Rainfall Field Modelling For European Satellite Networks

By

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This thesis is submitted in partial fulfilment of the requirements for the reward of the degree of Doctor of Philosophy of the University of Portsmouth.
Dedicated to Mingdao Yang and Zhengying Li and in memory of Dr Boris Grémont
Abstract

This thesis provides a new space-time statistical rain model and a novel space-time interpolation approach for planning and dimensioning wide area high frequency satellite communication networks.

Key characteristics of rainfall rate fields are modelled. These include detailed description of: (i) the first order statistical distribution, (ii) the spatial and temporal correlation functions of rainfall rate and, and (iii) the probability of rain/no-rain. With a focus on their relevance to satellite and terrestrial microwave network design, the key contribution of this study is the assessment of the impact of varying spatial and temporal integration lengths on these quantities. The issue of how these key characteristics of rainfall rate field change with different area sizes are analysed in this thesis and it is novel.

A simple but accurate interpolation approach of the key characteristic parameters is presented in this thesis. The novelty of the proposed technique is that it does not rely directly on the radar/raingauge derived rainfall rate data like traditional models do but rather on fitted coefficients and computed rain characteristics. This thesis proposes rain parameter contour maps and databases covering the whole of Western Europe from which users can conveniently obtain the key rain characteristic parameters at any location within the studied area. More speculatively, the 3D space-time interpolation approach can extrapolate to rain parameters at space-time resolutions shorter than those in the NIMROD databases. The results have been validated by comparing them with those from ITU Rec model and measurements by NIMROD rain radar.
In addition, a Graphical User Interface (GUI) software has been provided that allows users to interact with the proposed model. The user can easily obtain the information of the key rain characteristics at different space-time scales by simply inputting the longitude, latitude, space resolution and time resolution of the location of interest. The detailed results are then automatically calculated and displayed by the software significantly facilitating rain rate study.
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Declaration: *Whilst registered as a candidate for the above Degree, I have not been registered for any other research award. The results and conclusions embodied in this thesis are the work of the named candidate and have not been submitted for any other academic award.*
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List of Symbols

\( \gamma \)  
Rain attenuation

\( l \)  
Length of link

\( t \)  
Time

\( R \)  
Point rainfall rate

\( a(f) \)  
Frequency-dependent parameter for the calculation of \( \gamma \)

\( k(f) \)  
Frequency-dependent parameter for the calculation of \( \gamma \)

\( P_0 \)  
Probability of rain occurrence

\( \mu \)  
Mean

\( \sigma \)  
Standard deviation

\( d \)  
Separation between two locations

\( x \)  
Arbitrary location

\( \tau \)  
Time lag

\( \rho \)  
Cross-correlation factor

\( \text{cov}(\ ) \)  
Covariance

\( N_{\text{rainy}} \)  
Rainy sample amount

\( N \)  
Total sample amount at one location

\( A_{\text{rainy}} \)  
Rainy area within a rain field

\( A_r \)  
Total area of the rain field

\( N_r \)  
Total sample amount within an area

\( M \)  
Total number of maps

\( L \)  
Spatial integration length

\( T \)  
Time integration length

\( \lambda \)  
Parameter of space scale

\( \varphi \)  
Parameter of time scale
\( L' \) Integrated space scale

\( T' \) Integrated time scale

\( \chi^2 \) Chi-squared statistic

\( P_i \) Predicted data frequencies

\( E_i \) Predicted data frequencies

\( P_i \) Predicted data frequencies

\( P_i \) Predicted data frequencies

\( S \) Size of rain map

\( R_i(\mu, \sigma) \) Rain rate exceeded with the given probability

\( W_i \) Weights

\( H \) Hessian matrix

\( a \) Coefficient of proposed model for correlation function

\( q \) Coefficient of proposed model for correlation function

\( Corr_{fitted} \) Fitted correlation function

\( Corr_{measured} \) Measured correlation function

\( V \) Advection vector

\( b \) Coefficient for calculating the \( P_0 \)

\( c \) Coefficient for calculating the \( P_0 \)

\( e \) Coefficient for calculating the \( P_0 \)
Chapter One: Introduction

An introduction to rainfall rate and rain-induced attenuation is provided in this chapter. Signal attenuation caused by rain is the main problem for wireless communication links at frequencies above 10 GHz. New models are required to accurately predict rain characteristics this forms the motivation of this study. The objectives are presented in this chapter as well as the author’s main contributions.

1.1 Background

In recent decades, radio networks have become an indispensable part of most major communication systems. Many countries, especially developing countries, that had poor or little wired network infrastructure, have invested heavily in radio communication networks to meet the demand for mobile communications and wireless data access [1][2][3].

Rainfall, as a complex phenomenon, has been one of the most actively investigated elements in weather due to its significant role in a broad range of processes. It involves the interplay of many physical processes in the atmosphere and exhibits a strong variability in space and time; hence its stochastic modelling is not an easy task [4]. There is a close interrelation and interaction between rain and communication systems operating at high frequencies [5]. At frequencies above 10 GHz satellite systems widely used in the radiowaves are significantly attenuated by rain that can lead to link unavailability situations [2][5]. The next generation of satellite systems will need to allocate communication resources taking into account the users' communication needs and the weather conditions. Because of this, the
study of rainfall rate variability is critical for the design and planning of satellite networks.

However, rain and rain intensity vary in both space and time [6], and hence the level of signal attenuation is also varies. This variability makes the whole system design more challenging and therefore models are required for effective network planning. An appropriate rain attenuation model is critical to the implementation of efficient fade mitigation techniques (FMT) [7] such as, site diversity systems [8], high altitude platforms (HAPs) [9]. These require accurate knowledge of rainfall rate variability.

### 1.2 Rain-Induced Attenuation

Super High Frequency (SHF) and Extremely High Frequency (EHF) are the ITU designation for the band of radio frequencies. The SHF, which is in the range between 3 GHz and 30 GHz, is used for most radar transmitters, wireless LANs, satellite communication and numerous short range terrestrial data link. Compared to lower bands, radio waves in the electromagnetic spectrum from 30 GHz to 300 GHz (range for EHF) have high atmospheric attenuation. The largest dynamic fade experience by almost all extremely high frequency atmospheric radio-links, both terrestrial and Earth-space, are caused by rain attenuation which results from absorption and scattering of microwave energy by rain drops. Rain attenuation is often thought of as due to scattering and absorption by all the raindrops within the first Fresnel zone. At any particular moment, the rain attenuation experienced by a link of length $l$, may be approximated by the path integral of the specific attenuation $\gamma(x, t)$ in dB/km:

\[
A(t) = \int_0^l \gamma(x, t) \, dx \quad (1.1)
\]
The specific attenuation may be calculated from the raindrop size distribution but an adequate approximation may be formed from the rain rate and rain phase. Rec. ITU-R P.838 [10] provides a power-law model linking specific attenuation and rain rate:

\[ y \equiv aR^k \]  

(1.2)

where the parameter \( a \) and \( k \) are frequency dependant. For slant-path terrestrial links, Rec. ITU-R P.530 [11] includes a specific attenuation scaling factor that allows for the changes in precipitation phase, from liquid to mixed to solid, as the link path passes through the atmospheric layer around the zero degree isotherm. For Earth-space links, Rec.ITU-R. P.618 [12] assumes precipitation is liquid up to level known as the rain height, above which there is no rain attenuation.

For the design of radio links, the long-term or average annual distribution of rain fade is required. This is usually the average annual distribution of rain rate along with information on annual variation of rain height. Rec. ITU-R P.837 [13] provides the rain rate exceeded for 0.01% of an average year for any point on Earth. These estimates have been produced from Numerical Weather Prediction (NWP) models and are used if better results from measurements at those locations do not exist. Rec. ITU-R P.530 and P.618 provide methods to estimate the rain fade exceeded for 0.01% of an average year, and they use the 0.01% exceeded rain rate in this calculation.

Typically, the frequency-dependent parameters \( a(f) \) and \( k(f) \) for a given location are well studied. Their corresponding values at different frequencies are documented in published works, e.g. [14]. However, it should be noted that the link between rain rate and specific attenuation is statistical due to variation in drop size distribution for
the same rain rate. Also the effective length of the slant path can be easily calculated from the link geometry. The attenuation prediction therefore only requires accurate measurement of rain precipitation.

1.3 Rainfall Rate Measurement

The measurement and modelling of rainfall precipitation has been implemented by employing various methods over many decades. The raingauge (including weighting gauges, and tipping bucket gauges) is the most commonly used apparatus for measuring point rainfall rate at a specific location. The basic principle is to collect rain over a time period, the integration time. The rain rate is the mean precipitation value over that time interval and the measurement is recorded in millimetre per hour ($mm/h$). It provides good time resolutions but its main application drawback is that it is costly to characterise rain over large areas. In addition, rainfall exhibits spatial variability and is never evenly distributed over the study area [15]. Because of this, many rainfall stations are required in order to measure the actual precipitation over a large area. However, this is normally very difficult due to factors, such as budget constraints, inaccessibility of certain areas e.g. mountains and so on [16][17]. The distribution of ground-based raingauge networks, therefore, is often too sparse to capture the fine-scale spatial variability of rainfall, and it is difficult and expensive to cover wide areas with suitable space resolutions [18]-[20].

The advent of rain radar network systems has compensated for the drawbacks of raingauge networks and can provide the rainfall measurements over large areas spanning different climatic regimes. Radar systems have enabled the establishment of spatial distribution of rainfall databases, which can be used to study rain [21]-[23]. Particularly, some radar measurements can offer good spatial resolution, e.g. for the
UK, the radar observations are provided on a nominal 1 km grid. Radar-derived data can provide continuous spatial coverage thereby reducing the uncertainty in measured rainfall variability compared to those that are derived from raingauges. Thus, the use of meteorological radar has become more widespread and gained acceptance in rainfall study as a source of input data [24]. This has enabled a better understanding of spatial variability of rainfall and has boosted the development of models. However, weather radars tend to have longer sampling times compared to raingauges. This is because the rain radars need time to complete its physical rotation to scan the area being mapped [25].

1.4 Objectives

The measurement of rain precipitation using either rain radars or raingauges is time-consuming and costly. Following the information in Section 1.1, one can note that each apparatus, used for the rainfall intensity measurement, has its strengths and limitations. Numerical models have therefore attracted a lot of attention and are seen as solutions to address the limitations in rain measurement. Based on this the objectives of this study can be briefly summarised as follows;

- The first objective is to characterise the rainfall field over different climatic zones in Western Europe using radar measurements. As rain is irregular in both space and time, its intensity varies from location to location. The aim is to obtain detailed knowledge of key characteristics with varying space-time scales at different locations in order to capture its variability features.
- The second objective is to develop a new space-time statistical rain model. Critical elements to be considered are the spatial correlation and temporal
variation of rain over the whole of Western Europe. Higher order moments of rain fields and models of the rain/no rain areas will also need to be studied.

- The third objective is to study interpolation techniques to provide high resolution data for rain fields. The statistical model addresses some problems; but the resolution of model output is poor and it cannot meet the demands of ever-changing satellite communication systems. An appropriate interpolation model, therefore, is needed to provide reasonable estimates at finer scales or at locations where measurements are unavailable.

