TOWARDS A TYPOLOGY OF SPATIAL DECISION PROBLEMS

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Abstract. The aim of this paper is to establish a typology of spatial decision problems. The proposed typology is based on the nature of the spatial entities that we assign to potential spatial alternatives. Indeed, one alternative may be assimilated to one atomic spatial entity (i.e., point, line, polygon, network), or to a combination of several atomic spatial entities. The association between spatial entities and potential alternatives implies that these last ones have all the characteristics and the dimensions of the first and that are subject to the same spatio-temporal changes. The proposed typology is particularly useful to develop tools for multicriteria spatial decision-aid.

Spatial problems, Spatial decision-aid, Decision alternatives, Problem formulation, Typology, Multicriteria analysis.

1. Introduction

Spatial problems involve several geographic objects and phenomena, which have an inherent dynamic nature. In addition, these problems have a multidisciplinary characteristics that require the consideration of several quantitative and qualitative, generally non commensurable, evaluation criteria, and imply many individuals and institutions having conflicting preferences and objectives. All of these elements confer a dynamic and multicriteria complex nature to spatial problems. Along with the nature of decision problems, and whether real-world dynamic nature is considered or not and the manner by which this dynamic nature is handled, spatial decision problems may be addressed according to one of four possible formulations: static, temporal, sequential and dynamic. Each of these formulations requires a different form of data and call for different modelling and resolution techniques. It follows thus that the adoption of a specific formulation will have major effects on problem modelling and resolution.

On the other hand, most of spatial decision problems correspond to Roy’s (1985) choice problem-atic, where a restrained set of potential alternatives should be selected from an initially large set of feasible alternatives. In spatial context, decision alternatives are usually assimilated to geographic entities. Consequently, they have all the dimensions and the characteristics of these last ones and are subject to the same spatio-temporal changes. In addition, the type of the spatial entity by which alternatives are represented and modelled determines largely the temporal and spatial operators and analyzes routines which one can apply to evaluate and then compare these alternatives. Additionally, spatial decision problems are the concern of researchers from diverse disciplines that have different concepts and paradigms. Thus, to improve the understanding of the aspects and the specificities of these problems and to better select the adequate modelling and resolution techniques, we think that the elaboration of a classification of spatial problems in a purely abstract form, devoid of any socio-economic, political and environmental contexts, is a good starting point. The objective of this paper is thus to present an alternative-based typology of spatial problems which is obtained by crossing the different types of alternatives (as spatial entities) with the different formulations of spatial problems. The proposed typology is based on simple concepts and paradigms on which researchers from different domains agree. Consequently, it constitutes an adequate framework for multidisciplinary researches.

The rest of the paper is structured as follows. In section 2 we present some existing typologies. Section 3 is devoted to present some characteristics of spatial entities and to illustrate their inherent dynamic nature. Next in section 4, we distinguish the different possible formulations of spatial decision problems. Section 5 shows how spatial decision alternatives can be represented
with spatial entities. Then section 6 is devoted to present our alternative-based typology. Following in section 7, we briefly show some uses of the typology. Finally, section 8 concludes the paper.

2. Some existing typologies

An intuitive classification of spatial decision problems is the one based on the socio-economic and/or environmental domain to which the problem refers. We distinguish, for instance, site selection, land suitability, land use planning, service coverage, resource allocation and shortest path problems.

Spatial problems, as aspatial ones, can also be regrouped according to the quantity and the type of available information into (Leung, 1988; Munda, 1995; Malczewski, 1999): (a) deterministic problems (b) stochastic problems, and (c) fuzzy problems, which are respectively based on the use of perfect, probabilistic, and imprecise information.

Keller (1990) classifies spatial decision problems according to the number of criteria and the number of deciders that they involve. He identifies four classes: one class contains some problems that involve only one criterion and only one decision-maker, two classes containing some more problems which involve respectively only one criterion and several decision-makers or several criteria and only one decision-maker, and a class that contains problems that involve several criteria and several decision-makers. Keller (1990) points out that most of spatial decision problems belong to this last class.

Jankowski (2003) subdivides land-related decision problems into four common types: (a) site (or location) selection, concerned with the rank of a set of sites in priority order for a given activity (e.g. what site might best be for a particular type of business?), (b) location allocation concerned with stating a functional relationship between the attributes of the land and the goal(s) of the decision-maker(s) (e.g. where to locate new fire station so that the least amount of the population has no more than a 10 minute response time?), (c) land use selection (or alternative uses) looking in ranking the uses for a given site in priority order (e.g. given a property, what can it be used for?), and (d) land use allocation looking in defining the best uses for an array of sites (e.g. how much of the land should be allocated for the following uses: forestry, recreation, and wild life habitat?).

The first classification seems to be of general interest. However, it tells nothing about the specificity of each family of problems and focalizes only on the socio-economic and/or environmental contexts to which problems belong. The two next typologies are not specific for spatial contexts. Besides, groups in both of them are large, making it difficult to extract out their common and general characteristics. Moreover, in the second classification one problem can be assigned indifferently to the three groups, according to the accuracy of the available data, which may evolve over time. Additionally, in real-world problems we usually make use of a mixture of deterministic, probabilistic and fuzzy data, which complicate the assignment of the problem under study to a given class. The fourth typology focuses only on land use and consequently ignores a large spectrum of spatial problems. Finally, all typologies do not include explicitly the temporal context of spatial decision problems and ignore the specificities of spatial decision alternatives.

3. Spatial entities and their dynamics

Maguire et al. (1991) distinguish four families of spatial entities: (i) physical objects such as towns, buildings, houses, highways, etc., (ii) administrative units like communes, provinces, counties, parks, etc., (iii) geographic phenomena such as temperature, disease distribution, wind fields, etc., and (iv) derived information representing non-real phenomena such as ecological and environmental impacts of a nuclear central, criminality rate, poverty rate, suitability for cultivation, etc. Notice that in geographical information system (or GIS) community, geographic phenomena (or non-real phenomena) are considered as spatial entities when they are geographically located with their proper attributes (e.g. temperature or precipitation in Paris), with the geometric forms that represent them on maps (e.g. Severe Acute Respiratory Syndrome, or SARS, disease distribution in southern-east Asia may be represented with several polygons corresponding to the affected countries or zones), or with both of them [Bedard, 1991; cited in Laaribi (2000)].

