03-17	6.92E-04	5.09E-01	4.92E+01	4.10E+01	6.05E-01	6.76E-01	7.73E-05	2.00E-03	2.02E-04	3.42E-03	5.45E-05	2.96E-04
2016-03-18	3.65E-04	4.29E-01	4.76E+01	3.87E+01	6.12E-01	6.79E-01	1.08E-04	1.83E-03	1.38E-04	8.36E-04	1.51E-05	2.91E-04
2016-03-21	1.52E-04	4.05E-01	4.75E+01	3.84E+01	5.94E-01	6.98E-01	1.20E-04	2.99E-03	2.63E-04	7.37E-04	6.83E-05	2.93E-04

1 The estimates are performed in Matlab 2015b.

Table C1. ***Cont.***

Date	μ	λ	η_1	η_2	p	κ_σ	$\bar{\sigma}$	σ_σ	$\sigma(1)$	κ_r	$\bar{r}$	σ_r
2016-03-22	6.10E-04	4.77E-01	4.89E+01	3.96E+01	6.20E-01	7.01E-01	9.17E-05	1.30E-03	1.42E-04	1.42E-03	3.83E-05	2.91E-04
2016-03-23	5.41E-04	4.56E-01	4.81E+01	3.86E+01	6.21E-01	6.78E-01	1.01E-04	2.16E-03	2.27E-04	2.31E-03	8.41E-05	2.89E-04
2016-03-24	4.76E-04	4.42E-01	4.80E+01	3.90E+01	6.19E-01	6.46E-01	1.06E-04	2.06E-03	1.41E-04	2.29E-03	5.89E-05	2.99E-04
2016-03-25	7.88E-04	5.10E-01	4.99E+01	4.06E+01	6.23E-01	6.82E-01	8.10E-05	2.33E-03	2.01E-04	1.73E-03	8.55E-05	2.91E-04
2016-03-28	1.67E-04	2.90E-01	4.42E+01	3.62E+01	5.83E-01	6.99E-01	1.80E-04	4.67E-03	2.01E-04	1.45E-03	3.53E-05	2.89E-04
2016-03-29	9.79E-04	5.09E-01	4.95E+01	4.05E+01	6.23E-01	7.19E-01	8.18E-05	2.50E-03	1.73E-04	2.56E-03	5.18E-05	2.98E-04
2016-03-30	1.13E-03	5.49E-01	5.05E+01	4.14E+01	6.37E-01	6.29E-01	6.99E-05	1.68E-03	1.38E-04	3.95E-03	8.57E-05	2.89E-04
2016-03-31	6.43E-04	4.70E-01	4.88E+01	3.97E+01	6.20E-01	6.86E-01	9.34E-05	1.48E-03	6.27E-04	3.60E-03	4.48E-05	2.95E-04
2016-04-01	4.72E-04	4.24E-01	4.79E+01	3.84E+01	6.15E-01	6.14E-01	1.18E-04	2.28E-03	1.80E-04	6.59E-04	7.79E-05	2.84E-04
2016-04-05	6.07E-04	4.85E-01	4.89E+01	4.02E+01	6.04E-01	6.98E-01	9.45E-05	2.18E-03	1.68E-04	1.40E-03	5.65E-05	2.64E-04
2016-04-06	4.56E-04	4.27E-01	4.81E+01	3.85E+01	6.22E-01	7.28E-01	1.13E-04	3.18E-03	2.61E-04	1.30E-03	6.51E-05	2.57E-04
2016-04-07	5.63E-04	4.46E-01	4.84E+01	3.88E+01	6.27E-01	6.93E-01	1.03E-04	2.70E-03	2.70E-04	1.01E-03	4.81E-05	2.42E-04
2016-04-08	3.13E-04	3.73E-01	4.66E+01	3.69E+01	6.27E-01	7.20E-01	1.31E-04	2.86E-03	1.85E-04	8.43E-04	5.08E-05	2.46E-04
2016-04-11	7.11E-04	4.87E-01	4.92E+01	4.00E+01	6.18E-01	7.34E-01	8.63E-05	2.52E-03	1.57E-04	6.08E-04	7.19E-05	2.37E-04
2016-04-12	1.12E-03	5.66E-01	5.12E+01	4.22E+01	6.27E-01	6.42E-01	6.60E-05	1.56E-03	1.13E-04	9.11E-04	6.61E-05	2.43E-04
2016-04-13	4.65E-04	4.34E-01	4.78E+01	3.89E+01	6.13E-01	6.94E-01	1.04E-04	2.05E-03	1.55E-04	1.62E-03	4.94E-05	2.37E-04
2016-04-14	3.01E-04	3.78E-01	4.73E+01	3.72E+01	6.29E-01	6.43E-01	1.34E-04	1.81E-03	1.40E-04	4.61E-04	3.73E-05	2.37E-04
2016-04-15	2.59E-04	4.16E-01	4.74E+01	3.88E+01	6.03E-01	6.95E-01	1.16E-04	2.46E-03	1.33E-04	9.12E-04	6.27E-05	2.38E-04
2016-04-18	2.17E-04	3.80E-01	4.69E+01	3.79E+01	6.06E-01	6.53E-01	1.36E-04	2.28E-03	1.37E-04	1.16E-03	6.15E-05	2.33E-04
2016-04-19	7.21E-04	4.70E-01	4.88E+01	3.98E+01	6.22E-01	7.07E-01	9.53E-05	2.62E-03	1.65E-04	9.52E-04	6.13E-05	2.37E-04
2016-04-20	2.47E-04	4.03E-01	4.77E+01	3.82E+01	6.06E-01	7.16E-01	1.23E-04	2.73E-03	1.44E-04	2.52E-03	6.10E-05	2.42E-04
2016-04-21	2.79E-04	3.83E-01	4.66E+01	3.77E+01	6.04E-01	7.32E-01	1.28E-04	3.00E-03	2.20E-04	1.49E-03	4.80E-05	2.36E-04
2016-04-22	6.92E-04	4.55E-01	4.85E+01	3.94E+01	6.23E-01	7.20E-01	9.89E-05	2.79E-03	1.69E-04	2.76E-04	8.10E-05	2.34E-04
2016-04-25	7.29E-04	4.55E-01	4.83E+01	3.92E+01	6.22E-01	7.13E-01	9.69E-05	2.91E-03	1.99E-04	5.87E-04	4.75E-05	2.38E-04
2016-04-26	7.64E-04	4.91E-01	4.96E+01	4.05E+01	6.19E-01	6.88E-01	8.49E-05	2.00E-03	1.85E-04	2.49E-03	5.17E-05	2.36E-04
2016-04-27	7.92E-04	4.94E-01	4.88E+01	4.03E+01	6.15E-01	7.13E-01	7.89E-05	2.50E-03	2.16E-04	1.81E-03	5.94E-05	2.37E-04
2016-04-28	4.24E-04	4.10E-01	4.72E+01	3.80E+01	6.14E-01	6.45E-01	1.16E-04	2.51E-03	2.47E-04	8.83E-04	4.79E-05	2.35E-04
2016-04-29	1.01E-03	5.45E-01	5.10E+01	4.19E+01	6.21E-01	6.81E-01	7.03E-05	1.83E-03	1.47E-04	5.88E-04	4.54E-05	2.30E-04
2016-05-03	7.12E-04	4.58E-01	4.88E+01	3.92E+01	6.25E-01	7.11E-01	9.36E-05	2.25E-03	1.46E-04	8.05E-04	5.16E-05	2.34E-04
2016-05-04	3.62E-04	4.06E-01	4.75E+01	3.89E+01	6.00E-01	7.06E-01	1.16E-04	3.08E-03	1.54E-04	1.62E-03	4.33E-05	2.30E-04
2016-05-05	5.09E-04	4.41E-01	4.82E+01	3.90E+01	6.11E-01	6.65E-01	1.02E-04	2.73E-03	3.16E-04	1.75E-03	5.37E-05	2.28E-04
2016-05-06	7.88E-04	5.21E-01	5.02E+01	4.15E+01	6.06E-01	6.57E-01	7.92E-05	1.94E-03	1.75E-04	7.17E-04	4.47E-05	2.26E-04