1.5 Author’s Main Contribution of This Project

Rainfall fields spanning the whole of Western Europe have been characterised based on five years of rain radar data. The main contributions of the authors’ work are summarised as follows:

1) key characteristics of rain (first order statistics, spatial and temporal correlation functions of rain rate and probability of rain occurrence) have been studied to understand the variation of rain in different locations in Europe and climatic zones;

2) a new space-time statistical rain attenuation model has been proposed that can reasonably estimate the studied rain characteristics;

3) a numerical method has been proposed that shows how to integrate the radar-derived data from short integration length to a longer one. This is important for studying how the key rain characteristics change with varying space and time scales;

4) a detailed multi-resolution parameter database and software have been created for predicting the statistical/dynamic parameters that characterise
European precipitation rate fields. This has been applied to the development of the proposed statistical model;

5) rainfall map size has been investigated to show how the size of rain map affect the characteristics of rain rate;

6) an algorithm and software have been developed that produce contour maps of rain parameters over the whole of Western Europe. These allow users to efficiently obtain the values of rain characteristics at any location covered by the studied map;

7) a new interpolation approach has been proposed that can provide estimates of the studied rain characteristics at locations with a wide range of space-time resolutions; and

8) a graphical user interface (GUI) has been developed that allows users to interact with the proposed model and obtain detailed information of rain characteristics at any location for given space and time resolutions.

1.6 Thesis Organisation

In this chapter, the background of rainfall rate and rain-induced attenuation has been discussed. The author described the importance of rain studies to satellite communications. The main objectives and contributions have been presented. The rest of the thesis is organised as follows:

Chapter 2 reviews the development of the rain models. The related work done by other researchers is reviewed. The main gaps in the rainfall study are also presented. This forms the motivation, which was helpful in guiding this study.

Chapter 3 describes the experimental data used in this study. The proposed space-time stochastic rain model is presented. In addition, a numerical method is proposed
that shows how to integrate the radar-derived rain precipitation from short integration length to a longer one in both space and time domains.

Chapter 4 presents the outcome of an extensive experimental study of rain radar information spanning the whole of Western Europe. The key characteristics of rain at different locations are studied. A detailed multi-resolution database and associated software is presented for predicting the statistical/dynamic parameters characterising European rainfall rate fields. In particular, empirical equations are proposed that can accurately estimate the corresponding rain characteristics through a large range of spatial and temporal integration lengths.

Chapter 5 presents a simple but efficient interpolation approach. Databases with estimated parameter values, and maps for Europe, have been created to allow users to access the key rain characteristics at any location within the study area. This provides great assistance to users, as the rain characteristics can be easily obtained without long computation. In particular, an approach to interpolate the fitted coefficients and/or rain characteristics in space-time domain with arbitrary integration length has been proposed.

Chapter 6 concludes the main findings of this research. The reason why the author developed two models and combined them together for modelling rain is also presented. Future work is also given in this chapter.
Chapter Two: Literature Review and Related Works

The development of rain models is described in this chapter. Related works is reviewed and some classic models are discussed. It shows that there is a lack of knowledge of how the key rain characteristics vary in space and time over a large area covering many locations. Another gap in knowledge is how a space-time interpolation model could be used effectively without applying it to the fundamental data to achieve a higher accuracy whilst minimising the computation time.

2.1 Review of Related Works

The dynamic statistical characteristics of rain play an important role in the reliable design of radio communication (terrestrial and satellite) systems operating at frequencies above 10 GHz [26]-[29]. To further the development of rain-induced radio-wave attenuation models, and to provide more accurate performance prediction of satellite links over wide areas, there is an increasing need for a good understanding of the space-time characteristics of rainfall rate.

As discussed in Section 1.3, the most commonly used devices to measure rainfall rates are raingauges and radars, which can both provide rainfall rate measurements for the design of satellite networks. However, each method has its own limitations. Numerical models have been introduced to address the limitations of the above two rain measurement methods and integrate their strengths appropriately. Extensive studies of rain have been carried out in the last few decades. According to [30] and [31], rainfall rate models have been developed through several generations and
studies on the modelling of rainfall rate over the past few decades are already well documented. For example, Maseng and Bakken [32] proposed the stochastic-dynamic time-series model for rain attenuation field simulation. This was later extended to two locations in [8]. A study in [33] shows that Maseng’s model also applies to the rainfall rate with the same dynamic parameters. The first space-time rainfall processes model was proposed by LeCam [34]. It is based on the study of random structure of rainfall fields and it introduced a numerical method that converts the description of rain behaviour into formulae. Many other popular models are also well documented, including alternating renewal models [35], Markov chain models [36], clustered point process models [37], etc. A classic model was developed by Menabde [38] who used a discrete random cascade to generate a field with a desired statistical structure. A model of particular interest was developed by Bell in 1987 [39]. His work assumed that rainfall intensities in a field exhibit lognormal distribution and this was confirmed by Crane in 1990 [2].

Rainfall processes are highly variable in space and time. Better understanding of rainfall behaviour at multi-resolutions is important for the design of an accurate rain model [40]-[42]. To achieve this, knowledge of how the rain characteristics change with varying space and/or time resolutions is required. The analysis of the spatial and temporal characteristics of rain processes in particular have been of interest for the design of rain attenuation model for many decades [43]-[48]. There is a broad agreement that point rainfall rate exhibits a lognormal distribution characteristics e.g. [39][40][49]. This feature is useful for rainfall field simulation. The study in [50] investigated the rainfall properties in central Italy. It examined the variability of rainfall at different time scales from 30 mins to 720 mins using both raingauge and weather radar data. However, it only studied the spatial and temporal correlation function of
rain rate at different time scales and did not link these to spatial variation. The decay in correlation is found to be a function of climatic region. In [51], rainfall data from 42 stations were analysed in order to investigate how its variability changed at monthly, seasonal and annual scales. A representative piece of work was undertaken by Yevjevich and Karplus [52] who proposed three common correlation models based on the assumption of homogeneity and isotropy in space. Another classic example is given by Fukuchi [53], he studied the rainfall rate at 23 locations in the UK and proposed an exponential model for the spatial correlation function. Studies conducted by Takeshi [54] concluded that the exponential distribution can be used to model horizontal spatial correlation of rain rate in Europe.

The probability of rain occurrence is another key property of rain [55][56]. It reflects the statistics of an area- and/or time- averaged rain rate. A reliable rain model can be developed by studying and understanding these key characteristics of rain. A good example is given in [39]. It shows that rain fields can be synthesised numerically in two main steps. First, a Gaussian field, $g(x)$, is synthesised that exhibits the desired Gaussian statistical characteristics at each grid point. This Gaussian field is given a space correlation function in such a way that the ultimate rainfall rate field has the desired characteristics. This is achieved by having the random Gaussian field $g(x)$ converted to a rain field $r(x)$ using a nonlinear transformation so that rain has the required first order statistics. The probability of rain occurrence ($P_0$) in the area of interest should be based on the results of measured rain data. The remaining fraction, $1 - p_0$, of time is the proportion when there is no rain. For this, Bell selects a threshold value $g_0$. When the Gaussian field exceeds this threshold, a lognormal rainfall rate is produced after a suitable rescaling of the Gaussian field otherwise a zero rainfall rate is produced.
The traditional rain models (e.g. stochastic models, Markov chain models) can be used to aid the planning of satellite networks. However, there are some limitations inherent in such models and the two major ones are:

1) Data availability. The models are only applicable to areas where rainfall precipitation with the necessary integration volume has been observed and the accuracy of the models in areas where no data is available is difficult to verify.

2) Integration volume. The application of the traditional models is limited by the integration length. The modelling of rain and simulated rainfall fields can only be limited to the space-time resolution derived from rain radar/gauge measurements. Rainfall fields simulation at finer space-time scales is often possible but cannot be verified.

Based on this information, it is clear that the application range of stochastic models is limited by the above problems. Improvements, thus, are needed to compensate, enhance and extend the performance of stochastic models. In particular, an increase in the use of high frequency over short communication links has led to an increase in the need to predict rainfall rates at finer resolutions. Current stochastic models cannot satisfy this demand. As a result, interpolation techniques have attracted a lot of attention in recent decades.

According to [47], Drozdov and Sephelevskii [57] developed a spatial interpolation technique to analyse the spatial variations of a process over an area. Then later, a modified interpolation technique called Kriging was developed based on the theory of regionalised variables to estimate area averages considered as realisations of a stochastic process introduced by Matheron [58]. Since then significant progress has been made and two-dimensional (2D) space rainfall rate interpolation models have
been developed, e.g. [59]-[63]. The Random Midpoint Displacement algorithm (RMD) developed by Voss [64] in 1985 is one of the most popular interpolation algorithms. The basic idea of the technique is to introduce new rain rate samples with the same underlying distribution as existing measurements at new locations or times. The one-dimensional (1D) time interpolation is also of interest as network planners and designers of physical layer fade mitigation techniques [65] require knowledge of rain variation over much shorter time scales (of the order of seconds or less). Some excellent models have been published in [66]-[68]. One of such models proposed by Kevin Paulson [69] is a stochastic numerical model that can interpolate the point rain rate for short time durations down to 10 s.

The downscaling model, based on the space-time averaging theory, is another model that has also attracted significant attention. According to [70], there are two fundamental requirements for precipitation downscaling models, which are: 1) understanding of the statistical properties and scaling laws of rainfall fields, and 2) validation of downscaling models that are able to preserve statistical characteristics observed in real precipitation. Typically, based on the information given in [71], downscaling algorithms can generally be grouped into three main families with some simplification: 1) point process based on the random positioning of a given number of rain bands and rain cells [37]; 2) autoregressive processes passed through a static nonlinear transformation [72], and; 3) fractal cascades. In particular, the theory of fractals, which was first introduced by Mandelbrot in 1967 [73] has attracted great attention. This theory was not applied to rainfall study until the mid-1980s [74]. Rain has been shown to hold fractal properties over a range of scales. The intermittence and discontinuous nature of rain is reproduced by the fractal based models, which are strongly favoured for rainfall modelling. Many studies have been carried out to
interpolate the radar/raingauge measurement data to finer scales using the fractal theory, such as in [75]-[78]. Multifractal models, which may be simulated using random cascades, can easily capture any moment of the observed signal; especially higher order moments have attracted a lot of attention [79]. Because of their link with multifractal theory, multiplicative cascade models first proposed by Yaglom [80], appeal to rainfall simulations. The rainfall series have been shown to exhibit scaling invariance properties over a large range of space [81]-[85] and time [86]-[88] steps. Some multifractal models use discrete cascade algorithms to produce data at finer scales from original sparse observations, for example in [89]-[92]. A classic work is given by Menabde [93] who used a discrete random cascade to generate a rain field with the desired statistical structure and then applied a power law filter, thereby removing some of the blockiness resulting in a more realistic looking rain field.

The prediction at finer space-time resolution however, has long been a challenging issue in rainfall field modelling. Results from 3D interpolation studies are quite poor as it is very difficult to consider both space and time variability and irregularity of rainfall in an appropriate way. The basic idea of published models is to try to find the underlying principle of how the space-time transformation can be achieved. A representative model was developed by Deidda [94] based on the assumption that Taylor’s hypothesis [95] can be applied. The space-time rainfall field is assumed to be a three-dimensional (2D space and 1D time) homogeneous and isotropic process. An advection velocity parameter is introduced to connect the space scale and time scale. With the help of a velocity parameter, the statistical properties of rain at finer scales can be deduced from larger ones. Similar studies can be found in [96]-[100] in which rain has been studied in a range of space-time scales to define the transformation parameter. In particular, Kundu and Bell [101] developed a model that
can provide the correlation function of rain in $3D$ space-time domain but in a very complicated form.

Finally, it is necessary to mention that there are also many other hydrological rain models (e.g. [102]-[104]) for some special cases, such as flood caused by heavy rain storms, flood frequency estimation, flood hazard mapping, etc, but these typically focus on much larger integration volumes than those required for radio system modelling.