Whatever their natures, spatial entities have several characteristics that distinguish them from aspatial data. These characteristics are generally regrouped into several dimensions. Stefanakis and Sellis (2000) enumerate six dimensions along which spatial entities are defined. The ones
that are relevant in this paper are: (a) thematic (or descriptive) dimension which describes the aspatial characteristics of entities (e.g. soil type, parcel number, color, PH, civil address, etc.), (b) spatial dimension that describes the spatial characteristics of geographic entities in terms of position, geometry and topology, and (c) temporal dimension which describes the temporal characteristics of geographic entities in terms of temporal position that represents the occurrence (e.g. date of facility construction), or duration (e.g. period of land ownership) of spatial entities in or over time, temporal behavior which refers to the evolution of geographic entities in time, and temporal topology that describes the spatial and functional relationships between geographic entities induced by their temporal position or behavior.

The association of thematic and spatial dimensions with temporal dimension confers an inherent dynamic nature to spatial entities. This dynamic nature is the result of a series of changes (natural or not) that touch one or several of thematic and/or spatial characteristics of geographic entities. Basing on the works of Claramunt et al. (1997) and Frank (1999), Lardon et al. (1999) distinguish three types of changes induced by time over spatial entities:

- **Thematic changes.** They refer to the ones that affect the descriptive characteristics of geographic entities without modifying their existence and their spatial extensions (e.g. evolution of the population of a town).
- **Spatial changes.** They refer to changes in the spatial characteristics (i.e., position, geometry and topology) of entities that modify their spatial extensions (e.g. successive extensions of urban fabric of an agglomeration) or result in the movement of these entities (e.g. movement of an ambulance in service), without altering their existence.
- **Identity changes.** They refer to changes that modify the identity of geographic entities: entities may be divided, regrouped, combined, etc., (e.g. definition of new cadastral or new pasture units).

Actually, spatial entities are subject to a mixture of changes. A forest fire front or a flock of animals moves and changes form, velocity, and direction. Yet, a plane in navigation or an ambulance in service move without changing its form. In the management of a hydraulic system, it is generally the level of water that changes. In a forest management problem, changes may affect the form (as a result of the plantation of new zones, or disappearance of some other zones following, for example, an excessive exploitation or a severe drought), the topological relations (as a result of constructing new routes) and/or the thematic characteristics (introduction of new animal or vegetal species). Nevertheless, in practice attention is generally limited to only some aspects of changes because some ones are not pertinent to the problem in focus or because some others are conducted in a very low rhythm in comparison with human life (e.g. movement of continents).

In literature, dynamics of real-world is essentially addressed in database-oriented contexts, where the main objective is to represent and digitize geographic entities and their dynamics. However, here this dynamic nature should be appreciated in terms of the spatio-temporal evolution of the consequences and impacts of spatial decision alternatives. Indeed, these consequences and impacts are usually measured via thematic and spatial characteristics of spatial entities. Accordingly, changes that affect spatial entities will necessarily affect their impacts and consequences.

### 4. Different Formulations of Spatial Decision Problems

Along with the nature of the problem and the objective of the study, spatial entities may be classified into static or dynamic. Static entities are those which have neither space nor time-varying characteristics (e.g. buildings, mountains). On the contrary, dynamic entities have at least one of their spatial or descriptive characteristics that varies over space and/or in time (e.g. lacks, rivers, highways where traffic rate changes dynamically, etc.).

In practice, spatial decision problems may involve static or dynamic entities. This correspond to two different perceptions of decision environment:

- **a static perception** in which the inherent dynamic nature of geographic entities and of their functional and spatial interactions are not recognized because they have no significant effects on the achievement of the decision-making process, or because their handling is expensive and/or complicated, or
- **a dynamic perception** in which the evolutionary nature of decision environment is explicitly integrated in the problem formulation and modelling.
In several situations, the dynamic nature of real-world is considered to have no effects on the outcomes of the decision-making process. In practice, however, the high equity of spatial decisions and their long-term impacts on population and environment impose, to some extent, an explicit incorporation of the dynamic nature of real-world into problem formulation, modelling and resolution. One possible way to take into account the dynamic nature of real-world while preserving a static vision is to make large enough all decision variables and parameters in order to be able to respond to any future evolution of real-world. This idea may be applicable in some simple situations. However, it will be unsuitable in several practical situations where changes are not linear and/or unpredictable.

An explicit integration of real-world dynamics requires the availability of perfect predictions of real-world changes. In some situations changes of real-world may be captured quite accurately through forecasting and projection of current trends. However, in several other practical situations, it is not possible (or difficult and/or expensive) to produce such predictions. The solution generally adopted in similar situations consists in a decomposition of the initial problem into several sub-problems which are addressed sequentially in time. This permits to reduce partially the uncertainty related to unpredictable changes through a sequential information-acquisition process. The incorporation of the dynamic nature of real-world may also be imposed by the dynamic nature of the spatial system to which the decision problem refers.

Consequently, along with the nature of decision problems, and whether real-world dynamic nature is considered or not and the manner by which this dynamic nature is handled, spatial decision problems may be addressed according to one of four possible formulations: static, temporal, sequential and dynamic. The following paragraphs provide brief descriptions of these formulations.

**Static formulation.** In static situation, the attention is focalized on a unique decision, which should be tackled basing on punctual information that is available in the moment of making the decision and which represents a snapshot view of current (or predicted) real-world. In such situation, we suppose that the decision environment is stable. Accordingly, static formulation applies to problems with no or less equity and involves short or medium-term spatial decisions.

**Temporal formulation.** Here attention is focalized on a unique decision which, contrary to the previous formulation, should be tackled on the basis of dispersed information. This dispersed information represents predictions of changes that are often represented through time series-like functions. Thus, temporal formulation can be seen as an extension of a static one. However, it differs with its strategic aspect which imposes a deeper understanding of spatial actions and requires the consideration of their future socio-economic, environmental and political impacts. Practically, this formulation applies to problems with high equity that involve long-term spatial decisions.

**Sequential formulation.** In spatial context, it is usually the high level of uncertainty or the considerable financial, economic and human requirements that makes deciders behave as sequential decision-makers. In both cases, the initial problem is decomposed into a series of related sub-problems; each one is treated as a static decision problem, where a unique decision should be tackled basing on the available punctual information. Hence, we assist to a series of decisions dispersed over time, offering to the decider substantial opportunities of apprenticeship. This enables him/her to take into account impacts of previous decisions and to receive additional information, and consequently, to reduce (partially) the effects of uncertainty.