Table C2. Option Prices by the Double Exponential Jump-Diffusion Model with SV and SIR[2]

Contract / Date	50ETF/ Call/June/ 1.8 *	50ETF/ Call/June/ 2.0	50ETF/Call /June/2.2	50ETF/Cal l/June/2.4	50ETF/Cal l/June/2.6	50ETF/Put/ June/1.8 **	50ETF/Put /June/2.0	50ETF/Put /June/2.2	50ETF/ Put/June/ 2.4	50ETF/Put /June/2.6
2016-02-01	0.2270	0.1222	0.0582	0.0242	0.0083	0.0716	0.1653	0.2998	0.4643	0.6469
2016-02-02	0.2532	0.1422	0.0717	0.0324	0.0129	0.0654	0.1530	0.2812	0.4404	0.6195
2016-02-03	0.2323	0.1252	0.0594	0.0245	0.0082	0.0667	0.1581	0.2910	0.4546	0.6370
2016-02-04	0.2487	0.1358	0.0652	0.0270	0.0091	0.0579	0.1435	0.2715	0.4319	0.6125
2016-02-05	0.2534	0.1447	0.0754	0.0360	0.0159	0.0726	0.1625	0.2917	0.4510	0.6294
2016-02-15	0.2210	0.1132	0.0491	0.0169	0.0034	0.0593	0.1500	0.2845	0.4510	0.6361
2016-02-16	0.2701	0.1530	0.0776	0.0352	0.0141	0.0564	0.1379	0.2612	0.4173	0.5949
2016-02-17	0.2769	0.1569	0.0792	0.0355	0.0138	0.0515	0.1301	0.2511	0.4060	0.5829
2016-02-18	0.2734	0.1535	0.0763	0.0333	0.0124	0.0503	0.1291	0.2506	0.4063	0.5840
2016-02-19	0.2598	0.1384	0.0627	0.0229	0.0054	0.0419	0.1192	0.2422	0.4011	0.5823
2016-02-22	0.3067	0.1780	0.0922	0.0425	0.0173	0.0414	0.1114	0.2244	0.3735	0.5469
2016-02-23	0.2834	0.1583	0.0776	0.0329	0.0115	0.0424	0.1161	0.2341	0.3882	0.5656
2016-02-24	0.2966	0.1712	0.0884	0.0410	0.0169	0.0456	0.1189	0.2348	0.3860	0.5606
2016-02-25	0.2194	0.1159	0.0542	0.0224	0.0080	0.0718	0.1670	0.3040	0.4710	0.6554
2016-02-26	0.2276	0.1185	0.0534	0.0204	0.0061	0.0601	0.1497	0.2833	0.4490	0.6335
2016-02-29	0.2174	0.1130	0.0514	0.0204	0.0068	0.0688	0.1631	0.3003	0.4679	0.6530
2016-03-01	0.2425	0.1313	0.0630	0.0268	0.0101	0.0609	0.1484	0.2790	0.4416	0.6237
2016-03-02	0.2924	0.1613	0.0765	0.0303	0.0091	0.0330	0.1007	0.2147	0.3674	0.5450
2016-03-03	0.3029	0.1730	0.0873	0.0389	0.0150	0.0389	0.1078	0.2210	0.3713	0.5462
2016-03-04	0.3636	0.2222	0.1225	0.0611	0.0277	0.0310	0.0885	0.1876	0.3250	0.4904
2016-03-07	0.3450	0.2012	0.1024	0.0448	0.0162	0.0235	0.0786	0.1786	0.3199	0.4901
2016-03-08	0.3427	0.1952	0.0946	0.0374	0.0105	0.0171	0.0685	0.1667	0.3083	0.4803
2016-03-09	0.3460	0.2051	0.1080	0.0505	0.0209	0.0287	0.0867	0.1885	0.3298	0.4992
2016-03-10	0.3005	0.1670	0.0802	0.0328	0.0110	0.0316	0.0970	0.2091	0.3606	0.5377
2016-03-11	0.3078	0.1732	0.0848	0.0359	0.0129	0.0318	0.0961	0.2066	0.3566	0.5324
2016-03-14	0.3202	0.1855	0.0953	0.0436	0.0179	0.0346	0.0988	0.2075	0.3548	0.5279
2016-03-15	0.3371	0.1971	0.1018	0.0465	0.0188	0.0291	0.0880	0.1916	0.3352	0.5064
2016-03-16	0.3709	0.2254	0.1225	0.0598	0.0265	0.0264	0.0798	0.1759	0.3122	0.4778
2016-03-17	0.3666	0.2206	0.1180	0.0564	0.0242	0.0253	0.0783	0.1747	0.3120	0.4788
2016-03-18	0.3671	0.2158	0.1101	0.0482	0.0178	0.0178	0.0655	0.1587	0.2958	0.4643
2016-03-21	0.4119	0.2499	0.1311	0.0583	0.0213	0.0105	0.0475	0.1276	0.2538	0.4157