### 2.2 Description of Gaps in Research Literature

From the literature review no study has been done to systematically analyse the characteristics of rain in detail. Most of the published works have only focused on part of the rain characteristics at limited locations. Although Bell’s model shows a good example of rain field simulation by studying the key characteristics of rain, it also has some limitations. For example, Bell in [39] assumed that all locations within a wide area share the same rain statistics. Such an assumption serves to reduce the parameters that need to be specified. This is clearly unrealistic for large area communication networks as rainfall rate tends to vary from location to location caused by many uncertain factors, such as climate, topography, wind, etc. Detailed studies of the rain characteristics are needed to obtain a better understanding of the variability of rain, especially those at multiple space and time resolutions. Following the pointer of Bell’s model, this study will look into the key characteristics of rainfall rate to investigate their spatial and temporal variability in detail.

In addition, the absence of high resolution rainfall data at desired space and time scales is another main knowledge gap. Deidda in [94] pointed out that most of the existing rainfall studies at finer scales are purely focused on either space modelling
or time modelling [69]. However, both of these approaches have limitations. For example, the statistical behaviour of rain in time has inexplicit consideration of the spatial distribution and extension of the rain field itself; and the study in space is normally based on fixed time duration whilst the evolution in time of spatial patterns is ignored. Accurate rainfall field simulation requires knowledge of rainfall rate variability in both space and time domains. There is not enough research in the area of space-time interpolation apart from the work in [96]-[100]. Thus, an appropriate space-time interpolation model that can preserve the underlying statistical properties at finer scales is needed. The absence of knowledge of rain characteristics at high space and time resolution is another important gap and is the second objective of this study.

Kundu in [101] showed that the characteristics of rain depend on the space and time scales over which rain data is averaged. However, all the existing interpolation and/or multifractal models directly focus on rain precipitation and no work has been found that studied the characteristics of rain at scales better than the one provided by rain radars. The study in this thesis therefore will look into this issue to investigate the variability of rain characteristics at random space-time integration length.

### 2.3 Summary

In Summary, this chapter has reviewed the development and related studies in rainfall rate modelling. Some classic works have been discussed. From the literature review, the author has found that more detailed study of spatial and temporal variability of rain properties is needed. In particular, the interpolation in 3D space-time domain has received relatively little attention and need to be studied further.
This forms the main objectives of this study. A new statistical model has been proposed and the results are presented in following chapters.
Chapter Three: Description of Space-Time Statistical Rain Model and Characterisation of Rainfall Fields

The experimental data used in this study is described in this chapter. The proposed space-time statistical rain model is described and rain characteristics are explained. In addition, a numerical method of how to integrate the radar-derived data into longer integration length is also presented. This is applied to study the key rain characteristics at different spatial and temporal integration lengths.

3.1 Data Description

The rainfall rate data for the European area used in this study was obtained from Metrological Office NIMROD radar system in the form of composite spatial maps produced every 15 mins and binned into pixels with spatial integration length of 5 km × 5 km. The NIMROD data is one of the datasets available from the British Atmospheric Data Centre (BADC). BADC continuously collects rainfall rate data and images over the whole of Western Europe. As one of the Natural Environment Research Council (NERC) centres for atmospheric sciences, it is also used to ensure the long-term integrity of atmospheric data produced by NERC projects. Particularly, the NIMROD data has been validated through compared with raingauge data conducted by some researchers. The difference between NIMROD data and raingauge data is expected due to spatial averaging: NIMROD provides averages.
over areas at least 1 km square while raingauges yield point values. However, there is no evidence that the NIMROD estimate is biased.

The BADC weather radar network has 15 C-band rainfall radar sites, which cover the whole of the British Isles. These rain radars scan at high space and time resolution over long distances. Four or five radars repeat the scan at different elevations to build a 3-D scan of the area from which the best possible estimates of rain rates on the ground are established. A series of composite rain field maps are then produced of rainfall rate samples distributed on a 5 km squared Cartesian grid covering Western Europe. Each map contains 700 × 620 data cells. However, only a 400 × 400 grid in each radar image has been analysed in this study, see Fig. 3.1 in which the outline is the studied area. This is because rain rate estimates are only available within this area due to limited radar scan range. The study area ranges from 43.1938° to 59.4306° in latitude and −9.7370° to 19.8364° in longitude. Five complete years (2005 to 2009) of composite rain maps have been analysed for the development of a generic space-time statistical model and interpolation approach in this thesis. The full dataset consists of more than 166000 radar maps with data availability of over 90% for each year. Fig. 3.1 shows a typical NIMROD radar scan image. The outline is the studied area for which the size is 2000 km × 2000 km. The grey colour is the area outside the range of the radar network where no rain data is available, and the black colour represents the scanned area where NIMROD radar data is available. The interpolation model that is presented in Chapter 5 is based on the study of key characteristics of rainfall rate at each location within the studied area. Furthermore, rainfall rate measurement on a 1 km grid is held in the NIMROD database for the British Isles. This data is the best estimate of the precipitation in
space domain and is acquired with a sampling time of 5 mins. A typical radar scan image for UK is shown in Fig. 3.2. The validation of a model’s performance normally requires comparable observational data from apparatus (e.g. raingauge or rain radar) [105]. UK data, therefore, will helpful for validating the prediction of the proposed interpolation approach by comparing the interpolated estimates with the computed values based on historical radar data in the UK.
Figure 3.1: Composite radar scan image for Western Europe. The outline is the studied area.
Figure 3.2 : Composite radar scan image for the British Isles.
3.2 Space-Time Stochastic Rain Model

As a detailed understanding of the space-time characteristics of rain is essential for the planning of wireless satellite network systems, an accurate model is therefore needed for simulating rainfall fields.

It is well accepted that rainfall rate, $R$ in $mm/h$, at one location is well modelled as a lognormal process with mixed probability density function (PDF) [8]:

$$A_R(\mu, \sigma, P_0) = \begin{cases} 1 - P_0 & \text{no rain} \\ \frac{P_0}{\sqrt{2\pi \sigma^2}} e^{-\left(\frac{\ln R - \mu}{\sigma}\right)^2} & \text{for rain} \end{cases}$$

(3.1)

where $P_0$ denotes the probability of rain occurrence ($R > 0$) and $\{\mu, \sigma\}$ are the lognormal parameters describing the statistics of rainfall rate at that location of interest. In this study, one can see that statistical parameters $\{P_0, \mu, \sigma\}$ depend on the location $x = (x_1, x_2)$, and the spatial and temporal integration lengths. Another critical element that is needed to characterise rain fields is the spatial correlation function for two chosen locations. Experimental data suggests that it is reasonable to assume the field to be space homogeneous i.e. that the spatial correlation function of rain rate only depends on the separation distance between two locations and the 2D spatial correlation function can be assumed to be isotropic [1][2] i.e. it takes the form $c_S(x, y) \equiv E[R(x, t_1)R(y, t_1)] \equiv c_S(d)$ where $d = |x - y|$. The temporal correlation function between two rainfall rate samples at the same location but separated by a duration of $\tau = t_2 - t_1$, $c_T(x, y) \equiv E[R(x, t_2)R(y, t_2)] \equiv c_T(\tau)$ is also considered in this study.

Such assumptions can greatly reduce the complexity of rain modelling while the accuracy in matching the experimental data can still be acceptable for some
applications. The point statistics (including rain/no-rain) and the spatial and temporal correlation function form a basic set from which rain field can be synthesised numerically. In short, these quantities in this study are called the “key characteristics”.

3.3 Theory of Characterising the Key Properties of Rain Rate

Some key characteristics of rain need to be studied and used to model rain to yield useful results. Following the lead in [39], four key parameters are considered important for the development of a useful space-time statistical rain model. These include a detailed description of (i) the first order statistical distribution, (ii) the spatial correlation function of rain rate, (iii) the temporal correlation function of rain rate, and (iv) the probability of rain/no rain. The detailed descriptions are given in the following sections.

3.3.1 Statistics of Rain

The first principal parameter for the characterisation of rainfall fields is the probability distribution of rain, which is an indispensable element for modelling rainfall rate. In the past decades, many studies have been carried out which showed that point rainfall rate (when the rain is actually occurring, $R > 0$) is well modelled as a log-normal random variable (e.g. [1][8]) with a probability density function given by:

$$f(R) = \frac{1}{\sqrt{2\pi}R\sigma} \exp\left(-\frac{1}{2} \left(\frac{\ln R - \mu}{\sigma}\right)^2\right)$$  \hspace{1cm} (3.2)

where $\mu$ and $\sigma$ are the mean and standard deviation of log rainfall rate, respectively. In practice, the distribution is often presented as a complementary cumulative distribution function (CCDF). If the rain rate is log-normally distributed, then it is possible to transform the CCDF to a linear relationship. This technique is well
described by Filip [106]. The applicability of this statistical technique for both different spatial integration lengths and temporal integration lengths are shown in Section 4.3.

Estimation of the log-normal parameters \{μ, σ\} requires a trade-off between acquiring a large enough sample to yield significant estimates while remaining within a homogenous climate region of rain regime.

3.3.2 Correlation Function of Rain Rate

An understanding of the correlation function is central to the design of a reliable model of rainfall rate [107]. The study of the correlation function in both the space and time domains is very useful for investigating the horizontal rain field structure and the evolution of the spatial pattern in time, respectively.

3.3.2.1 Spatial Correlation Function of Rain Rate

The spatial correlation function, which is equal to the inverse Fourier transform of the power spectral density (PSD), is the second important rain characteristic. It can be expressed as:

\[
ρ = \frac{cov(R_1, R_2)}{\sqrt{σ_1σ_2}}
\]  

(3.3)

where \(R_1\) and \(R_2\) are the rainfall rates (millimetre/hour) at two locations 1 and 2, \(ρ\) is the cross-correlation factor between \(R_1\) and \(R_2\) and \(cov(\quad )\) and \(σ\) are the covariance and standard deviation, respectively.

According to [99], it is reasonable to assume that the spatial correlation function of rain rate only depends on the separation distance i.e. that the rainfall field is spatially homogeneous and isotropic and stationary in time. These assumptions are more likely to be valid over small distances and times that are less affected by the shape
and intermittence of rain events. This assumption has limits but it is necessary to simplify the model. For example, fronts and squalls have large linear features which are not homogeneous or isotropic over event scales, and temporal stationarity would imply negative exponential inter-event times which are not observed etc. Theoretically, the correlation in space will be the same in any horizontal direction according to the isotropic assumption. The spatial correlation function of rain rate therefore can be computed using pairs of rain rates in any orientation.

3.3.2.2 Temporal Correlation Function of Rain Rate

Rain events are also highly variable in time [108], so the third characteristic considered is the temporal correlation. Typically, intense rain showers, which cause extreme attenuation, are of short durations [109]. The temporal correlation function is therefore essential for investigating how the rain intensity at one particular location is correlated over different times. The development of effective route diversity or site diversity networks requires detailed knowledge of both the temporal and spatial correlation functions of rain rate. The temporal correlation function can be expressed as:

$$
\rho = \frac{\text{cov}(R_{t_1}, R_{t_2})}{\sqrt{\sigma_1 \sigma_2}}
$$

(3.4)

where \( R_{t_1} \) and \( R_{t_2} \) are the point rainfall rates (\( mm/h \)) at two different times \( t_1 \) and \( t_2 \) but the same location.