**Dynamic formulation.** Contrary to the three previous formulations, this one is oriented towards the control and tracking of spatial dynamic systems. It applies to problems where: (a) a series of decisions should be tackled over time to reach a global objective, (b) these decisions are interdependent, and (c) the decision environment is dynamic; it is subject to several changes that may result from natural phenomena and/or induced by decision-maker’s previous decisions. The main objective of dynamic decision-making is to maintain the spatial system under consideration in a dynamic equilibrium situation.

Static, sequential and temporal formulations are more suitable to Simon’s (1960) decision-making process phases (i.e., intelligence, design and choice or selection) because they represent situations where "we have time to act". On the contrary, dynamic formulation requires that decisions are
made under time pressure. Hence, it is concerned mainly with choice phase rather than intelligence or design ones.

On the other hand, these formulations respond to different objectives. Selecting a particular formulation will depend on both the nature of the problem and the objective of making a decision. Static and temporal ones occur generally in decision-aid perspectives. In static situation we suppose that the decision environment is stable, minimizing hence its effects on the decision-making process. In turn, in temporal formulation, the dynamic nature of decision environment is explicitly integrated in the problem modelling and resolution. Consequently, static and temporal formulation respond respectively to short-term decision-aid and long-term decision-aid perspectives. Sequential formulation applies to spatial context essentially when spatial management and planning problems are seen as pure investment ones in which financial aspects are the more relevant elements for the decision-maker(s). In this case, the elaboration of a sequential decision logic for handling the problem permits to reduce progressively the uncertainty, which will ameliorate the achievement of the decision-makers’ objectives. The dispersed nature of sequential decisions does not means that dynamic aspects of real-world are put in consideration. In fact, each sub-problem represents a static decision situation but the fact that these problems are resolved successively implies that the decision-maker takes the new decision after appreciating the consequences of pervious ones. Accordingly, sequential formulation of (spatial) decision problems are mainly interested in the sequential search for information to be used in the decision-making process (Diederich, 1999). Unlike the previous situations, dynamic one manifests essentially in a problematic of control and tracking of dynamic spatial systems and/or of moving objects and/or phenomena and, contrary to the temporal formulation, operates in continually changing decision environment.

Finally, it is important to notice that these formulations, whose their characteristics are summed up in Table 1, may apply also to non spatial decision problems and are not always crisply defined.

<table>
<thead>
<tr>
<th>Formulation</th>
<th>Static</th>
<th>Temporal</th>
<th>Sequential</th>
<th>Dynamic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decision environment</td>
<td>Stable</td>
<td>Dynamic</td>
<td>Stable</td>
<td>Dynamic</td>
</tr>
<tr>
<td>Nature of information</td>
<td>Punctual</td>
<td>Dispersed</td>
<td>Punctual</td>
<td>Dispersed</td>
</tr>
<tr>
<td>Type of decision</td>
<td>Unique decision</td>
<td>Unique decision</td>
<td>Series of decisions</td>
<td>Series of decisions</td>
</tr>
<tr>
<td>Objectives</td>
<td>Short-term decision-aid</td>
<td>Long-term decision-aid</td>
<td>Sequential search for information</td>
<td>Control of dynamic systems</td>
</tr>
</tbody>
</table>

5. Representing spatial decision alternatives

Spatial decision alternatives are defined with at least two elements (Malczewski, 1999): (a) action (what to do?) and (b) location (where to do it?). In a dynamic decision situation, a third element is required to define spatial decision alternatives: (c) time (when to do it?). Even though in temporal situation (as it is defined here) the temporal dimension is explicitly integrated into the problem formulation and modelling, attention is essentially focalized on one decision, and the time when this decision is performed is a priori with no importance. This is because the major objective of the temporal formulation of spatial problems is to take into account the future impacts of spatial decisions. In sequential formulation, the time dimension intervenes implicitly in the problem modelling because it is not formally expressed in the model but it can be deduced. It has simply a repartitioning role and different temporal points correspond in fact to the logic succession of a series of activities, events, phenomena or, in a decision-aid context, a series of decisions.

In each (spatial) decision problem, we assign to each alternative one or several decision variables, permitting us to measure the performance of this alternative. These variables may be binary, discrete or continuous. Illustrating this with some examples inspired from Malczewski (1999). In a nuclear waste deposit location problem, for instance, the decision "locate the deposit at site
“is an action geographically located (via the site address, for example). The binary variable associated to each potential alternative (site) is the binary decision “construct the deposit at site $x$” or “not construct the deposit at site $x$”. In a school location problem, we may be concerned with the size of the school in terms of the number of students to be affected to it. So, to each alternative and in addition to the binary locational variable, we assign a discrete variable which determines the size of the school. If we return to the nuclear waste deposit location problem, one may also be called to use a new continuous variable to measure the deposit area.

The performances of spatial alternatives according to different decision variables are essentially determined by their spatial and descriptive characteristics. As we have mentioned above, these characteristics vary across space and time. Hence, a better evaluation (and then comparison) of these alternatives imposes an explicit undertaking of their dynamic characteristics during problem modelling and resolution phases.

On the other hand, in (multicriteria) spatial decision-aid activity, we generally represent alternatives through one of four atomic geographic entities, namely point, line, polygon and network. Therefore, in a facility location problem, alternatives take the form of points representing different potential sites; in a linear planning problem (e.g. highway construction), alternatives take the form of lines representing different possible routes; and in the problem of identification and planning of a new industrial zone, alternatives are assimilated to a set of polygons representing different candidate zones. Table 2 below contains some more examples of typical problems where these atomic alternatives may intervene.

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Typical problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point</td>
<td>Site selection: points represent different potential sites</td>
</tr>
<tr>
<td>Line</td>
<td>Highway layout identification: lines represent possible routes</td>
</tr>
<tr>
<td>Polygon</td>
<td>Evaluation of construction zones: polygons represent different construction zones</td>
</tr>
<tr>
<td>Network</td>
<td>Goods distribution: networks are the different distribution policies</td>
</tr>
</tbody>
</table>

Even though a ‘network’ entity is a composed one, it is introduced here as an atomic primitive in order to handle some applications in which attention is focalized on the identification of some distribution networks. In a petroleum distribution problem, for instance, networks always refer to different policies of distribution, where nodes represent demand points and arcs represent routes between these demand points. Networks define different system of routes, each one is characterized with its level of coverage, transportation cost, deliverance rapidity and so on. The same remark holds for public services distribution (e.g. electricity and heat) where networks are the different possible configurations which differ with their implementation costs, coverage levels, their responses to congestion and saturation situations, etc.