2 The estimates are performed in Matlab 2015b.

Table C2. ***Cont.***

Contract / Date	50ETF/Call/June/1.8 *	50ETF/Call/June/2.0	50ETF/Call/June/2.2	50ETF/Call/June/2.4	50ETF/Call/June/2.6	50ETF/Put/June/1.8 **	50ETF/Put/June/2.0	50ETF/Put/June/2.2	50ETF/Put/June/2.4	50ETF/Put/June/2.6
2016-03-22	0.3952	0.2401	0.1284	0.0603	0.0248	0.0171	0.0610	0.1482	0.2791	0.4427
2016-03-23	0.3958	0.2396	0.1269	0.0585	0.0234	0.0156	0.0583	0.1447	0.2753	0.4391
2016-03-24	0.3654	0.2130	0.1070	0.0459	0.0166	0.0166	0.0632	0.1562	0.2941	0.4638
2016-03-25	0.3736	0.2224	0.1160	0.0532	0.0215	0.0197	0.0676	0.1602	0.2964	0.4638
2016-03-28	0.3334	0.1750	0.0705	0.0180	-0.0012	0.0039	0.0446	0.1392	0.2857	0.4656
2016-03-29	0.3364	0.1924	0.0956	0.0414	0.0158	0.0240	0.0791	0.1814	0.3263	0.4997
2016-03-30	0.3853	0.2322	0.1233	0.0580	0.0245	0.0188	0.0648	0.1549	0.2886	0.4542
2016-03-31	0.3786	0.2224	0.1123	0.0486	0.0180	0.0146	0.0575	0.1466	0.2820	0.4505
2016-04-01	0.3836	0.2230	0.1094	0.0444	0.0143	0.0104	0.0489	0.1344	0.2684	0.4375
2016-04-05	0.3957	0.2358	0.1210	0.0532	0.0200	0.0128	0.0521	0.1364	0.2677	0.4336
2016-04-06	0.3826	0.2211	0.1072	0.0428	0.0136	0.0098	0.0475	0.1328	0.2675	0.4375
2016-04-07	0.3595	0.2033	0.0964	0.0379	0.0121	0.0130	0.0559	0.1482	0.2888	0.4622
2016-04-08	0.3409	0.1836	0.0794	0.0263	0.0056	0.0094	0.0514	0.1463	0.2924	0.4709
2016-04-11	0.3655	0.2092	0.1014	0.0415	0.0144	0.0142	0.0571	0.1484	0.2877	0.4598
2016-04-12	0.3659	0.2120	0.1055	0.0455	0.0173	0.0167	0.0619	0.1547	0.2938	0.4648
2016-04-13	0.3838	0.2200	0.1046	0.0407	0.0127	0.0088	0.0442	0.1281	0.2634	0.4346
2016-04-14	0.3854	0.2163	0.0973	0.0333	0.0075	0.0044	0.0346	0.1148	0.2501	0.4235
2016-04-15	0.3910	0.2232	0.1040	0.0385	0.0107	0.0063	0.0377	0.1177	0.2514	0.4229
2016-04-18	0.3656	0.1990	0.0852	0.0269	0.0051	0.0049	0.0376	0.1230	0.2640	0.4415
2016-04-19	0.3765	0.2129	0.0992	0.0377	0.0117	0.0090	0.0447	0.1302	0.2680	0.4413
2016-04-20	0.3606	0.1954	0.0832	0.0266	0.0056	0.0063	0.0403	0.1274	0.2701	0.4484
2016-04-21	0.3577	0.1916	0.0797	0.0244	0.0046	0.0054	0.0387	0.1261	0.2700	0.4495
2016-04-22	0.3698	0.2046	0.0911	0.0323	0.0090	0.0076	0.0417	0.1275	0.2680	0.4440
2016-04-25	0.3593	0.1959	0.0855	0.0295	0.0080	0.0083	0.0441	0.1331	0.2765	0.4543
2016-04-26	0.3662	0.2019	0.0897	0.0320	0.0093	0.0084	0.0435	0.1306	0.2723	0.4489
2016-04-27	0.3591	0.1966	0.0869	0.0311	0.0092	0.0094	0.0463	0.1359	0.2795	0.4570
2016-04-28	0.3532	0.1863	0.0752	0.0224	0.0045	0.0056	0.0382	0.1265	0.2730	0.4545
2016-04-29	0.3502	0.1889	0.0816	0.0285	0.0083	0.0098	0.0478	0.1399	0.2862	0.4654
2016-05-03	0.3699	0.2005	0.0846	0.0274	0.0069	0.0057	0.0357	0.1192	0.2615	0.4403
2016-05-04	0.3617	0.1899	0.0741	0.0205	0.0035	0.0037	0.0313	0.1150	0.2608	0.4433
2016-05-05	0.3609	0.1908	0.0764	0.0226	0.0048	0.0050	0.0344	0.1194	0.2651	0.4468
2016-05-06	0.3174	0.1591	0.0608	0.0179	0.0042	0.0096	0.0508	0.1520	0.3086	0.4943

Note: * SSE 50ETF call option expiring on the fourth Wednesday in June, with strike price at 1.8￥. ** SSE 50ETF put option expiring on the fourth Wednesday in June, with strike price at 1.8￥.