3.3.3 Probability of Rain Occurrence

The last important parameter considered in this study is the probability of rain occurrence (\( P_0 \)) in a geographical area, which is chosen to be of similar size as the
spot-beam or footprint of the satellite network [39]. Theoretically, the $P_0$ (for which $R > 0$) represents equally well the probability of rain occurrence at one point over a long period of time, or, the expected fraction of the rainy area that one can expect in a satellite network [39]. The calculation is presented individually as follows:

1) $P_0$ calculation for a location

The $P_0$ for an individual location is equivalent to the proportion of rainy time over the whole studied period. It can be calculated by:

$$P_0 = \frac{N_{\text{rainy}}}{N}$$  \hspace{1cm} (3.5)

where $N_{\text{rainy}}$ is the rainy sample amount at the point of interest over a long period of time, and $N$ represents the total sample amount (including rain and no rain) at that point.

2) $P_0$ calculation for a rain field

The $P_0$ for a rain field is the fraction of the rainy area over the whole area studied. It can be expressed as:

$$P_0 = \frac{A_{\text{rainy}}}{A_T}$$  \hspace{1cm} (3.6)

where $A_{\text{rainy}}$ is the rainy area within a rain map, and $A_T$ represents the total area of the rain map of interest. One should note that Eq. (3.6) is used to calculate the $P_0$ of a single map.

In particular, the $P_0$ for an area over a long period (multiple rain radar maps) is either the mean $P_0$ value of all locations within the rain field of interest, or, the average $P_0$
value of this rain field at all snaps. The general equation for the calculation of $P_0$ can be described as:

$$
P_0 = \frac{\sum_{N_T}^{N_{rainy}}}{N_T} = \frac{\sum_{M}^{T_{rainy}}}{M}
$$

(3.7)

where $N_T$ is the total sample amount within $A_T$, and $M$ is the total number of maps taken over a certain period of time.

### 3.4 Integration of Rainfall Rate Data

The planning and development of high frequency wireless networks can be improved by using the radar-derived space-time rainfall statistics spanning a large area. It will provide more detailed information on the spatial distribution of rainfall rate compared to raingauge networks [110][111]. However, the problem lies in the reduced quality originating from the coarse integration length of measured rainfall rate [70]. Thus, it is important to gain a better knowledge of key rain characteristics at finer resolutions. One of the key objectives of this study is the assessment of the impact of varying spatial integration lengths ($L$) and temporal integration lengths ($T$) on the studied quantities. This means looking at how the key characteristics of rain change with both varying spatial and temporal integration lengths. This will be useful to enhance the ability to predict the rain-induced attenuation at any arbitrary spatial-temporal resolution.

Let $R(x, y, t)$ be the measured rainfall rate with space resolution $L$ and time resolution $T$. $R(x, y, t) = R_{ijk}$ if $iL \leq x \leq (i + 1)L$, $jL \leq y \leq (j + 1)L$, and $iT \leq t \leq (k + 1)T$, otherwise $R(x, y, t) = 0$. In this definition, the array of measured rain rate values with averaging regions of size $L \times L$ and interval of $T$, is $R_{ijk}$ for $1 \leq i, j, k \leq N$. The 3D indicator function
can be used to define how the rain rate changes with integration volume, where $|(x,y,t)|_\infty$ is the infinite norm. The multi-scale rain rate fields are defined as

$$R_\lambda(x,y,t) = \frac{1}{\lambda^2 L^2 T} \int \int \int R(x',y',t') I \left( \frac{x-x'}{\lambda L}, \frac{y-y'}{\lambda L}, \frac{t-t'}{\lambda T} \right) dx'dy' dt'$$

(3.9a)

where $R_\lambda(x,y,t)$ is the rain rate at position $(x,y)$ derived from a spatial integration region of linear size $\lambda L$ and temporal integration time $\lambda T$, where $\lambda > 1$ is known as the scale parameter. More generally, the spatial and temporal regions could have different scale parameters e.g.:

$$R_\lambda(x,y,t) = \frac{1}{\lambda^2 \varphi L^2 T} \int \int \int R(x',y',t') I \left( \frac{x-x'}{\lambda L}, \frac{y-y'}{\lambda L}, \frac{t-t'}{\varphi T} \right) dx'dy' dt'$$

(3.9b)

In particular, the integration can be divided into three categories, which are 2D spatial integration (see the example given in Fig. 3.3), 1D temporal integration (see the example given in Fig. 3.4), and 3D spatial-temporal integration. The 2D spatial integration only changes the integration length of rain rate in space domain while time resolution is fixed, and vice-versa for the temporal integration. The 3D integration is particularly interesting as the rain event is actually evolving in both space and time simultaneously. The integration in 3-dimensional domain is clearly the combination of both spatial and temporal integration.
Figure 3.3: Method for increasing the spatial integration length, here the space resolution of rain map in (a), (b), and (c) is $5\ km$, $10\ km$, and $20\ km$, respectively.

Figure 3.4: Methodology for increasing the temporal integration length from $T = 15\ mins$ to a longer one.

It is important to highlight that each grid point will be used only once for each integration and no overlapping regions are considered. The integrated data will be tiled up without changing the size of original rain map but new dataset with larger integration scale will be achieved. Note that the larger the integration length the smaller number of data samples will be.
3.5 Summary

A space-time statistical rain model has been described in this chapter. Following the lead in [39], the key characteristics of rain, including the first order statistics, spatial/temporal correlation function of rain rate and probability of rain/no rain have been described in order to model the rain in an appropriate way.

In particular, the numerical method to integrate the radar measurements from short to longer integration lengths, including 2D spatial integration, 1D temporal integration as well as 3D spatial-temporal integration, has been presented in this chapter. It focuses on the space-time variability of rain and enables a better understanding of its key characteristics, which can be studied in a broad range of space-time resolutions. In addition, this is also important for the development of an interpolation approach that is presented in Chapter 5. The outcome of a comprehensive experimental study of rain radar spanning the whole of Western Europe will be discussed in following chapters.
Chapter Four: Experimental Results of Rainfall Rate Modelling

Detailed knowledge of the space-time characteristics of rainfall rate fields is an increasing requirement for planning and dimensioning wide area high frequency satellite communication networks. This chapter presents comprehensive empirical results obtained from a Europe-wide rain radar network. In particular, this chapter presents detailed multi-resolution databases and software for predicting the statistical/dynamic parameters characterising European precipitation rate fields. These include description of the: (i) first order statistical distribution, (ii) spatial and temporal correlation function of rainfall rate and, (iii) probability of rain/no-rain, with a focus on their relevance to satellite and terrestrial microwave network design. The key contribution of this chapter is the assessment of the impact of varying spatial and temporal integration lengths on these quantities. The dependence of key statistical parameters on integration volumes is also discussed.

4.1 Introduction

Rain-induced attenuation is the dominant dynamic impairment of microwave wireless signals often used for high capacity satellite and terrestrial links. The experimental characterisation of rain based on measurements (e.g. using raingauges or rain radars) at different integration lengths, is extremely important to communication systems design. This is because integration over different space and time scales changes the rain rate statistics. Radio engineers need to work with rain rates measured over a range of spatial-temporal integration volumes, and use the results
to predict fading on links for a particular temporal integration interval, [41][42]. This chapter therefore addresses the issue of making available rain characteristics at multiple space ($\lambda L$) and time ($\varphi T$) scales to communication systems engineers. Realistic planning and design of systems requires descriptions of rain characteristics at smaller scales $L'$ and $T'$ than are typically available from radars or raingauges. However, network planners and designers of physical layer fade mitigation techniques [65] require knowledge of rain variation over much shorter time scales: of the order of seconds or less. This requirement provides impetus for the development of a rain model which can be used to predict rain precipitation rates at fine scales.

Research has shown that the relationship between rain characteristics at different integration lengths is not trivial due to the range of physical processes that lead to spatial and temporal rain rate variation, e.g. [112][113]. Rainfall rates exhibit dramatic increases in magnitude when the integration length is reduced, particularly in the time domain. This indicates that the characteristics of rain will not be constant, but exhibit changes with varying integration lengths both in space and time domains, e.g. the probability of rain occurrence decreases with decreasing integration length approaching to zero as the integration length approaches zero (this point is expanded upon in Section 4.3.3). Thus, detailed knowledge of the space-time characteristics of rainfall rate is important for planning and dimensioning wide area high frequency satellite communication networks, particularly at smaller scales than those at which they are commonly measured [114].

As previously mentioned in Section 3.2 the space-time statistical rain model proposed in this thesis has been developed by studying four key characteristics of rain, as used in Bell's model [39]. His model is constructed with several constraints that serve to simplify the complexity. One of them is that the statistics within the rain
fields are assumed to be homogeneous and isotropic in space and stationary in time. According to this assumption, every location within the rain fields has the same rain characteristics, so that fewer parameters need to be specified. For example, when it rains, on average, any one grid point within the studied rain map has the same probability of rain occurrence, see details in [39]. However, according to [115]-[117], rain statistics are not homogeneous over a region due to topography, etc. In this thesis rain precipitation information at all points in the studied areas that cover a range of different topographies as well as climates have been investigated to obtain the representative statistics of typical rain conditions in Western Europe. The whole of Western Europe has been studied to investigate how the characteristics of rain change with varying locations. In this chapter, the statistics of the rain fields at Portsmouth are presented as an example to discuss the key characteristics of rain in detail.

4.2 Goodness of Fit Test

The goodness of fit test describes how well a statistical distribution fits a set of observations. Such tests are commonly used to evaluate the association between the predictions and the observed data. A null hypothesis is proposed that the observations come from a specific probability distribution. A test statistic is then calculated from the observations. The probability of the observed test statistic value occurring by chance, assuming the null hypothesis is true, is then calculated. If this probability is small then this is evidence that the null hypothesis is not correct. The Chi-squared test has been chosen to carry out the goodness of fit test in this project. A Chi-squared test is a statistical hypothesis test in which the distribution of the test statistic is a Chi-square distribution when the null hypothesis is true; it also referred to as $\chi^2$ test. The Chi-squared statistic is normally defined as:
\[ \chi^2 = \sum_{i=1}^{n} \frac{(P_i - E_i)^2}{E_i} \] (4.1)

Here \( P_i \) and \( E_i \) are predicted and expected data frequencies, respectively. For a chi-square goodness of fit test, the hypothesis takes the following form.

\[ H_0: \text{The data are consistent with a specified distribution.} \]
\[ H_a: \text{The data are not consistent with a specified distribution.} \]

Typically, the null hypothesis (\( H_0 \)) specifies the proportion of observations at each level of a categorical variable, such as a histogram frequency.

In this project, the \( P_i \) are the predictions of the proposed model and \( E_i \) are the measured data. Taking the significance level of 0.05 as standard, if the calculated \( P \)-value is less than 0.05, then this is interpreted as evidence that the null hypothesis is not acceptable. It means the predictions of the proposed model are inconsistent with the rainfall field data. However, if the calculated \( P \)-value is greater than 0.05, it indicates that model predictions are acceptable. Noticeably, the higher the \( P \)-value, the closer the measured and predicted data are.

For the Chi-squared statistic to follow a Chi-squared distribution, the probability of the categorical outcomes needs to be nearly the same. For test of rain rate distributions, this means that the histogram bins should be close to equi-probable. Given a log-Normal rain rate distribution, equi-probable bin boundaries can be estimated from the cumulative probability distribution function. The probability is divided into several equal ranges (based on the requirement). Ideally, the frequency of corresponding measured rainfall rates within each range should be the same. This forms the equal-probability bin histogram of rainfall rate.
The requirement for near equi-probable bins reduces the utility of the Chi-squared test for radio applications. The design of radio systems is determined by the incidence of rain rates at exceedance percentages of 0.01% or less. For histogram bins to resolve the distribution shape at these low exceedance probabilities required a very large number of bins spanning small rain rate ranges, often smaller than the measured rain rate quantisation error is required. Some compromise is required in order to use a standard and recognised goodness of fit test for radio engineering applications. It is useful to study the rainfall rates from 30 mm/h to 60 mm/h, which is the range that is important in the satellite communications. See the results in following sections.