The association between spatial entities and spatial alternatives means that these last ones have all the characteristics and the dimensions of the first, and that are subject to all the changes mentioned in §3. In the rest of this paper, an alternative refers implicitly to a spatial entity. At this level, it is important to notice that punctual, linear, polygonal and network alternatives are better handled through vector-based GISs while raster-based GISs are more suitable for map-based alternatives (see §6.3 for alternatives based on map structures). Generally, problems of land use and those related to the management of public infrastructures such as water pipes, telephone and electricity networks, highways, etc., can be handled more efficiently by using the vector-based GISs, while in problems related to soil science, the raster-based GISs is an appropriate tool. However, the recent development of new technologies and algorithms, permitting the conversion from one structure to another, allows vector and raster alternatives to be used at the same time. Finally, notice that in raster-based GISs, punctual, linear and polygonal alternatives will be represented respectively with an individual pixel, a set of linearly adjacent pixels and a collection of pixels. Networks require combination of individual pixels with several pixel-based linear entities.

6. Proposed typology

One way to classify spatial decision problems is the one based on the type of decision alternatives that they imply. Accordingly, we may distinguish four basic families of spatial decisions
problems which correspond to the four atomic types of alternatives (see Table 2). This classification is mainly useful to define the types of operators and spatial routines susceptible to be used in the evaluation and comparison of potential alternatives. However, this classification ignores the inherent dynamic nature of spatial problems. Earlier, we have seen that depending on the nature of the problem and the objective of the study, a spatial problem may be formulated as a static, temporal, sequential or a dynamic decision problem. The adoption of a given formulation will determine largely the problem modelling and resolution. Thus, to improve the understanding of the aspects and the specificities of spatial problems and to better select the adequate modelling and resolution techniques and to better define the spatial operators and routines susceptible to be used, an alternative-based typology of spatial decision problems is detailed hereafter.

The proposed typology is based on a crossovering of the different types of alternatives with the different formulations of spatial problems. It is summed up in Figure 3. Two major classifications can be distinguished in this figure: a problem formulation-oriented typology and an alternative-oriented typology.

6.1. Problem formulation-oriented typology. The first typology contains four major families of spatial problems which map to the four possible formulations of these problems. In the following paragraphs we will focus only on data structures and modelling and resolution techniques required for each family of problems. Notice that the following descriptions apply to all types of alternatives; and their presentation here will avoid redundancy.

Spatio-static decision problems. This family regroups problems with less equity, where the dynamic nature of real-world is ignored. Technics used to resolve spatio-static problems are fundamentally static because they do not integrate explicitly the dynamic aspects of real-world. Examples of techniques include linear programming, multicriteria analysis, network analysis models, simulated annealing, neural networks, graph theory, multi-agents systems, genetic algorithms, flow analysis, etc. In all cases, evaluation and comparison of potential alternatives are based on punctual information issued from direct and punctual (in time) measurements of alternatives’ attributes. These punctual information may also be issued from spatial and/or temporal total aggregations of dispersed data (e.g. average annual precipitation in a given region), an extrapolation of past data in the present time or a projection of current trends of real-world in the future. These information can be supported easily by conventional spatial data management systems (e.g. GIS).

Spatio-temporal decision problems. This family regroups problems with high equity, where the dynamic nature of real-world is explicitly integrated in the problem modelling and resolution. This requires anticipations of future facts and events. Several techniques may be combined with static models in order to predict future and integrate effects of natural, social, economic, etc., transformations in modelling spatial problems as, for instance, those based on probabilistic representations or those that use belief functions or fuzzy sets. However, these techniques are based on a total temporal aggregation, which generates problems of temporal compensation. Some more elaborated techniques are also available: animation (morphing) techniques, spatio-temporal Makovian models, time series, regression equations, etc. The explicit integration of time dimension in spatial decision-aid context requires the use of dispersed and evolutionary information that permit to retrace spatial and temporal evolution of spatial entities. These dispersed information may take the form of a series of time-indexed values (e.g. population of a town taken at different dates, mensual precipitations of a given region, etc.) or the form of a discrete function (e.g. representing population evolution of a town with a function \( p(t), t \in \mathbb{Z} \)). In both cases, a temporal spatial data management system (e.g. temporal GIS) is necessary for handling evolution of facts and events over time.

Spatio-sequential decision problems. Most of sequential problems are not spatial ones. However, several spatial problems may be formulated as sequential decision ones (especially by risk-averse deciders) mainly when there is a high level of uncertainty. Tools as strategic choice approach and robustness analysis are often used to deal with non spatial problems characterized by a significant level of uncertainty. These tools may apply also to spatial problems essentially when only financial aspects of these problems are considered. However, they are of limited use when a variety of different social and environmental criteria should be included in the study. In addition, the two tools are more graphical technologies rather than formal mathematical formulations.
are many other more formal tools based on solid mathematical formulations such as Markovian decision models, dynamic programming tools, discounted utility-based models, etc. Nevertheless, most of these tools do not exploit the spatial characteristics of the problem components because they are initially conceived and used for non-spatial decision problems and they focalize only on the financial aspects of the problem. As in the first family, this one does not consider the dynamic nature of real-world and evaluation and comparison of potential alternatives are based on punctual information. However, in this case this information is often characterized by a high level of uncertainty and/or fuzziness, which require tools that are able to handel uncertain and fuzzy spatial data.

Spatio-dynamic decision problems. This last family regroups problems related to the control of dynamic systems or to the tracking of moving objects or phenomena. In dynamic system-related problems, we usually consider that geographic position does not vary over time while at least one of the other characteristics (description, geometry or topology) is time-varying. In these problems we are interested in the study of trajectories followed by spatial entities in order to analyze their evolution and behavior. In problems of tracking of moving objects or phenomena, the geographic position is time-varying; the other characteristics may or not vary over time. In both cases, we consider that real-world changes continuously. Equally in both cases, decisions need to be made under time pressure, especially in tracking problems where the timing of decisions has important effects on the achievement of objectives. Techniques used to handel spatio-dynamic problems include dynamic programming, multiobjective dynamic optimization, differential equations, cellular automata, simulation techniques, system dynamics, etc. In dynamic situation, the required information evolves continuously and often represented through continuous functions (e.g. representing movement of spatial engine with function \(m(x, y, z, t)\), \(t \in \mathbb{R}\) which gives positions \((x, y, z)\) taken by the engine in different time instants \(t\)). A better handling of this kind of information necessitates the development of real-time spatial data management systems.