Table C3. Option Prices by BS Model[3]

Contract / Date	50ETF/ Call/June/ 1.8 *	50ETF/ Call/June/ 2.0	50ETF/Call /June/2.2	50ETF/ Call/June/ 2.4	50ETF/Cal l/June/2.6	50ETF/Put/ June/1.8 **	50ETF/Put /June/2.0	50ETF/Put /June/2.2	50ETF/ Put/June/ 2.4	50ETF/Put /June/2.6
2016-02-01	0.2468	0.1442	0.0780	0.0396	0.0190	0.0886	0.1842	0.3162	0.4759	0.6536
2016-02-02	0.2695	0.1604	0.0885	0.0457	0.0223	0.0784	0.1676	0.2939	0.4493	0.6242
2016-02-03	0.2529	0.1480	0.0801	0.0405	0.0194	0.0840	0.1773	0.3077	0.4664	0.6435
2016-02-04	0.2699	0.1601	0.0878	0.0450	0.0218	0.0763	0.1647	0.2907	0.4461	0.6212
2016-02-05	0.2619	0.1538	0.0835	0.0422	0.0202	0.0784	0.1687	0.2966	0.4536	0.6298
2016-02-15	0.2473	0.1429	0.0761	0.0377	0.0176	0.0830	0.1769	0.3084	0.4684	0.6466
2016-02-16	0.2843	0.1698	0.0935	0.0480	0.0231	0.0683	0.1520	0.2741	0.4268	0.6004
2016-02-17	0.2925	0.1755	0.0971	0.0500	0.0242	0.0646	0.1460	0.2659	0.4171	0.5897
2016-02-18	0.2901	0.1733	0.0953	0.0487	0.0234	0.0644	0.1460	0.2663	0.4181	0.5912
2016-02-19	0.2854	0.1694	0.0925	0.0468	0.0222	0.0649	0.1474	0.2688	0.4215	0.5953
2016-02-22	0.3217	0.1967	0.1106	0.0577	0.0282	0.0535	0.1268	0.2392	0.3847	0.5536
2016-02-23	0.3021	0.1813	0.0999	0.0510	0.0243	0.0581	0.1357	0.2527	0.4022	0.5740
2016-02-24	0.3083	0.1856	0.1025	0.0523	0.0250	0.0554	0.1311	0.2465	0.3948	0.5659
2016-02-25	0.2319	0.1295	0.0660	0.0311	0.0137	0.0822	0.1783	0.3133	0.4768	0.6579
2016-02-26	0.2450	0.1384	0.0713	0.0339	0.0151	0.0755	0.1674	0.2988	0.4599	0.6396
2016-02-29	0.2307	0.1280	0.0646	0.0300	0.0130	0.0804	0.1762	0.3113	0.4753	0.6568
2016-03-01	0.2541	0.1442	0.0745	0.0354	0.0157	0.0699	0.1586	0.2875	0.4469	0.6257
2016-03-02	0.3129	0.1876	0.1027	0.0518	0.0243	0.0509	0.1242	0.2378	0.3855	0.5566
2016-03-03	0.3152	0.1889	0.1032	0.0519	0.0243	0.0494	0.1217	0.2346	0.3819	0.5528
2016-03-04	0.3710	0.2322	0.1329	0.0700	0.0343	0.0364	0.0962	0.1955	0.3312	0.4941
2016-03-07	0.3610	0.2237	0.1265	0.0658	0.0317	0.0376	0.0989	0.2003	0.3382	0.5028
2016-03-08	0.3635	0.2253	0.1272	0.0659	0.0317	0.0363	0.0966	0.1972	0.3346	0.4990
2016-03-09	0.3560	0.2188	0.1224	0.0626	0.0297	0.0369	0.0984	0.2006	0.3395	0.5052
2016-03-10	0.3158	0.1872	0.1004	0.0492	0.0222	0.0449	0.1149	0.2268	0.3743	0.5460
2016-03-11	0.3205	0.1903	0.1021	0.0499	0.0225	0.0428	0.1112	0.2217	0.3682	0.5395
2016-03-14	0.3278	0.1954	0.1052	0.0515	0.0232	0.0402	0.1065	0.2150	0.3600	0.5304
2016-03-15	0.3452	0.2084	0.1136	0.0563	0.0256	0.0358	0.0977	0.2016	0.3430	0.5111
2016-03-16	0.3759	0.2322	0.1296	0.0658	0.0307	0.0296	0.0846	0.1808	0.3158	0.4794
2016-03-17	0.3725	0.2290	0.1270	0.0639	0.0295	0.0294	0.0846	0.1814	0.3171	0.4814
2016-03-18	0.3787	0.2334	0.1296	0.0652	0.0300	0.0277	0.0812	0.1762	0.3106	0.4742
2016-03-21	0.4238	0.2697	0.1552	0.0810	0.0386	0.0210	0.0657	0.1500	0.2746	0.4311