4.3 Experimental Results

4.3.1 Statistics of Rain

Generally, rainfall rate measurements are carried out at particular locations at uniformly distributed time intervals. For a particular location, a histogram of measurements may be formed and a cumulative distribution of the measured occurrences of different rain intensities can be generated. Here Portsmouth (UK) is taken as an example to discuss the results in detail.
4.3.1.1 Log-Normal Distribution Test in Space

Fig. 4.1(a) illustrates the cumulative distribution function (CDF) of rainfall rate conditioned on the occurrence of rain for rainfall rate ranging from $1 \text{ mm/h}$ to $150 \text{ mm/h}$. The values at $150 \text{ mm/h}$ count all occurrences of rain rates higher than this threshold. Over the period of observation (five complete years), it is clear that the rainfall rate is usually less than $30 \text{ mm/h}$ and the probability of heavier rain is very low for any spatial integration length.

Fig. 4.1(b) is the normalised CDF for different spatial integration lengths. Simply dividing each histogram by the total number of sample at that integration scale, the y-axis is then transformed to be the proportion within a bin.

The complementary cumulative distribution function (CCDF) of rainfall rate with spatial integration lengths ranging from $L = 5 \text{ km}$ to $L = 75 \text{ km}$ ($T = 15 \text{ mins}$) have been estimated. Using the technique described in [106], the experimental CCDF is then transformed to test its log-Normality. According to [106], a Normal distribution leads to a straight line given by:

$$Q_{\text{inv}} = \frac{\ln(R)}{\sigma} + \frac{\mu}{\sigma} \quad (4.2)$$

where $\mu$ and $\sigma$ are the mean and standard deviation, respectively, of the log-Normal PDF given in Eq. (3.1).
Figure 4.1: (a) the CDF of rainfall rate for different spatial integration lengths, and (b) normalised CDF, here ($T = 15 \text{ mins}$) and rain rate bins are evenly distributed with $1 \text{ mm/h}$ width.
Example results are shown in Fig. 4.2. The symbols are the measured data and Least Squares (LSQ) linear regression lines are also plotted. These results appear to support the hypothesis that rain rate distributions are well approximated by log-Normal distributions. However, it should be noted that, due to the log rain rate parameter, the plot is dominated by the incidence of low rain rates. There does appear to be a consistent "s" shaped variation from linear across all integration region sizes. This is consistent with the findings of other researchers who have found that rainfall intensity is well modelled as log-Normally distributed, for example in [118].

**Figure 4.2:** The test for log-Normality of rainfall rate distribution for different spatial integration lengths, here $T = 15 \text{ mins}$. Lines are the fitted curve for each integration length.
Table 4.1 lists all the derived coefficients $\mu$ and $\sigma$ for Portsmouth with different spatial integration lengths. Notably, the value of $\mu$ gradually increases with increasing integration length while for $\sigma$, the coarser the resolution the smaller the $\sigma$ value. In particular, such systematic change indicates that $\{\mu, \sigma\}$ values at finer resolutions can be predicted based on the tendency. This assumption has been tested using the 1 km spatial grid UK NIMORD data. According to the Fig. 4.3, one can see that when the spatial integration length is 1 km ($\lambda = 0.2$), $\mu$ and $\sigma$ are approximately equal to $-3.5$ and $2.1$, respectively. Meanwhile, the measurement of $\mu$ from UK NIMROD is $-3.85$ and $\sigma$ is $2.04$. Although the $\{\mu, \sigma\}$ values of both are different, the associated 0.01% exceeded rain rates are similar: 24 mm/h for the 1 km data and 22 mm/hr for the interpolation from the 5 km data. Details can be seen in Section 4.3.1.3. It appears that the systematic variation seen in Fig. 4.3 may be used to predict rain distributions at other spatial integration areas. This demonstrates that it might be acceptable to take advantage of such a regular change to produce predictions of $\{\mu, \sigma\}$ at finer scales both in space and time domains. It will significantly reduce the computation time and make it significantly easier to interpolate to finer integration volumes. Details are provided in Chapter 5.

**Table 4.1:** Experimental coefficients value of log-Normal distribution parameters $\{\mu, \sigma\}$ for different spatial integration lengths at Portsmouth ($T = 15\,mins$).

<table>
<thead>
<tr>
<th>$L$ (km)</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
<th>55</th>
<th>65</th>
<th>75</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>-3.35</td>
<td>-3.21</td>
<td>-3.00</td>
<td>-2.97</td>
<td>-2.79</td>
<td>-2.67</td>
<td>-2.60</td>
<td>-2.52</td>
<td>-2.45</td>
<td>-2.41</td>
<td>-2.33</td>
<td>-2.28</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1.99</td>
<td>1.84</td>
<td>1.71</td>
<td>1.66</td>
<td>1.59</td>
<td>1.52</td>
<td>1.48</td>
<td>1.49</td>
<td>1.43</td>
<td>1.39</td>
<td>1.31</td>
<td>1.22</td>
</tr>
</tbody>
</table>
4.3.1.2 Log-Normal Distribution Test in Time

Similarly, Fig. 4.4 and Fig. 4.5 illustrate the experimental CDF results and log-Normality test for different temporal integration lengths. In particular, Fig. 4.1(a) and Fig. 4.4(a) show that the occurrence of rain gradually decreases with increasing rain rate and integration length. Fig. 4.1(b) and Fig. 4.4(b) show that the probability of a spatial-temporal volume containing rain gradually increases with the increasing integration length both in space and time domains up to around 65 mm/h. It inverses if rainfall rate is higher than this threshold due to the occurrence of heavy rain event is very low. The systematic variation between fitted lines in Fig. 4.2 and Fig. 4.5 indicate that the statistics of rain rate changes with varying integration lengths.

Figure 4.3: Experimental coefficients value of log-Normal distribution parameters \{μ, σ\} for increasing \( \lambda \) at Portsmouth (\( T = 15 \) mins).
Figure 4.4: (a) the CDF of rainfall rate for different temporal integration length, and (b) normalised CDF, here ($L = 5 \text{ km}$) and rain rate bins are even distributed with width $1 \text{ mm/h}$.
Figure 4.5: The test for log-Normality of rainfall rate distribution for different temporal integration lengths, here $L = 5\ km$. Lines are the fitted curve for each integration length.

Similar to the space domain, the lognormal distribution parameters $\{\mu, \sigma\}$ also exhibits a systematic change with increasing temporal integration length. Table 4.2 and Fig. 4.6 show that the longer the temporal integration length, the bigger the magnitude of $\mu$, and the smaller the $\sigma$ i.e. the distribution become narrower and centred on the mean rain rate. The prediction at finer scales has been validated by comparing the predicted parameters with those derived from UK NIMROD data (with $T = 5\ mins$). The results of exceeded rain rate at 0.01% are provided in Section 4.3.1.3.
Figure 4.6: Experimental coefficients value of log-Normal distribution parameters \( \{\mu, \sigma\} \) for increasing \( \varphi \) at Portsmouth \( (L = 5 \text{ km}) \).

Table 4.2: Experimental coefficients value of lognormal distribution parameters \( \{\mu, \sigma\} \) for different temporal integration lengths at Portsmouth \( (L = 5 \text{ km}) \).

<table>
<thead>
<tr>
<th>( T (\text{min}) )</th>
<th>15</th>
<th>30</th>
<th>45</th>
<th>60</th>
<th>75</th>
<th>90</th>
<th>105</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu )</td>
<td>-3.35</td>
<td>-3.16</td>
<td>-3.00</td>
<td>-2.95</td>
<td>-2.82</td>
<td>-2.78</td>
<td>-2.77</td>
<td>-2.64</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>1.99</td>
<td>1.81</td>
<td>1.72</td>
<td>1.67</td>
<td>1.60</td>
<td>1.56</td>
<td>1.49</td>
<td>1.43</td>
</tr>
</tbody>
</table>

Fig. 4.7 gives values of \( \mu \) and \( \sigma \) for different linear map dimensions, \( S \), from \( 5 \text{ km} \times 5 \text{ km} \) to \( 530 \text{ km} \times 530 \text{ km} \), with \( L = 5 \text{ km} \) and \( T = 15 \text{ mins} \). It is evident that both \( \mu \) and \( \sigma \) values change significantly for small areas \( S < 200 \text{ km} \times 200 \text{ km} \); and then
gradually become stable and converge to constant values when the size of the map is greater than 400 km × 400 km. This happens for any other spatial and temporal integration lengths ranging from \( T = 15 \text{ mins} \) to \( T = 120 \text{ mins} \) although all figures are not presented. In this thesis, the study was carried out based on 400 km × 400 km.

**Figure 4.7:** Experimental values of parameters of fitted lines for different map sizes \( S \) centred at Portsmouth with \( L = 5 \text{ km} \) and \( T = 15 \text{ mins} \).

### 4.3.1.3 Validation

The Chi-squared test was applied to test how well the proposed model fits the observations. Equal probability histogram bins are required for the Chi-squared test. Fig. 4.8(a) shows the comparison between equal-probability bin histograms of rainfall rates observed and expected, here \( L = 5 \text{ km} \) and \( T = 15 \text{ mins} \). The calculations
based on these data yields the probability of different rainfall rates being exceeded, see Fig. 4.8(b). As stated in previous section, the rainfall rates ranging from 1 mm/h to 150 mm/h have been binned into 151 bins with the first bin for 0 mm/h, which means no rain. The last bin is the occurrence when \( R \geq 150 \text{ mm/h} \). It is obvious that the number in each bin decreases with increasing rain rate as the heavier the rain rate the lower the probability. Bins containing non-zero rain rates are combined to yield 10 near equi-probable bins, based on the fitted log-Normal distribution. The bins are not exactly equi-probable due to the quantisation of the original histogram bins and the large probability of low rain rates. Some errors are also introduced as the first bin contains an unknown mixture of zero and low rain rates (rain below the sensitivity of the radar). Given the rainfall rates from 30 mm/h to 60 mm/h is the range that is important for satellite communications, the result in Fig. 4.8(b) shows that the probability of rain \( P \) is between 0.01% and 0.001%. This fit appears plausible up to 35 mm/h, after which systematic divergence occurs. However, it should be noted that the probability of exceedance at these higher rain rates are based on a very small number of samples i.e. less than 10 samples for 55 mm/h; and so the uncertainty is large.
Figure 4.8: Comparison between observed and expected results: (a) near equal probability histograms of rainfall rate, (b) rainfall rates exceedance distribution, here $L = 5\ km$ and $T = 15\ mins$. 
Fig. 4.9 presents the rain rate exceeded for 0.01% of the average year given by Rec. ITU-R P. 837-6 [13] using 1-min temporal integration length of rainfall rate, which is useful for the prediction of rain attenuation in radio communications. This rainfall rate statistic is in the range between 30 $mm/h$ and 60 $mm/h$ in Western European. In the Portsmouth area the 0.01% exceeded rain rate is approximately 30 $mm/h$, which is somewhat larger than the 5 km-instantaneous 0.01% exceeded rain rate consistent with the distribution presented in Fig 4.8(b).
Figure 4.9: Rain rate (mm/h) exceeded for 0.01% of the average year given by Rec. ITU-R P. 837-6.
The results of the Chi-squared test are presented in Fig. 4.10. It is clear that the proposed model can give reasonable fits to rain rate exceedances around the 0.01% level important for radio systems. However, the Chi-squared test statistic is very high, with value of 130421.2275 (see the “Chi-squared” in Fig. 4.10). Assuming independent rain rate samples, the probability of observing the measured distribution given the proposed underlying distribution, is exceedingly small. However, the rain rate samples are not independent. In particular, the heavy rain rates are associated with events and the incidence of these events varies considerably from year-to-year. If the rain rate time series is sub-sampled to yield less dependency between samples, then the important exceedances cannot be estimated. Also note that as the Chi-Squared test assumed equi-probability histogram bins, the test is highly insensitive to the distribution of the higher rain rates i.e. the ones that radio system engineers are particularly interested in. A different method is required to determine the usefulness of the results developed in this project.