6.2. Alternative-oriented typology. The second typology is an alternative-oriented one. It is detailed hereafter. It contains four major families which map to the four types of alternatives. Each of these families contains four sub-families which correspond to the four problem formulations. In the list below, each family is briefly described but only sub-families of punctual and polygonal-based decision alternatives are detailed. It is important to notice that several didactic examples are discussed in the following paragraphs and the data used in all of them are artificial.

6.2.1. Punctual alternative-based spatial decision problems. This family regroups punctual alternatives-based decision problems. Punctual alternatives are usually used to represent potential sites in location (or selection)-related problems, particularly when only geographic location is considered. Punctual alternative-based spatial decision problems may be further decomposed into four sub-families.

Punctual alternative-based spatio-static decision problems. This sub-family contains location problems in which the dynamic nature of the decision environment is neglected. Figure 1.(a) represents the static formulation of a didactic example of a nuclear waste deposit location problem. Two siting criteria are considered: implementation cost and impacts on environment. Thus, we associate to each cell two values representing measurements of the two siting factors (values in cells of Figure 1.(a) represent respectively implementation costs and level of impacts on environment). Here, the two factors are considered stable over time. If we use a weighed sum (regards to its limits) as a decision rule and we consider that the two factors are of equal importance, we obtain three cells that minimize the average sum (circled cells in Figure 1.(a)); each one will give a sum of 11 (we suppose that equal weights of 1 are used). In practice, however, the selection of the suitable location is more delicate than this didactic example, and usually multicriteria evaluation procedures are used.

Punctual alternative-based spatio-temporal decision problems. Actually, most of practical location applications are handled as static decision problems. This may be true for facilities with no or less equity and with no or limited impacts on population and environment. However, facilities like airports, nuclear centers, hypermarkets, hospitals, universities, stadiums, and so on, have inevitably long-range impacts and depend on several socio-economic, environmental, ecological and political criteria which evolve over time. Figure 1.(b) represents the temporal formulation of
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Figure 1. Punctual alternative-based spatial decision problems: (a) Static formulation, (b) Temporal formulation, (c) Sequential formulation, and (d) Dynamic formulation

The nuclear waste deposit location problem cited above. Which makes the difference between the situation in Figure 1.(a) and the situation in Figure 1.(b) is that in the latter one, implementation costs (we suppose that costs in $t_2$ and $t_n$ are relative to operating and maintenance of the facility to be located because the implementation cost is considered to be committed in $t_1$) and impacts on environment are considered as time-varying ones and they are expressed as series of values representing measurements of the two factors in different dates (only three dates are represented in Figure 1.(b)). Here, we recognize that siting a nuclear waste deposit is a long-term investment decision implying several impacts on environment that vary across space and in time. For instance, the type of soil will have major roles in reducing or increasing impacts on environment. Taking into account the soil type will reduces substantially long-term impacts on environment. As an example, the double circled cell in Figure 1.(b) has better long-term performances than the

(a)

(b)

(c)

(d)
three cells selected in the previous paragraph. In terms of modelling, the question that holds is: knowing the predicted states of real-world and looking at minimizing long-term costs and impacts, which cell(s) should one select at time $t_1$ for implementing the deposit?

**Punctual alternative-based spatio-sequential decision problems.** As we have signaled above, spatial problems are formulated as sequential decision problems when there is a high level of uncertainty and/or when they are of high equity. Considering the example illustrated by Figure 1.(c) and suppose that a multinational company looks to open three new car assembling factories in three foreign neighbors countries. Due to the fact that this investment project is of high equity and due to the high level of uncertainty (which may be related to demand, competition, foreign governments policies, etc.) that characterizes it, the responsible(s) has (have) adopted a sequential decision-making strategy, where three-period planning horizon is defined. The original problem is thus decomposed into three inter-related static location decision problems, one for each period. The specificity of this situation, compared with the situation where a series of three non related static problems are considered, is that the objectives of the three sub-problems are the same: minimize total implantation costs and maximize total coverage. Basing on the information concerning implementation costs and potential average demands (we suppose that only these criteria are considered) available on the beginning of the first period, the decider(s) should select the site where to implement the first factory. If we suppose that the weights of implementation cost and potential average demand are respectively .75 and .25, and we retain the weighted sum as a decision rule, the site (i.e. cell) that minimizes implementation cost and maximizes coverage in period 1 is circled in Figure 1.(c). Among the utilities of such behavior are: minimizing the lost (if the project is not profitable) and acquiring additional information, which will be very useful in the two next periods.

**Punctual alternative-based spatio-dynamic decision problems.** This sub-family regroups problems related to the control and tracking of dynamic or moving spatial punctual objects such as controlling and guiding ambulances and fire-fighter vehicles, military navies or aircrafts in navigation, etc. Figure 1.(d) illustrates a military navy siting problem. Here, the decision environment changes continuously in response to changes of climatic conditions, evolution of enemy and compatriots positions and of fighting. The deciders must respond in real-time to the environment dynamics in order to avoid enemy attacks and climatic changes, and to ameliorate the navy attack’s positions. Contrary to the temporal formulation, here changes of real-world are instantaneous and a series of inter-dependent location decisions are required. At time $t_{i-1}$, the captain of navy V detects three enemy navies. Consequently, he must change the location of the navy to avoid their attacks. Suppose that the captain has contacted his compatriots and at the same time moves back. At time $t_i$ and with the help of his compatriots, the captain decides to attack the enemy navies. Then at time $t_{i+1}$ the situation is complicated because the enemy navies $E_1$ and $E_3$ return to their places and the three of them decide to attack. The captain and his compatriots are called then to respond to this new situation.

Even though these different problems have the same nature (location), they have different objectives and require different data structures and different modelling and resolution tools. These differences have crucial impacts on the development of spatial decision-aid tools and their understanding will have good results on the efficiency of these tools. All these elements legitimacy, to some extent, the subdivision of punctual location problems into the different sub-families detailed above.