3 The estimates are performed in Matlab 2015b.

Table C3. ***Cont.***

Contract / Date	50ETF/Call/June/1.8 *	50ETF/Call/June/2.0	50ETF/Call/June/2.2	50ETF/Call/June/2.4	50ETF/Call/June/2.6	50ETF/Put/June/1.8 **	50ETF/Put/June/2.0	50ETF/Put/June/2.2	50ETF/Put/June/2.4	50ETF/Put/June/2.6
2016-03-22	0.4027	0.2518	0.1418	0.0722	0.0335	0.0230	0.0710	0.1598	0.2890	0.4492
2016-03-23	0.4037	0.2522	0.1416	0.0717	0.0331	0.0223	0.0695	0.1578	0.2868	0.4470
2016-03-24	0.3756	0.2288	0.1246	0.0609	0.0270	0.0253	0.0774	0.1720	0.3072	0.4722
2016-03-25	0.3792	0.2311	0.1257	0.0613	0.0271	0.0241	0.0749	0.1684	0.3028	0.4675
2016-03-28	0.3576	0.2134	0.1131	0.0536	0.0229	0.0266	0.0814	0.1800	0.3193	0.4875
2016-03-29	0.3422	0.2008	0.1042	0.0482	0.0201	0.0284	0.0859	0.1883	0.3311	0.5019
2016-03-30	0.3886	0.2371	0.1285	0.0620	0.0270	0.0209	0.0684	0.1587	0.2912	0.4551
2016-03-31	0.3860	0.2344	0.1261	0.0603	0.0259	0.0206	0.0679	0.1586	0.2917	0.4562
2016-04-01	0.3933	0.2398	0.1293	0.0618	0.0265	0.0190	0.0644	0.1529	0.2844	0.4481
2016-04-05	0.4016	0.2459	0.1330	0.0637	0.0273	0.0175	0.0608	0.1469	0.2765	0.4391
2016-04-06	0.3919	0.2374	0.1266	0.0595	0.0249	0.0179	0.0624	0.1506	0.2826	0.4470
2016-04-07	0.3678	0.2172	0.1121	0.0507	0.0204	0.0200	0.0685	0.1624	0.3000	0.4687
2016-04-08	0.3538	0.2056	0.1039	0.0458	0.0179	0.0212	0.0720	0.1693	0.3103	0.4814
2016-04-11	0.3707	0.2183	0.1118	0.0499	0.0196	0.0183	0.0649	0.1575	0.2946	0.4634
2016-04-12	0.3682	0.2157	0.1095	0.0482	0.0187	0.0179	0.0645	0.1573	0.2952	0.4647
2016-04-13	0.3909	0.2333	0.1208	0.0542	0.0213	0.0148	0.0563	0.1429	0.2754	0.4416
2016-04-14	0.3957	0.2365	0.1224	0.0547	0.0214	0.0138	0.0537	0.1387	0.2702	0.4359
2016-04-15	0.3987	0.2383	0.1229	0.0547	0.0212	0.0129	0.0517	0.1355	0.2663	0.4320
2016-04-18	0.3761	0.2193	0.1096	0.0469	0.0174	0.0145	0.0569	0.1463	0.2827	0.4524
2016-04-19	0.3817	0.2232	0.1115	0.0475	0.0175	0.0133	0.0539	0.1414	0.2766	0.4458
2016-04-20	0.3692	0.2124	0.1037	0.0429	0.0153	0.0139	0.0563	0.1468	0.2853	0.4568
2016-04-21	0.3666	0.2096	0.1013	0.0413	0.0144	0.0135	0.0557	0.1466	0.2858	0.4582
2016-04-22	0.3750	0.2154	0.1043	0.0424	0.0147	0.0121	0.0518	0.1398	0.2772	0.4487
2016-04-25	0.3642	0.2061	0.0976	0.0386	0.0129	0.0124	0.0536	0.1443	0.2846	0.4582
2016-04-26	0.3698	0.2097	0.0992	0.0390	0.0129	0.0113	0.0504	0.1391	0.2782	0.4514
2016-04-27	0.3618	0.2025	0.0939	0.0359	0.0115	0.0114	0.0514	0.1421	0.2834	0.4582
2016-04-28	0.3592	0.1997	0.0914	0.0343	0.0107	0.0110	0.0508	0.1418	0.2840	0.4597
2016-04-29	0.3521	0.1932	0.0866	0.0315	0.0095	0.0110	0.0514	0.1442	0.2884	0.4657
2016-05-03	0.3736	0.2095	0.0961	0.0357	0.0109	0.0087	0.0439	0.1299	0.2688	0.4434
2016-05-04	0.3673	0.2036	0.0916	0.0331	0.0098	0.0086	0.0443	0.1316	0.2725	0.4485
2016-05-05	0.3648	0.2008	0.0890	0.0315	0.0090	0.0082	0.0436	0.1312	0.2730	0.4499
2016-05-06	0.3198	0.1648	0.0669	0.0213	0.0054	0.0114	0.0558	0.1573	0.3111	0.4946

Note: * SSE 50ETF call option expiring on the fourth Wednesday in June, with strike price at 1.8￥. ** SSE 50ETF put option expiring on the fourth Wednesday in June, with strike price at 1.8￥.