![Results](image)

**Figure 4.10:** Detailed results of model prediction and Chi-squared test result.
The log-Normal distributions are fitted to the data by minimising an error functional: \( \text{Error}(\mu, \sigma) \).

\[
\text{Error}(\mu, \sigma) = \sum_{i=1}^{3} W_i |\ln \left( \frac{R_i}{R_i(\mu, \sigma)} \right) |
\]  

(4.3)

where \( R_i \) are the measured rain rates exceeded with probability 0.1\%, 0.01\% and 0.001\% for \( i = 1,2 \) and 3. The function \( R_i(\mu, \sigma) \) is the rain rate exceeded with the given probability using the model (Eq. (3.1)). The weights \( W_i \) are required to allow for the greater uncertainty in the rain rates exceeded with lower probabilities. For this project the weights used were \( W_i = \{2,1,1\} \). These weights have been chosen from experience as the actual uncertainty in these rain rates is unknown. This error function approximates the Rec. ITU-R P.311 [119] goodness of fit metric and is approximately the mean relative error.

Using the multi-dimensional Taylor expansion, at the minimum, the error functional can be approximated by:

\[
\text{Error}(\mu + \Delta \mu, \sigma + \Delta \sigma) \approx \text{Error}(\mu, \sigma) + \frac{1}{2} \left( \begin{array}{c} \Delta \mu \\ \Delta \sigma \end{array} \right)^T H \left( \begin{array}{c} \Delta \mu \\ \Delta \sigma \end{array} \right)
\]  

(4.4)

where \( H \) is the Hessian matrix:

\[
H \equiv \begin{bmatrix}
\frac{\partial^2 \text{Error}}{\partial \mu^2} & \frac{\partial^2 \text{Error}}{\partial \mu \partial \sigma} \\
\frac{\partial^2 \text{Error}}{\partial \mu \partial \sigma} & \frac{\partial^2 \text{Error}}{\partial \sigma^2}
\end{bmatrix}
\]  

(4.5)

Around the minimum, the error function is approximately bi-quadratic with contours that are ellipses. The Singular Value Decomposition (SVD) of the Hessian gives the lengths and directions of the axes of the ellipses. The values of the second derivatives in Fig 4.10 indicate ellipse axis directions along \( \Delta \mu + \Delta \sigma \) and \( \Delta \mu - \Delta \sigma \) with an axis length ratio of approximately 100. This indicates that the fit is very insensitive
to changes where $\Delta \mu + \Delta \sigma \approx 0$. This is because the exceedance probability of rain rate is determined by the $z$ value: $z = (\ln(R) - \mu)/\sigma$. When $\mu$ becomes larger and more negative and $\sigma$ becomes smaller and more positive, the $z$ value can remain the same. The best fit of $\mu$ and $\sigma$ is determined by the poorly defined shape of the exceedance distribution for low-probability high rain rates.

The UK NIMROD data, which are provided with a sampling of 1 km in space and 5 mins in time, have been integrated over 5 km by 5 km regions to approximate the EU NIMROD data. The rain rate exceedance distributions at Portsmouth (UK) were compared with the one achieved from radar-derived EU NIMROD data, see Fig. 4.11.

**Figure 4.11**: Comparison of rainfall rates exceedance distribution at Portsmouth using EU and UK data.
The figure compares the observed and expected results at Portsmouth using EU and UK data, respectively. From the plot one can note that after the integration, the rain rate exceedance distributions generated from different databases are nearly the same, although the distributions given by UK NIMROD data is slightly lower. The observed 0.1%, 0.01% and 0.001% exceeded rainfall rates from both EU and UK data at Portsmouth are \( \{7.85 \text{ mm/h}, 23.9 \text{ mm/h}, 67.9 \text{ mm/h}\} \) and \( \{7.6 \text{ mm/h}, 21.7 \text{ mm/h}, 66.3 \text{ mm/h}\} \), respectively.

### 4.3.2 Correlation Function of Rain Rate

The study of the spatial and temporal correlation function of rain rate is important for rainfall field modelling [126]. Such correlation functions can be used to generate rain rate samples in a computer simulation, e.g. the space-time stochastic model given by Bell [39] and the autoregressive model by Hendrantoro [120]. Many studies show that the spatial and temporal correlation functions vary from one location to another depending on climate, topography, rainfall type, etc [115]-[117]. Furthermore, the impact of space and time averaging on the autocorrelation function is of particular interest [42][122]. All these factors should be taken into account in the prediction of rain-induced attenuation.

#### 4.3.2.1 Spatial Correlation Function of Rain Rate

One of the important factors for planning satellite system networks utilising high frequencies is the requirement for detailed knowledge of parameters related to the horizontal structure of rainfall fields [53]. From the literature review, many empirical mathematical formulae have been proposed to model the spatial correlation function of rain rate. Some widely used models are presented here to show the development
of spatial correlation function models in the last few decades. In general, these models can be divided into two groups based on the data collection methodology.

The first group uses the measurements obtained from raingauge networks. The study of spatial correlation based on raingauge observations has been conducted by many researchers. A representative piece of work was undertaken by Yevjevich and Karplus [52] in 1973 who proposed three common models based on the assumption of homogeneity and isotropy in space. The models are as follows:

Reciprocal Model:

\[ \rho = \frac{1}{1 + d/c_o} \]  \hspace{1cm} (4.6)

Square Root Model:

\[ \rho = \frac{1}{\sqrt{1 + d/c_o}} \]  \hspace{1cm} (4.7)

Exponential Model:

\[ \rho = \exp \left( -\frac{d}{c_o} \right) \]  \hspace{1cm} (4.8)

where \( \rho \) is the cross-correlation factor, \( d \) represents distance and \( c_o \) is a parameter with units of distance which needs to be estimated. These three models were evaluated and extended by Tabios and Salas [46] for a study of rain property in north central continental United States.

Later, the work of Morita and Higuti [122] demonstrated that the spatial correlation coefficient for a Japanese climate can be expressed as:

\[ \rho = \exp \left( -\sqrt{\frac{d}{c_o}} \right) \]  \hspace{1cm} (4.9)
Here, $\rho$ is the cross-correlation factor, $d$ represents distance and $c_o$ is the parameter that needs to be determined.

In 1988, Fukuchi [52] proposed another exponential model for space correlation characteristics after studying the rainfall rate at 23 locations in the UK. The correlation coefficient as a function of the separation distance was derived from the study of 253 pairs of the locations. It is given by:

$$\rho = \exp \left( - \left( \frac{d}{c_o} \right)^\alpha \right)$$  \hspace{1cm} (4.10)

where $\rho$ is the cross-correlation factor, $d$ represents distance in km, $\alpha$ and $c_o$ are the parameters derived using the least-squares method.

The second group of models are based on radar-derived observations. In 1985, Capsoni [129][130] proposed the functional dependence of spatial correlation coefficient, equation 4.14, after studying radar observations in Italy. He proposed the Eq. (4.11):

$$\rho = \exp \left( - \frac{d}{c_o} \right)$$  \hspace{1cm} (4.11)

The biggest advantage of his model is its simplicity as only one coefficient $c_o$ is needed. However, single parameter models may not be adjustable to a wide variety of climates. It is reasonable to say that the fewer the number of parameters, the narrower the range of climates it will be applicable to. Hence, Bell [38] proposed another model with several parameters:

$$\rho = \frac{1}{(c_o d + c_o)^\alpha}$$  \hspace{1cm} (4.12)
Here $d \geq 4 \text{ km}$ (for information, $4 \text{ km}$ is the space resolution of the data used for Bell's study). It has been shown that Bell's model greatly improves the fitting accuracy at the cost of adding two more coefficients than Capsoni's. The work done by Kundu and Bell [131] developed a model that can provide the correlation function of rain rate in space and time domains simultaneously, but in a very complicated form. Although the accuracy is high, it however involves 10 parameters. The equation is given by:

$$\rho(d, t) \approx \omega(s)\exp\left\{-[\pi t(s)]^{\mu(s)}\right\}$$  \hspace{1cm} (4.13)

Here,

$$\omega(d) = \begin{cases} (a_1 d + a_2)^{-a_3} e^{\frac{a}{a_4}} & d \geq 4 \text{ km} \\ d = 0 \text{ km} \end{cases}$$  \hspace{1cm} (4.14)

$$t(s) = \begin{cases} b_1 d^{b_2} + b_3 & d \geq 4 \text{ km} \\ t(0) & d = 0 \text{ km} \end{cases}$$  \hspace{1cm} (4.15)

$$\mu(s) = \begin{cases} c_1 d^{c_2} + c_3 & d \geq 4 \text{ km} \\ \mu(0) & d = 0 \text{ km} \end{cases}$$  \hspace{1cm} (4.16)

A summary of the models is presented in Table 4.3. Each model is derived from the study of different regions covering different continents, and for rain rate measurements made with a range of integration volumes. The differences amongst these models indicate that the spatial structure of rainfall fields depends strongly on climate, topography, etc. This finding is consistent with the conclusions of [53]. Therefore, it is reasonable to conclude that simple models with one or two parameters will not accurately fit the spatial correlation coefficient for the whole world. However, the table shows that the exponential model is probably the most popular for modelling the spatial correlation function of rain rate. This is in agreement with
the opinion of Manabe and Kobayashi [53] who argued that the exponential distribution can give good agreement for European regions. Other exponential models can be found, such as in [127][128].

Table 4.3: List of the popular space correlation function models

<table>
<thead>
<tr>
<th>Model</th>
<th>Author</th>
<th>Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho = \frac{1}{1 + d/c_o}$</td>
<td>Yevjevich &amp; Karplus</td>
<td>1973</td>
</tr>
<tr>
<td>$\rho = \frac{1}{\sqrt{1 + d/c_o}}$</td>
<td>Morita and Higuti</td>
<td>1976</td>
</tr>
<tr>
<td>$\rho = \exp\left(-\frac{d}{c_o}\right)$</td>
<td>Fukuchi</td>
<td>1988</td>
</tr>
<tr>
<td>$\rho = \exp\left(-\left(\frac{d}{c_o}\right)^{\alpha}\right)$</td>
<td>Capsoni</td>
<td>1985</td>
</tr>
<tr>
<td>$\rho = \frac{1}{(c_o d + a_o)^\alpha}$</td>
<td>Bell</td>
<td>1987</td>
</tr>
<tr>
<td>$\rho(d, t) \approx \sigma^2 \omega(s)\exp\left{-[\tau(\mu(s))]^{\mu(s)}\right}$</td>
<td>Bell and Kundu</td>
<td>1996</td>
</tr>
</tbody>
</table>

Research reported in this thesis, has produced another single general empirical equation that fits both the spatial correlation and the temporal correlation functions of rain rate. The common model is given by:

$$\rho = \frac{a}{a + x^q}$$  \hspace{1cm} (4.17)
Where $a > 0$ and $q > 0$ are coefficients to be determined from the data, and $x$ can either be $d$ (here $d$ represents the distance in km) or $t$ (here $t$ is the time lag in mins). It is clear that the proposed model in this study balances the advantage and disadvantage of Capsoni’s and Bell’s models as there are two coefficients involved. However, the fitting accuracy is still high and it can be easily tuned, as will be seen in the following sections.