**6.2.2. Linear alternative-based spatial decision problems.** This family regroups linear alternative-based decision problems. Linear objects may represent several types of real-world decision alternatives such as highways and routes, rivers, gasolines, etc. Equally, problems of this category can be further decomposed into four sub-families:

1. Linear alternative-based spatio-static decision problems
2. Linear alternative-based spatio-temporal decision problems
3. Linear alternative-based spatio-sequential decision problems
4. Linear alternative-based spatio-dynamic decision problems

A highway construction, for instance, can be formulated as static, temporal or sequential spatial decision problem. The static and temporal formulations differ essentially on the information on which decision is taken: in the first case, we suppose that the highway has no long-term impacts
on population, environment, ecology etc., and static information are supposed to be sufficient for representing these impacts, while in the second case these long-term impacts are explicitly incorporated in the problem modelling and resolution via the use of time-dispersed spatial information that reflect future evolution of these impacts. Sequential formulation intervenes often when the linear planning problem has considerable financial impacts. In such a situation, the solution is to subdivide the original problem into several parts, which will be constructed in different dates. Finally, dynamic formulations manifest, for example, in the management of linear hydraulic systems (e.g. river) or highways traffic regulation, in shortest path problems for moving objects, etc.

6.2.3. Polygonal alternative-based spatial decision problems. This family regroups polygonal alternative-based decision problems. Polygons, which describe topological proprieties of areas in terms of their shapes, neighbors and hierarchy (Burrough and McDonnell, 1998), are often used in regional planning and land use-related problems, which Jankowski (2003) subdivides into four types:

- location selection problems concerned with the rank of a set of sites (i.e. polygons) in priority order for a given activity,
- location allocation problems concerned with stating a functional relationship between the attributes of the land and the goal(s) of the decision-maker(s),
- land use selection problems looking in ranking the uses for a given site in priority order, and
- land use allocation problems looking in defining the best uses for an array of sites.

In the first type of problems, polygons represent candidate areas for a given industrial, commercial or social activity. On the contrary, the three next types of problems involve only one area and polygons will represent different suitability measures of the area regarding different objectives or uses. In this case, map-based alternatives may apply better (see §6.3). Polygons may also be useful in several environmental applications such as the management of hydraulic dynamic systems. However, in such applications polygons intervene as a decision space rather than a decision alternative.

Polygons are also suitable for problems related to the control of moving phenomena (e.g. fire fronts, diseases dispersion, aquatic pollution, etc.). In such problems, a series of polygons will represent the temporal evolution of geometry and spatial pattern of the phenomena under focus. Decisions will concern, for instance, the selection of the front of fire to handle first, the region which is more affected with the diseases and which will be treated immediately, etc.

Identically to punctual or linear alternative-based decision problem families, this one can be subdivided into four sub-families that will be explained through some didactic examples in the following paragraphs.

Polygonal alternative-based spatio-static decision problems. This sub-family contains regional planning and land use related problems, where the dynamic nature of the real-world is not considered. Figure 2.(a) depicts an hypothetical example which is inspired from Jankowski (2003). The figure represents 7 districts, each one is candidate for primacy health care funding. Two evaluation criteria are considered: low birth-weight rate and poverty rate. Values respectively for low birth-weight and poverty rates are shown for all districts in Figure 2.(a). These values represent average rates obtained from past censuses. A simple weighted sum decision rule with identical weights for both criteria permits to select district $d_6$ for funding primacy health care.

Polygonal alternative-based spatio-temporal decision problems. Regional physical planning and land use related problems are usually characterized by impacts and consequences which are dispersed over space and in time. A static formulation of these problems may be suitable to take into account the spatial dispersion of impacts and consequences. But it is insufficient for handling the temporal evolution of these impacts and consequences. Figure 2.(c) represents a temporal formulation of the primacy health care funding example. Here, we suppose that both criteria vary across time because they depend on several other time-varying factors such as level of instruction, population growth, economic development, etc. In Figure 2.(c) three snapshots representing current and predictive rates of low birth-weight and poverty criteria. Basing on these new data, a weighted sum decision rule indicates that funding a primacy health care in district $d_1$ is more urgent than district $d_6$. This may be explained by the amelioration of social and sanitary conditions that has
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Polygonal alternative-based spatio-sequential decision problems. As for the two previous paragraphs, some physical planning which are characterized with high equity and/or involve parameters of high uncertainty can be approached sequentially in time. We suppose that the map in Figure 2.(b) represents historic residential zone where an ambitious project of habitat restoration is envisaged. The project has considerable financial requirements. Local authorities have defined an action plan of fifteen years divided into three periods, each of five years. The problem now is to classify the 7 districts into three groups in priority order. If we suppose that the two criteria considered are: the total number of sites to be restored and the number of highly priorate sites. Measurements of the two criteria are depicted in Figure 2.(b). Here, polygons represent potential alternatives which should be regrouped into three ordered classes. In the first five years, the priorer districts (d_5 and d_7) are restored. Designing which are the next districts to restored may be defined in the beginning of the first period or in future time. In the first case, we may select districts d_2, d_3 and d_4 to be restored in period 2 and districts d_1 and d_6 to be restored in period 3.

Polygonal alternative-based spatio-dynamic decision problems. This sub-family regroups problems related to the control of moving phenomena. Figure 2.(d) presents an example of fire fighting problem which is inspired from Brehmer (1990). The figure schematizes the evolution of fire fronts over space and in time. The decider is called to coordinate different firefighter groups in order to extinct fires as soon as possible and to minimize damages. He continuously receives

Figure 2. Polygonal alternative-based spatial decision problems: (a) Static formulation, (b) Sequential formulation, (c) Temporal formulation, and (d) Dynamic formulation

been occurred in d_6 and which result from effective interventions of local authorities in this district.
information concerning the evolution of fronts from different controlling centers. Basing on these information, he commands his staff towards more important fronts. At time \( t_{i-1} \), three fire fronts are observed (\( f_1, f_2 \) and \( f_3 \)). Even though that fronts \( f_1 \) and \( f_2 \) are more important, the decider sends some groups to extinct front \( f_1 \) because there is an important touristic center in this part of the forest and because it is easier to extinct just started fire. This will reduce the number of firefighters available for struggling fronts \( f_1 \) and \( f_2 \). At time \( t_i \), front \( f_3 \) is overcome but a new fire front, \( f_4 \), is started. The decider sends firefighters of front \( f_3 \) to extinct this new front. Then at time \( t_{i+1} \), fronts \( f_1 \) and \( f_2 \) form a unique front, \( f_{12} \). And the process continuous until all fires are eliminated. In problems of this sub-family, the timing of decisions has an important role. The situation is complicated with the fact that the environment changes continuously. In the fire fighting example, for instance, fires start spontaneously (or accidentally) and disperse in function not only of decision-maker’s previous decisions but also in function of several exogen factors that the decider is not able to control such as wind direction and velocity, temperature, density and type of forest.