Table C4. Option Prices by Kou Model[4]

Date \ Contract	50ETF/Call/June/1.8 *	50ETF/Call/June/2.0	50ETF/Call/June/2.2	50ETF/Call/June/2.4	50ETF/Call/June/2.6	50ETF/Put/June/1.8 **	50ETF/Put/June/2.0	50ETF/Put/June/2.2	50ETF/Put/June/2.4	50ETF/Put/June/2.6
2016-02-01	0.3425	0.2416	0.1662	0.1121	0.0746	0.1452	0.2428	0.3660	0.5105	0.6715
2016-02-02	0.3668	0.2608	0.1807	0.1226	0.0819	0.1325	0.2251	0.3436	0.4842	0.6421
2016-02-03	0.3487	0.2469	0.1705	0.1156	0.0772	0.1441	0.2408	0.3631	0.5066	0.6668
2016-02-04	0.3624	0.2560	0.1761	0.1186	0.0786	0.1285	0.2207	0.3394	0.4805	0.6391
2016-02-05	0.3512	0.2449	0.1657	0.1094	0.0709	0.1261	0.2185	0.3379	0.4803	0.6404
2016-02-15	0.3360	0.2343	0.1589	0.1055	0.0689	0.1388	0.2356	0.3589	0.5041	0.6661
2016-02-16	0.3731	0.2630	0.1801	0.1204	0.0790	0.1184	0.2070	0.3227	0.4617	0.6190
2016-02-17	0.3820	0.2697	0.1846	0.1231	0.0805	0.1137	0.2000	0.3136	0.4509	0.6070
2016-02-18	0.3802	0.2676	0.1825	0.1213	0.0789	0.1128	0.1989	0.3125	0.4499	0.6063
2016-02-19	0.3775	0.2670	0.1835	0.1233	0.0813	0.1182	0.2064	0.3217	0.4601	0.6169
2016-02-22	0.4030	0.2861	0.1968	0.1319	0.0866	0.1027	0.1845	0.2940	0.4278	0.5812
2016-02-23	0.3910	0.2769	0.1902	0.1273	0.0835	0.1098	0.1944	0.3065	0.4424	0.5974
2016-02-24	0.3869	0.2712	0.1838	0.1212	0.0781	0.1026	0.1857	0.2971	0.4332	0.5890
2016-02-25	0.3142	0.2139	0.1411	0.0908	0.0574	0.1352	0.2336	0.3595	0.5080	0.6734
2016-02-26	0.3361	0.2320	0.1552	0.1012	0.0647	0.1310	0.2257	0.3476	0.4923	0.6546
2016-02-29	0.3166	0.2143	0.1401	0.0891	0.0554	0.1302	0.2267	0.3513	0.4990	0.6641
2016-03-01	0.3335	0.2274	0.1497	0.0959	0.0601	0.1201	0.2128	0.3339	0.4789	0.6419
2016-03-02	0.3941	0.2767	0.1878	0.1239	0.0800	0.0987	0.1802	0.2901	0.4250	0.5799
2016-03-03	0.3913	0.2724	0.1828	0.1190	0.0756	0.0927	0.1727	0.2819	0.4169	0.5723
2016-03-04	0.4463	0.3195	0.2210	0.1485	0.0974	0.0812	0.1533	0.2537	0.3801	0.5279
2016-03-07	0.4381	0.3129	0.2160	0.1448	0.0948	0.0849	0.1586	0.2606	0.3884	0.5373
2016-03-08	0.4392	0.3134	0.2158	0.1441	0.0939	0.0839	0.1569	0.2583	0.3855	0.5342
2016-03-09	0.4316	0.3046	0.2068	0.1359	0.0869	0.0781	0.1500	0.2512	0.3791	0.5291
2016-03-10	0.3904	0.2714	0.1816	0.1177	0.0744	0.0929	0.1727	0.2819	0.4169	0.5725
2016-03-11	0.4001	0.2795	0.1880	0.1226	0.0780	0.0905	0.1688	0.2763	0.4098	0.5642
2016-03-14	0.3996	0.2762	0.1832	0.1174	0.0731	0.0809	0.1565	0.2625	0.3956	0.5504
2016-03-15	0.4147	0.2889	0.1932	0.1249	0.0785	0.0777	0.1510	0.2543	0.3850	0.5376
2016-03-16	0.4493	0.3172	0.2149	0.1404	0.0891	0.0680	0.1349	0.2315	0.3561	0.5037
2016-03-17	0.4425	0.3114	0.2103	0.1370	0.0868	0.0692	0.1371	0.2350	0.3608	0.5096
2016-03-18	0.4482	0.3147	0.2117	0.1371	0.0862	0.0638	0.1294	0.2253	0.3498	0.4979
2016-03-21	0.4898	0.3498	0.2391	0.1572	0.1000	0.0556	0.1146	0.2030	0.3201	0.4619