In this study, five complete years of rainfall rate data have been analysed to estimate the spatial and temporal correlation function of rain rate. According to the homogeneous and isotropic assumption, the spatial correlation function of rain rate at one location is studied based on a fixed size of map, which shifts grid by grid horizontally and vertically until there is no more overlap.

Fig. 4.12 gives an example of the spatial correlation function of rain rate at Portsmouth for different spatial integration lengths ranging from $L = 5\ km$ to $L = 20\ km$. One can note that the spatial correlation function is greatly dependent on the integration length. The level of spatial correlation increases with increasing integration length up to about $150\ km$ due to the mixing of point covariance, with the larger covariance dominating. For example, let $C(z) = E[R(x)R(x + z)]$ be the second moment of the point rain rate process. For rain integrated over a line $R_d(x) = \frac{1}{2d} \times \int_{x-d}^{x+d} R(z)dz$ then the second moment is $R_d(x) = E[R_d(x)R_d(x + z)] = \frac{1}{(2d)^2} \times \int_{x-d}^{x+d} \int_{x+z-d}^{x+z+d} E[R(z_1) * R(z_2)]dz_1dz_2$. As long as $C(z)$ is decreasing and convex then $C_d(z) > C(z)$. The near linear sections of the curves (see Fig. 12(a)) suggest exponential correlations. In particular, the $5\ km$ data shows two exponential regions: from $0$ to $50\ km$ and $50\ km$ to $200\ km$. The larger integration regions mix up a range
of scales. The two exponential ranges might appear if the integration region becomes smaller.

An example of a fitted curve using the proposed model (Eq. (4.17)) is illustrated in Fig. 4.13 in which the dots are measured data with resolutions of 5 km in space and 15 mins in time, and the curve is the fitted line. Fitting has been accomplished by minimisation of the error function:

$$\text{Error} = \sum |\text{Corr}_{\text{fitted}} - \text{Corr}_{\text{measured}}|$$

(4.18)

Here $\text{Corr}_{\text{fitted}}$ is the fitted correlation function and $\text{Corr}_{\text{measured}}$ is the measured value.
Figure 4.12: Spatial correlation function of rainfall rate for different spatial integration diameters at Portsmouth (here $T = 15$ mins).
Figure 4.13: Example of fitted spatial correlation function of rain with $L = 5 \, km$ and $T = 15 \, mins$. 

(a) 

(b)
Note that the correlation function of rain rate falls off quickly with distance and the fitted curve is quite accurate up to 150 km. The measured correlation functions become negative for lags greater than approximately 200 km, and none of the proposed models in Table 4.3 produce negative correlations. Table 4.4 lists the fitted coefficients for different spatial integration lengths ranging from $L = 5$ km to $L = 20$ km. The best fit correlation models, Eq. (4.17), yields similar minimum error values for all spatial integration scales.

**Table 4.4:** Experimental coefficients value of spatial correlation functions of rain rate for each spatial integration length at Portsmouth ($T = 15$ mins).

<table>
<thead>
<tr>
<th>$L$ (km)</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>118</td>
<td>494</td>
<td>615</td>
<td>884</td>
</tr>
<tr>
<td>$q$</td>
<td>1.37</td>
<td>1.58</td>
<td>1.74</td>
<td>1.81</td>
</tr>
</tbody>
</table>

Fig. 4.14 shows the computed spatial correlation function of rain rate with different temporal averaging. The EU NIMROD data provides near-instantaneous rain rates at 15 mins sampling intervals. Averaging $n$ consecutive rain rate values for the same 5 km diameter region, yields a coarse estimate of the $(n - 1) \times 15$ mins temporal integration. It demonstrates that spatial correlation function is also greatly affected by time resolution. Similar to the results presented in Fig. 4.12, the spatial correlation gradually increases with increasing temporal integration length.
Particularly, Fig. 4.14(a) indicates that there are two ranges of exponential distribution for the 15 mins integration length but this is increasing merged into one range with increasing temporal averaging. The fitted parameter values for integration lengths ranging from 15 mins to 120 mins are given in Table 4.5.

**Table 4.5:** Experimental coefficients value of spatial correlation functions of rain rate for each temporal integration length at Portsmouth ($L = 5 \text{ km}$).

<table>
<thead>
<tr>
<th>$T \text{ (min)}$</th>
<th>15</th>
<th>30</th>
<th>45</th>
<th>60</th>
<th>75</th>
<th>90</th>
<th>105</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>117</td>
<td>165</td>
<td>206</td>
<td>221</td>
<td>222</td>
<td>205</td>
<td>218</td>
<td>214</td>
</tr>
<tr>
<td>$q$</td>
<td>1.37</td>
<td>1.41</td>
<td>1.43</td>
<td>1.42</td>
<td>1.40</td>
<td>1.37</td>
<td>1.36</td>
<td>1.35</td>
</tr>
</tbody>
</table>
Figure 4.14: Spatial correlation function of rainfall rate for different temporal integration lengths at Portsmouth (here $L = 5 \text{ km}$).
Five regions distributed across the Western European radar scanned area were investigated to see how the spatial correlation function of rain rate changes with locations. Fig. 4.15 illustrates the spatial correlation function for the five locations calculated using all five years of radar data. The spatial correlation is very close for all locations for the scales (0 – 15 km), which is a typical scale for rain storms. For larger scales, the spatial correlation is more variable and differs appreciably between different locations. This is consistent with the conclusions of Manabe’s work [53], in which the spatial correlation coefficients from 8 European locations were compared. The large difference in correlation values between Paris and Rennes is self-evident indicating a strong climatological or topographic dependence. In particular, Fig. 4.15 shows that all the distributions are close to exponential over the range 50 km to 200 km. The range beyond roughly 300 km yields a negative correlation. This observation could be utilised to optimise diversity gain when choosing locations for satellite ground stations. Further processing indicates that Eq. (4.17) is a good fit for all other studied locations. The only change required to encompass all the apparent variability of the correlation function is through the empirical coefficients $\alpha$ and $\eta$. The same conclusion can also be found in [53].
Figure 4.15: Comparison of spatial correlation functions of rain, for five European locations, derived from five years of radar data (here $L = 5\ km$ and $T = 15\ mins$).
4.3.2.2 Temporal Correlation Function of Rain Rate

Rain events are stochastic not only in space but are also highly variable in time. The temporal correlation function, as another critical rain characteristic, is significant for the statistical study of temporal evolution of spatial patterns of rainfall fields [39][40].

![Temporal Correlation Function of Rain Rate](image)

**Figure 4.16:** Temporal correlation function of rainfall rate at Portsmouth (averaging over five years), here $L = 5 \text{ km}$ and $T = 15 \text{ min}$.

Fig. 4.16 gives an example of temporal correlation function of rain rate at the Portsmouth location. It shows that the correlation in time is highly peaked and drops down quickly over short time lags (labelled $t$); then the correlation decreases more gently from time lags of about $360 \text{ mins} (6 \text{ h})$ down to 0 over time lags up to about $1100 \text{ mins} \approx 18.3 \text{ hours}$. In addition, it is worth highlighting that the exponential
section is between 200 mins and 1100 mins. After that the temporal correlations become irregular with high variations.

Fig. 4.17 compares the temporal correlation functions of rain rate for five European locations. This was computed from five years data, but only short time lags (up to 1440 mins or one day) are shown. Clearly, there are many similarities to the spatial correlation curves. The temporal correlation function changes significantly between locations. At a correlation value of 0.37 (equal to the value of \(1/e\)), correlation times of between 60 mins and 200 mins occur at different locations indicating a strong location-dependency. It should be noted that for lags greater than about 360 mins (6 h) the correlation value drops off slowly indicating that dependence extend over long periods. Hurst reported positive correlations in wet years extending over 100 years.

![Temporal correlation function of rainfall rate for five European locations](image)

**Figure 4.17:** Temporal correlation function of rainfall rate for five European locations, here \(L = 5 \text{ km}\) and \(T = 15 \text{ mins}\).
Fig. 4.18 shows that the temporal correlation function of rainfall rate changes significantly with temporal integration lengths between 15 mins and 120 mins. In particular, Fig. 4.18 indicates that the temporal correlation function, similar to spatial correlation, increases with increasing integration length because the variance is gradually compensated. This also applies to other locations although the results are not presented in this thesis. Similar results can be found in [132] but it only considered sampling periods of 10 s and 1 min using data measured from disdrometer. It is clear that for time lags greater than about 500 mins, irrespective of the time resolution, the temporal correlation value falls off very slowly and is below 0.2. The points in Fig. 4.19 are the correlations estimated from the data while the curves are the fitted lines for different temporal integration lengths. The fitted curves, based on Eq. (4.17), are in reasonable agreement with the measured data throughout the whole range of time lags up to roughly 1000 mins (more than 16 h). The fitted values of the coefficients in Eq. (4.17) for each temporal integration length are listed in Table 4.6.
Figure 4.18: Temporal correlation function of rainfall rate for different temporal integration lengths at Portsmouth, here $L = 5\ km$.

Figure 4.19: Fitted lines for the temporal correlation function of rainfall rate with different temporal integration lengths at Portsmouth, here $L = 5\ km$. 
Table 4.6: Experimental coefficients of temporal correlation function of rain for different temporal integration lengths at Portsmouth ($L = 5\ km$).

<table>
<thead>
<tr>
<th>$T$ (min)</th>
<th>15</th>
<th>30</th>
<th>45</th>
<th>60</th>
<th>75</th>
<th>90</th>
<th>105</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>24.1</td>
<td>45.5</td>
<td>60.6</td>
<td>84</td>
<td>102</td>
<td>136</td>
<td>193</td>
<td>215</td>
</tr>
<tr>
<td>$q$</td>
<td>0.86</td>
<td>0.93</td>
<td>0.96</td>
<td>0.99</td>
<td>1.01</td>
<td>1.05</td>
<td>1.09</td>
<td>1.10</td>
</tr>
</tbody>
</table>

Fig. 4.20 shows the short-lag temporal correlation function at Portsmouth for different spatial integration lengths ranging from $L = 5\ km$ up to $L = 75\ km$ with $T = 15\ mins$. One can note that the temporal correlation also increases with increasing spatial integration length and this is similar to Fig. 4.18. It shows that the temporal correlation function gradually becomes smoother with increasing integration length. For any given time lag, the larger the integration length, the smaller the variance, and the higher the correlation level will be.
**Figure 4.20**: Temporal correlation function of rainfall rate for different spatial integration lengths at Portsmouth, here $T = 15 \text{ mins}$.

**Table 4.7**: Experimental coefficients of temporal correlation functions of rain rate for each spatial integration length at Portsmouth ($T = 15 \text{ mins}$).