6.2.4. Network alternative-based spatial decision problems. This family regroups network alternative-based decision problems. Networks intervene in a variety of real-world problems including shortest path, minimal spanning tree, maximal flow, travelling salesman, multicast communication, transportation and commodity flow problems. The specificity of network alternatives in comparison with punctual or linear ones is that they have an inherent spatial information about connectivity such as might be required for road and transportation or drainage network analysis (Burrough and McDonnell, 1998). Identically to previous families, network alternative-based one may be subdivided into:

1. Network alternative-based spatio-static decision problems
2. Network alternative-based spatio-temporal decision problems
3. Network alternative-based spatio-sequential decision problems
4. Network alternative-based spatio-dynamic decision problems

In network related problems, we may be interested in the implementation of a network infrastructure or to its exploitation. Implementation of a network infrastructure has usually long-term impacts and should normally be approached through a temporal formulation. The implementation of a transportation network, for instance, should take into account urban growth, road expansions and soil type, in order to satisfy increasing demands, to avoid congestion and soil slippage or erosion problems. The same remark holds for several other public management and planning problems such as electricity and heat distribution, liquid waste evacuation, etc. Static formulation may be justified in small scale planning problems. In practice, however, networks management problems are usually of high equity and often approached according to a sequential formulation.

Network alternatives intervene also in a variety of socio-economic applications such as public transportation, automatic route finding in car and truck navigation, commodity flow problems, etc. In such problems, usually several networks are compared mainly in terms of travel time. The specificity of this type of problems is that there is not an explicit selection of a network alternative but only one alternative is dynamically constructed. Hence, a dynamic formulation will apply better.

6.3. Problems implying complex spatial alternatives. In many real-world applications, one may be called to represent alternatives with a combination of two or more atomic entities. In schools partitioning problem, for instance, decision alternatives can be assimilated into a combination of points and polygons where points represent schools and polygons represent zones to serve. A set of 'point-point' composed alternatives may represent potential paths in the shortest path identification problem. Equally, a set of composed alternatives of 'polygon-network' type may schematize different feasible locations, each one belongs to a different transportation network. Table 3 provides some other examples of problems where complex alternatives are required. Composed alternative-based spatial decision problems are not explicitly included in the typology detailed above. Nevertheless the typology is still adequate to describe most of these problems. Indeed, these last ones are usually decomposed into series of basic problems, each one involves only one atomic alternative type. The school partitioning problem, for instance, can be decomposed into polygonal alternative-based problem and several punctual alternative-based problems. Thus, the initial decision space is decomposed into several polygons, each one will represent the new
decision space for a punctual location problem. One particular composed alternative is the one based on a map structure. Map structures are relevant mainly in spatial problems that are related to the control of (non real) spatial phenomena. They are also useful in applications in which a strong spatial and/or temporal relation between elements of the decision space should be verified. An illustrative example of representing decision spatial alternatives using map structures is provided in Janssen and Herwijnen (1998). The authors have proposed several transformation and aggregation methods in order to represent the performance of different policies of antipollution fights in the 'Green Heart' region of the Netherlands. The results of the transformation and aggregation operations have been presented in performance maps, which represent the relative quality of the different policies along with their spatial patterns. These maps are than used as inputs to the evaluation step. Another example of using map structures to represent decision alternatives is furnished by Sharifi et al. (2002), where the authors have interested to the problem of relocating the boundary between the 'Tunari National Park' and the 'Cochabamba City' (in Bolivia) in order to avoid spontaneous illegal settlements in between the park and the city. Four different maps, each represents a possible approach to address the problem and satisfy the objectives of stakeholders, are generated and compared with current situation. Map-based alternatives are also suitable for applications in which a strong spatial and/or temporal relation between elements of the decision space (e.g. spatial contingency and adjacency in redistricting problems, spatial compactness in land use allocation problems, composition relations in administrative partitioning problems) should be verified. An example is the redistricting problem where attention is focalized on the definition of a zoning (representing, for example, administrative, commercial or service zones) that verifies at best the spatial contingency propriety between neighbor zones and, at the same time, eliminates intersections between these zones and avoids holes. In such a problem, maps constitute an excellent support for the presentation of potential partitions and for their visual evaluation and then their comparison.

### Table 3. Examples of complex spatial alternatives (some ones are reproduced from Malczewski (1999))

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Typical problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Map</td>
<td>Antipollution fight policy choice: each map represents a potential policy</td>
</tr>
<tr>
<td>Point-Point</td>
<td>Shortest path problem: pairs of points represents different paths</td>
</tr>
<tr>
<td>Point-Line</td>
<td>Bus stops location: lines represent routes and points candidate stations</td>
</tr>
<tr>
<td>Point-Polygon</td>
<td>School partitioning problem: points schematize schools while polygons represent zones to serve</td>
</tr>
<tr>
<td>Point-Network</td>
<td>Location in a distribution network: points represent different</td>
</tr>
<tr>
<td></td>
<td>distribution sites (e.g. supermarkets) in the distribution network</td>
</tr>
<tr>
<td>Line-Line</td>
<td>Routes intersection: linear objects schematize the different routes</td>
</tr>
<tr>
<td>Line-Polygon</td>
<td>Agriculture preservation: rivers are represented with lines and zones to</td>
</tr>
<tr>
<td></td>
<td>preserve with polygons</td>
</tr>
<tr>
<td>Line-Network</td>
<td>Adding a new arc in a distribution network: lines are potential arcs to</td>
</tr>
<tr>
<td></td>
<td>be included in a distribution network</td>
</tr>
<tr>
<td>Polygon-Polygon</td>
<td>Hierarchical zoning: administrative zoning where districts, departments, etc.,</td>
</tr>
<tr>
<td></td>
<td>take the form of hierarchical polygons</td>
</tr>
<tr>
<td>Polygon-Network</td>
<td>Industrial zone location: polygons schematize different potential zones in a</td>
</tr>
<tr>
<td></td>
<td>transportation network</td>
</tr>
<tr>
<td>Network-Network</td>
<td>Correspondence between networks in public transporting: between an underground</td>
</tr>
<tr>
<td></td>
<td>and bus networks, for instance</td>
</tr>
<tr>
<td>Map-Point</td>
<td>Regional planning problem: maps represent different potential regions for</td>
</tr>
<tr>
<td></td>
<td>implementing a new regional hospital and points represent potential</td>
</tr>
<tr>
<td></td>
<td>sites for locating the hospital.</td>
</tr>
</tbody>
</table>

Complex alternatives should verify several spatial relations (e.g. proximity, appurtenance, minimum distance separation for new development from existing livestock facilities) among its atomic entities. These relations will serve as inclusion/exclusion criteria by which one atomic alternative included or not to another atomic alternative representing the new decision space. Generally,
the generation of complex alternative begins by defining more complex atomic alternatives (e.g., networks) on which less complex atomic alternatives are defined (e.g., in the school partitioning problem cited above, we should normally define polygons and then associate to each polygon a punctual location alternative). In some cases, the order by which alternatives are defined may be imposed by the nature of the problem (e.g., locating restaurants in preexisting highways, where highways with high potential demands are selected first followed by punctual locations on these highways).