4 The estimates are performed in Matlab 2015b.

Table C4. ***Cont.***

Date \ Contract	50ETF/Call/June/1.8 *	50ETF/Call/June/2.0	50ETF/Call/June/2.2	50ETF/Call/June/2.4	50ETF/Call/June/2.6	50ETF/Put/June/1.8 **	50ETF/Put/June/2.0	50ETF/Put/June/2.2	50ETF/Put/June/2.4	50ETF/Put/June/2.6
2016-03-22	0.4664	0.3299	0.2233	0.1455	0.0918	0.0604	0.1229	0.2154	0.3365	0.4819
2016-03-23	0.4664	0.3289	0.2216	0.1435	0.0899	0.0582	0.1198	0.2116	0.3325	0.4780
2016-03-24	0.4422	0.3082	0.2050	0.1310	0.0810	0.0625	0.1276	0.2236	0.3487	0.4977
2016-03-25	0.4434	0.3077	0.2032	0.1285	0.0784	0.0588	0.1222	0.2168	0.3412	0.4902
2016-03-28	0.4210	0.2910	0.1920	0.1217	0.0746	0.0696	0.1388	0.2389	0.3677	0.5197
2016-03-29	0.4055	0.2767	0.1798	0.1119	0.0674	0.0688	0.1392	0.2413	0.3726	0.5272
2016-03-30	0.4476	0.3087	0.2019	0.1260	0.0757	0.0522	0.1125	0.2048	0.3281	0.4769
2016-03-31	0.4433	0.3051	0.1990	0.1237	0.0740	0.0532	0.1141	0.2071	0.3310	0.4805
2016-04-01	0.4489	0.3100	0.2029	0.1267	0.0761	0.0529	0.1132	0.2053	0.3282	0.4767
2016-04-05	0.4517	0.3103	0.2014	0.1243	0.0736	0.0477	0.1055	0.1958	0.3179	0.4664
2016-04-06	0.4454	0.3054	0.1979	0.1220	0.0721	0.0498	0.1090	0.2007	0.3239	0.4733
2016-04-07	0.4214	0.2852	0.1823	0.1107	0.0646	0.0546	0.1177	0.2139	0.3416	0.4947
2016-04-08	0.4063	0.2711	0.1704	0.1017	0.0583	0.0530	0.1170	0.2154	0.3460	0.5018
2016-04-11	0.4229	0.2848	0.1805	0.1085	0.0624	0.0508	0.1120	0.2070	0.3341	0.4873
2016-04-12	0.4202	0.2824	0.1786	0.1071	0.0617	0.0503	0.1117	0.2071	0.3349	0.4887
2016-04-13	0.4436	0.2996	0.1893	0.1125	0.0635	0.0431	0.0984	0.1873	0.3098	0.4601
2016-04-14	0.4423	0.2986	0.1887	0.1123	0.0635	0.0429	0.0984	0.1877	0.3106	0.4611
2016-04-15	0.4439	0.2987	0.1877	0.1109	0.0622	0.0402	0.0943	0.1826	0.3050	0.4557
2016-04-18	0.4220	0.2793	0.1719	0.0992	0.0543	0.0417	0.0982	0.1902	0.3167	0.4711
2016-04-19	0.4241	0.2806	0.1726	0.0993	0.0542	0.0409	0.0967	0.1879	0.3139	0.4680
2016-04-20	0.4148	0.2729	0.1670	0.0958	0.0523	0.0425	0.0999	0.1933	0.3215	0.4773
2016-04-21	0.4079	0.2658	0.1606	0.0910	0.0490	0.0404	0.0976	0.1918	0.3214	0.4788
2016-04-22	0.4168	0.2721	0.1638	0.0917	0.0484	0.0383	0.0929	0.1839	0.3112	0.4672
2016-04-25	0.4044	0.2608	0.1547	0.0852	0.0444	0.0384	0.0941	0.1874	0.3173	0.4757
2016-04-26	0.4090	0.2643	0.1572	0.0869	0.0454	0.0365	0.0912	0.1834	0.3125	0.4704
2016-04-27	0.3982	0.2530	0.1466	0.0784	0.0393	0.0348	0.0890	0.1820	0.3130	0.4734
2016-04-28	0.3945	0.2488	0.1423	0.0745	0.0364	0.0340	0.0876	0.1805	0.3121	0.4734
2016-04-29	0.3902	0.2461	0.1413	0.0746	0.0369	0.0361	0.0914	0.1859	0.3186	0.4803
2016-05-03	0.4077	0.2593	0.1496	0.0790	0.0389	0.0317	0.0827	0.1724	0.3012	0.4606
2016-05-04	0.4031	0.2559	0.1474	0.0779	0.0384	0.0333	0.0855	0.1765	0.3063	0.4664
2016-05-05	0.3954	0.2456	0.1365	0.0687	0.0320	0.0286	0.0783	0.1686	0.3002	0.4630
2016-05-06	0.3538	0.2111	0.1116	0.0532	0.0236	0.0338	0.0905	0.1905	0.3316	0.5014

Notes: * SSE 50ETF call option expiring on the fourth Wednesday in June, with strike price at 1.8￥. ** SSE 50ETF put option expiring on the fourth Wednesday in June, with strike price at 1.8￥.

References

Barndorff-Nielsen, O., 1977. Exponentially decreasing distributions for the logarithm of particle size. Proceedings of the Royal Society A 353, 401–419.

Barndorff-Nielsen, O.E., Shephard, N., 2006. Econometrics of testing for jumps in financial economics using bipower variation. Journal of Financial Econometrics 4, 1–30.

Beliaeva, N., Nawalkha, S., 2012. Pricing American interest rate options under the jump-extended constant-elasticity-of-variance short rate models. Journal of Banking and Finance 36, 151–163.

Bo, L., Wang, Y., Yang, X., 2010. Markov-modulated jump-diffusions for currency option pricing. Insurance: Mathematics and Economics 46, 461–469.

Cai, N., Kou, S.G., 2011. Option pricing under a mixed-exponential jump diffusion model. Management Science 57, 2067–2081.

Cai, N., Kou, S.G., 2012. Pricing Asian options under a hyper-exponential jump diffusion model. Operations Research 60, 64–77.

Chen, W.T., Yan, B.W., Lian, G.H., Zhang, Y., 2016. Numerically pricing American options under the generalized mixed fractional Brownian motion model. Physica A: Statistical Mechanics & Its Applications 451, 180–189.

Cheng, P., Scaillet, O., 2007. Linear-quadratic jump-diffusion modeling. Mathematical Finance 17, 575–598.

Chiarell, C., Kang, B., 2013. The evaluation of American compound option prices under stochastic volatility and stochastic interest rates. Journal of Computational Finance 17, 71–92.

Eraker, B., 2001. MCMC analysis of diffusion models with application to finance. Journal of Business & Economic Statistics 19, 177–191.

Espinosa, F., Vives, J., 2006. A volatility-varying and jump-diffusion Merton type model of interest rate risk. Insurance: Mathematics and Economics 38, 157–166.

Fajardo, J., Farias, A., 2010. Derivative pricing using multivariate affine generalized hyperbolic distributions. Journal of Banking & Finance 34, 1607–1617.

Geman, S., Geman, D., 1984. Stochastic relaxation, Gibbs distributions, and the Bayesian restoration of images. IEEE Transactions on Pattern Analysis and Machine Intelligence 6, 721–741.

Gil-Pelaez, J., 1951. Note on inversion theorem. Biometrika 38, 481–482.

Gilks, W.R., Best, N.G., Tan, K.K.C., 1995. Adaptive rejection Metropolis sampling within Gibbs sampling. Journal of the Royal Statistical Society. Series C (Applied Statistics) 44, 455–472.

Glasserman, P., 2004. Monte Carlo Methods in Financial Engineering. Springer, New York, NY, USA.

Glasserman, P, Kou, S.G., 2003. The term structure of simple forward rates with jump risk. Mathematical Finance 13, 383–410.

Gong, X.L., Zhuang, X.T., 2016. Option pricing for stochastic volatility model with infinite activity Lévy jumps. Physica A: Statistical Mechanics & Its Applications 455, 1–10.

Han, C.H., Molina, G., Fouque, J.P., 2010. MCMC estimation of multiscale stochastic volatility models with applications. Mathematics & Computers in Simulation 103, 1–11.

Hu, S.H., Zhang, S.Y., Zhang, T., 2006. Using MCMC method to estimate double exponential jump diffusion. Journal of Systems Engineering 21, 113–118.

Huang, J., Zhu, Wenli, W., Ruan, X., 2014. Option pricing using the fast Fourier transform under the double exponential jump model with stochastic volatility and stochastic intensity. Journal of Computational and Applied Mathematics 263, 152–159.

Johannes, M., 2004. The statistical and economic role of jumps in continuous-time interest rate models. Journal of Finance 59, 227–260.

Kiesel, R., Rahe, F., 2017. Option pricing under time-varying risk-aversion with applications to risk forecasting. Journal of Banking and Finance 76, 120–138.

Kim, B., Kim, J.H., 2011. Default risk in interest rate derivatives with stochastic volatility. Quantitative Finance 11, 1837–1845.

Kim, J.H, Yoon, J.H., Yu, S.H., 2014. Multiscale stochastic volatility with the Hull-White rate of interest. Journal of Futures Markets 34, 819–837.

Kleinert, H., Korbel, J., 2016. Option pricing beyond Black–Scholes based on double-fractional diffusion. Physica A Statistical Mechanics & Its Applications 449, 200–214.

Kou, S.G., 2002. A jump-diffusion model for option pricing. Management Science 48, 1086–1101.

Kou, S.G., Wang, H., 2004. Option pricing under a double exponential jump diffusion model. Management Science 50, 1178–1192.

Leippold, M., Wu, L., 2002. Asset pricing under the quadratic class. Journal of Financial & Quantitative Analysis 37, 271–295.

Leippold, M., Schärer, S., 2017. Discrete-time option pricing with stochastic liquidity. Journal of Banking and Finance 75, 1–16.

Leippold, M., Vasiljevi'c, N., 2017. Pricing and disentanglement of American puts in the hyper-exponential jump-diffusion model. Journal of Banking and Finance 77, 78–96.

Li, L., Mendoza-Arriaga, R., Mitchell, D., 2016. Analytical representations for the basic affine jump diffusion. Operations Research Letters 44, 121–128.

Lu, B., Hua, R.H., 2010. Research on collaborative herding behavior and market volatility: Based on computational experiments. Journal of Management Sciences in China 13, 98–106

Maekawa, K., Lee, S., Morimoto, T., Kawai, K., 2008. Jump diffusion model with application to the Japanese stock market. Mathematics and Computers in Simulation 78, 223–236.

Martino, L., Read, J., Luengo, D., 2015. Independent doubly adaptive rejection Metropolis sampling within Gibbs sampling. IEEE Transactions on Signal Processing 63, 3123–3138.

Merton, R.C., 1976. Option pricing when underlying stock returns are discontinuous. Journal of Financial Economics 3, 125–144.

Meyer, R., Cai, B., Perron, F., 2008. Adaptive rejection Metropolis sampling using Lagrange interpolation polynomials of degree 2. Computational Statistics & Data Analysis 52, 3408–3423.

Pillay, E., O'Hara, J.G., 2011. FFT based option pricing under a mean reverting process with stochastic volatility and jumps. Journal of Computational and Applied Mathematics 235, 3378–3384.

Ramezani, C.A., Zeng, Y., 1998. Maximum Likelihood Estimation of Asymmetric Jump-Diffusion Processes: Application to Security Prices. SSRN Working paper. Available online: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=606361 (accessed on 18 December 2017).

Simonato, J.G., 2011. Computing American option prices in the lognormal jump–diffusion framework with a Markov chain. Finance Research Letters 8, 220–226.

Shreve, S.E., 2004. Stochastic Calculus for Finance I & II. Springer-Verlag, New York, NY, USA.

Wang, X.Y, Xie, D.J., Jiang, J.J., Wu, X.X., He, J., 2016. Value-at-Risk estimation with stochastic interest rate models for option-bond portfolios. Finance Research Letters 21, 10–20.

Wong, H.Y., Lo, Y.W., 2009. Option pricing with mean reversion and stochastic volatility. European Journal of Operational Research 197, 179–187.

Yang, R.C., Qin, X.Z., Zhou, Y.Y., 2010. Parameter estimation of CNY exchange rates of jump-diffusion model using mixed algorithm of jump identification and MCMC. Systems Engineering-Theory & Practice 30, 2165–2171.

Yu, C.L., Li, H.T., Wells, M.T., 2011. MCMC estimation of Lévy jump models using stock and option prices. Mathematical Finance 21, 383–422.

Zhang, L., Zhang, W., Xu, W.J., Xiao, W.L., 2012. The double exponential jump diffusion model for pricing European options under fuzzy environments. Economic Modelling 29, 780–786.

Zhang, S.M., Wang, L.H., 2013. A fast numerical approach to option pricing with stochastic interest rate, stochastic volatility and double jumps. Communications in Nonlinear Science and Numerical Simulation 18, 1832–1839.

Zhou, W., He, J.M., Yu, D.J., 2013. Double-jump diffusion model based on the generalized double exponential distribution of the random jump and its application. System Engineering Theory and Practice 33, 2746–2756.