Finally, we notice that formulations defined in §4 and §6.1 apply also to problems implying complex spatial alternatives. We particularly notice that data structures defined above are suitable to these problems’ type. They, however, require more complex modelling and resolution techniques. Equally, evaluation and comparison of complex alternative require more complex operators and analysis routines.

7. Utility of the typology

The proposed typology overcomes several limits of typologies of section 2. Indeed, it has at least the following merits. First, it is useful for understanding the specificities of spatial decision problems. Second, the typology facilitates the choice of spatial and temporal operators and analysis routines required to each problem type. In fact, these operators and routines are essentially determined by the type of spatial objects by which real-world spatial entities (and consequently spatial alternatives) are conceived and digitally represented. Third, it provides tools for formulating spatial problems and representing their dynamics by explicitly integrating the temporal dimension. Fourth, its object-oriented nature makes it easy to handle the dynamics of spatial entities and, at the same time, to develop general re-usable spatial analysis routines. And, fifth, it facilitates overcoming semantic divergences and interoperability problems in the sense that decision problems are conceived and represented more naturally and independently of their socio-economic and/or environmental contexts. This is strengthened by the recent emergence of object-oriented-based GISs (e.g., Smallworld GIS, Graphic Data System, MapObjects) and by the successful implementation of many object-oriented GIS-based application.

Furthermore, our typology is particularly useful to develop multicriteria evaluation-based spatial decision-aid tools. In fact, the most used practice in multicriteria spatial decision-aid consists in representing decision alternatives in terms of spatial entities. Practically, the typology will provide a convenient framework (a) for generating, evaluating and comparing potential alternatives, and for (b) dealing with conceptual, methodological and technical questions related to the undertaking of the dynamics of basic elements of multicriteria evaluation models (i.e., alternatives, criteria and preferences) in many real-world applications.

The typology is also suitable in a visual decision-aid context. In fact, the basic ingredient of visual decision-aid making is an advanced cartographic map. Representing decision alternatives in terms of spatial entities enables decision-makers to express their preferences within a manner adapted to their simple reasoning mode and to appreciate visually the consequences and impacts of spatial decision alternatives.

Equally, the typology is useful to develop spatial decision support systems (or SDSS). Specifically, it helps to construct a general framework for classifying analysis, modelling and resolution techniques according to their suitability to different problem formulations. This framework will represent the first step towards the development of a model base management system to be incorporated into the SDSS. Its role is to assist analysts and decision-makers to select the adequate model for the problem in focus. In fact, a large variety of structured models including statistical methods, mathematical models, heuristic procedures, algorithms, and so on, are available in GIS-like tools, and usually analysts and decision-makers come across difficulty in selecting the relevant model to use. A well-established framework can therefore be used for a systematic organization of analysis and modelling tools through, for instance, IF-THEN rules or YES-NO questions (Leung, 1997), inside the SDSS.

8. Conclusion

This paper is a tentative for classifying spatial decision problems. Concretely, we have presented a typology obtained through a crossover of alternatives’ types with different possible formulations of spatial decision problems. The proposed typology covers most of spatial decision problems that
involve atomic alternatives. Also, a great number of spatial decision problems that involve complex decision alternatives may be modelled as a series of basic spatial problems. Notice, however, that it is by no means to suppose that the typology encompasses all spatial decision problems. The typology is mainly useful for understanding the specificities of spatial decision problems and for selecting the adequate modelling and resolution techniques for each category of problems. It also constitutes a suitable framework for multidisciplinary researches and for developing multicriteria spatial decision-aid tools.

References

Figure 3. The proposed typology

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Static</th>
<th>Temporal</th>
<th>Sequential</th>
<th>Dynamic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point</td>
<td>Punctual alternative-based spatio-static decision problems</td>
<td>Punctual alternative-based spatio-temporal decision problems</td>
<td>Punctual alternative-based spatio-sequential decision problems</td>
<td>Punctual alternative-based spatio-dynamic decision problems</td>
</tr>
<tr>
<td>Line</td>
<td>Linear alternative-based spatio-static decision problems</td>
<td>Linear alternative-based spatio-temporal decision problems</td>
<td>Linear alternative-based spatio-sequential decision problems</td>
<td>Linear alternative-based spatio-dynamic decision problems</td>
</tr>
<tr>
<td>Polygon</td>
<td>Polyhedral alternative-based spatio-static decision problems</td>
<td>Polyhedral alternative-based spatio-temporal decision problems</td>
<td>Polyhedral alternative-based spatio-sequential decision problems</td>
<td>Polyhedral alternative-based spatio-dynamic decision problems</td>
</tr>
<tr>
<td>Network</td>
<td>Network alternative-based spatio-static decision problems</td>
<td>Network alternative-based spatio-temporal decision problems</td>
<td>Network alternative-based spatio-sequential decision problems</td>
<td>Network alternative-based spatio-dynamic decision problems</td>
</tr>
<tr>
<td>Map</td>
<td>Map alternative-based spatio-static decision problems</td>
<td>Map alternative-based spatio-temporal decision problems</td>
<td>Map alternative-based spatio-sequential decision problems</td>
<td>Map alternative-based spatio-dynamic decision problems</td>
</tr>
<tr>
<td>Complex</td>
<td>Complex alternative-based spatio-static decision problems</td>
<td>Complex alternative-based spatio-temporal decision problems</td>
<td>Complex alternative-based spatio-sequential decision problems</td>
<td>Complex alternative-based spatio-dynamic decision problems</td>
</tr>
</tbody>
</table>

- Punctual alternative-based spatial decision problems
- Linear alternative-based spatial decision problems
- Polyhedral alternative-based spatial decision problems
- Network alternative-based spatial decision problems
- Map alternative-based spatial decision problems
- Complex alternative-based spatial decision problems