<table>
<thead>
<tr>
<th>$L$</th>
<th>5 km</th>
<th>10 km</th>
<th>15 km</th>
<th>20 km</th>
<th>25 km</th>
<th>35 km</th>
<th>40 km</th>
<th>45 km</th>
<th>50 km</th>
<th>55 km</th>
<th>65 km</th>
<th>75 km</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>24.1</td>
<td>24.6</td>
<td>44.2</td>
<td>59.9</td>
<td>74.8</td>
<td>87.4</td>
<td>141</td>
<td>176</td>
<td>131</td>
<td>376</td>
<td>471</td>
<td>121</td>
</tr>
<tr>
<td>$q$</td>
<td>0.86</td>
<td>0.87</td>
<td>0.92</td>
<td>0.96</td>
<td>0.98</td>
<td>0.99</td>
<td>1.07</td>
<td>1.10</td>
<td>1.04</td>
<td>1.22</td>
<td>1.24</td>
<td>1.00</td>
</tr>
</tbody>
</table>
The differences between the temporal correlation between $L = 5 \text{ km}$ and $L = 75 \text{ km}$ indicate a strong integration length dependency. However, the temporal correlation falls off quickly for any spatial integration length with short time lags to below 0.37 (roughly $1/e$) roughly 500 mins later. It keeps decreasing after that but at a slower rate. Interestingly, one can note that the temporal correlation becomes smoother with increasing spatial integration length due to the averaging of the notional point correlation. The proposed mathematical model Eq. (4.17) can give a good fit for the temporal correlation with any of the investigated integration lengths by tuning the coefficients $a$ and $q$. The fitted coefficients of temporal correlation function for each spatial integration length are given in Table 4.7.

In this study, the temporal correlation function of rain at one location is achieved by analysing five complete years (2005 to 2009) of data. Typically, the temporal correlation function of point rainfall rate will be affected by advection. The variation of rainfall rate at a point is due to two processes; the first process is the evolution of the rain event. This tends to be slow but dominates at time lags of 40 minutes or longer. The second cause of variation at a point is the movement of the rain event over that point. This effect dominates for short time lags. This variation is modelled as the movement (advection) of a fixed pattern of rain over a point by the frozen storm hypothesis, which states that the statistics of rain at a point are the same as the statistics of rain along a line parallel with advection. The interpretation of this is:

$$R(x_0, t) = R(x_0 - V(t - t_0), t_0),$$

where $V$ is the advection vector, $x_0$ and $t_0$ are an arbitrary position and time. This means that for time lags less than forty minutes, the temporal covariance of point rain rate for a lag $\tau = (t - t_0)$ is the same as the spatial covariance for a spatial lag of $|V \cdot \tau|$. This analysis assumes that the rainfall field is homogeneous and isotropic. The temporal correlation presented is the actual
correlation experienced at a point. At short time lags it averages over the range of advection speeds experienced. For lags greater than about 40 minutes, the results are determined by a combination of the evolution of large rain events and the clustering of rain events.

4.3.2.3 Validation

The predictions were subjected to extensive validation to test the agreement with measurements. Comparing the model predictions with observational data from apparatus (e.g. raingauge or rain radar) is commonly used to validate model performance. Therefore, the predicted rainfall rate and radar/raingauge-derived rain precipitations are needed. The predictions of the model proposed in this thesis, however, are not rainfall rate data but the derived key characteristics of rain. Because of this it is necessary to compute the observational data and compare the computed rain characteristics with the predicted ones to validate the model performance. In this study, the UK NIMROD data has been up-scaled to achieve the same integration length as EU data for which the spatial resolution is 5 km and time resolution is 15 mins. The data was then used to estimate the studied rain characteristics.

One complete year of UK NIMROD data (2008) has been studied to validate the model performance. The data resolution has been integrated up to 5 km in space and 15 mins in time. Fig. 4.21 presents the spatial correlation function of rain rate at Portsmouth estimated from both EU NIMROD database and UK NIMROD database. The dots are derived from UK data of year 2008, the circles are measurements from EU data of year 2008, and the crosses are the measurements from five complete years of EU NIMROD data (2005 to 2009). The up-scaled UK 1 km grid data from
2008 yields very similar results as the EU 5\,km data for the same period. This is reassuring but unsurprising as the two datasets are derived from the same radar measurements over the UK. The results derived from 5 years of EU data are significantly different showing that 2008 was not representative of the 5 year period and indicating the scale of year-to-year rainfall rate variation. The two near-exponential ranges, from 0\,km to 50\,km and between 50\,km and 200\,km, appear in both the single year of data and the five years of data. This suggests that the same processes are present but in different proportions. This finding is in accordance with the results presented in Fig. 4.12 - 4.15.

![Figure 4.21: Comparison of spatial correlation function of rain rate at Portsmouth achieved from different databases, here L = 5\,km and T = 15\,mins.](image)
A comparison of temporal correlation function of rain rate at Portsmouth achieved from both EU NIMROD database and UK NIMROD database for 2008 is presented in Fig. 4.22. Clearly, the temporal correlation achieved from each database decreases with time lag, although there is a small discrepancy. It shows that the proposed model can give reasonable prediction of the temporal correlation at one location for different years.

![Figure 4.22: Comparison of temporal correlation function of rain rate at Portsmouth achieved from both EU NIMROD database and UK NIMROD database, for $L = 5\ km$ and $T = 15\ mins$.](image)

**4.3.3 Probability of Rain Occurrence**

From the point of view of the design of rain models, the probability of rain occurrence is a vital rain property [131]-[136]. In this thesis, the probability of rain occurrence $P_0$
is studied by processing five complete years of radar data. As previously mentioned in Section 3.2, $P_0$ can be characterised as the percentage of time at one point over a long period when rain actually occurs ($R > 0$). In fact, studying the rain rate at each individual location is a good starting point for the probability of rain occurrence. Each single point has its own $P_0$ value, while the $P_0$ value for any size of map can be easily deduced by averaging the $P_0$ values for all points within that map. It should be noted that the value of $P_0$ is poorly defined, as it is very difficult to tell whether rain is very light or it is not raining at all. Its value is determined strongly by the way rain rate is measured as many instruments (such as raingauges) become unreliable at low rain rates. For radars, the incidence of light rain is either masked, or falsely generated, by noise within the radar equipment. In many studies, the value of $P_0$ is determined less by the measurements of very light rain but by the optimised fit of a rain rate distribution model to the full range of rain rates.

An example that reveals how the probability of rain occurrence changes with varying map sizes, $S$, (here the radar data resolution is $L = 5 \text{ km}$ and $T = 15 \text{ mins}$) is presented in Fig. 4.23. The probability of rain varies in both space and the time interval considered. If a sufficiently long time interval is studied, then spatial variation in $P_0$ will exist due to orographic effects and microclimates. In this study we assume that the 5 years of data is sufficiently long to estimate the long-term first order statistics of rain rate. The $P_0$ is estimated through studying the proportion of $5 \text{ km}$ cell with rain rates ($R > 0$) over 5 years period (detailed mechanism is given in Section 3.3.3). Fig. 4.23 shows that this spatial variation is such that points that are a few tens of kilometres apart can have significantly different rain probabilities. This is true for any other spatial and temporal integration lengths. It causes great difficulty in characterising $P_0$ using a generic mathematical equation encompassing three factors.
However, characterising $P_0$ on a point-by-point basis is a good initial start as it avoids the problems introduced by averaging over inhomogeneous regions. This is the difference between the approach proposed in this thesis and those reported elsewhere, where focus was on using the joint probability of rain to investigate the relationship between two locations [137]-[140]. These estimate the $P_0$ value for the un-measured location from measured ones. The work in this thesis however, studied and provides the $P_0$ values for any location within the studied area with five years data.

![Figure 4.23](image_url): The variation of probability of rain occurrence for five locations changes with increasing size of map, here $L = 5 \text{ km}$ and $T = 15 \text{ mins}$.

According to [141], there is no easy physical way to determine $P_0$. Here the author studied $P_0$ from a grid point of view (with small area of $S = 5 \text{ km} \times 5 \text{ km}$, the best estimate from NIMROD radar systems for Europe) over a five years period (2005
to 2009) with a range of spatial and temporal integration lengths at all locations. The results from the 5 study locations are provided in this section. A mathematical equation has been proposed that gives a useful fit to the curves derived from the radar measurements:

\[ P_0(x) = 100 - b \exp(c x^e) \]  

(4.19)

where \( b, c \) and \( e \) are experimental constants which can be determined from study and \( x \) denotes either spatial integration length \( L \) or temporal integration length \( T \). The equation indicates that \( P_0 \) is constrained by two factors: \( L \) and \( T \). Hence, the values of \( b, c \) and \( e \) also exhibit variability for different spatial and temporal integration lengths.

Another numerical algorithm, Eq. (4.20), for modelling the probability of rain occurrence is proposed by Kundu and Siddani [55] after investigating raingauge-derived data from Dec 1999 to Nov 2000.

\[ P_0(Y) = \alpha_0 \exp\left(-\left(\frac{Y}{Y_0}\right)^\chi\right) \]  

(4.20)

where \( \alpha_0 \) and \( \chi \) and are experimental parameters which should be defined from study and \( Y \) denotes either the spatial integration length \( L \) or the temporal integration length \( T \). \( Y_0 \) is the shortest considered integration length.

The variation of \( P_0 \) with increasing integration length, both in space and time, is shown in Fig. 4.24. The \( P_0 \) is calculated using the same algorithm described in Section 3.3.3 after integration. Noticeably, the value of \( P_0 \) for each location increases with an increase in the integration length. The points represent the data based on
radar measurements and the curves are the corresponding fitted lines. One can clearly see that the longer the integration length, the higher the probability of rain occurrence. It is evident (and logical) that \( P_0 \) values will drop close to 0 when integration length approaches 0 or increase up to 100 when the integration length is long enough. This is consistent with the result in [134].

In addition, the detailed coefficients for each location are also presented in Table 4.8. Note that the calculated parameter values in Table 4.8 have been rounded off to two or three significant figures. The results show that the proposed \( P_0 \) expression can give a good estimate of the measured parameter throughout the whole range of integration lengths, especially in the time domain, see Fig. 4.24(b). This is because there are a large number of samples (five years in total) for studying \( P_0 \) in the time domain but insufficient samples in the space domain as only one location are focused on during each processing time. Otherwise, the proposed model, Eq. (4.23), can give excellent estimates of the \( P_0 \) in space as well (see the results in [141], where \( P_0 \) is estimated from the study of all locations within an area with size of 256 km \( \times \) 256 km). However, Fig. 4.24(a) shows that the accuracy of estimates from Eq. (4.19) is usable for the whole range of spatial integration lengths considered.
Figure 4.24: The probability of rain occurrence for increasing integration length in Europe: (a) $P_0$ for different spatial integration lengths $L$ (km), and (b) $P_0$ for varying temporal integration lengths $T$ (mins).
Table 4.8: Equation 4.19 fitted coefficients for a range of spatial and temporal integration lengths.

<table>
<thead>
<tr>
<th></th>
<th>Space Domain</th>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L (km)</td>
<td>b</td>
<td>c</td>
<td>e</td>
</tr>
<tr>
<td>Glasgow</td>
<td>92.7</td>
<td>−0.0915</td>
<td>0.678</td>
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</tr>
<tr>
<td>Paris</td>
<td>95.3</td>
<td>−0.0143</td>
<td>0.770</td>
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</tr>
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<td>−0.0331</td>
<td>0.736</td>
<td></td>
</tr>
<tr>
<td>Rennes</td>
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<td>−0.313</td>
<td>0.373</td>
<td></td>
</tr>
<tr>
<td>Reims</td>
<td>96.2</td>
<td>−0.0126</td>
<td>0.781</td>
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<table>
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<th></th>
</tr>
</thead>
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<td></td>
<td>T (mins)</td>
<td>b</td>
<td>c</td>
<td>e</td>
</tr>
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<td>0.438</td>
<td></td>
</tr>
<tr>
<td>Paris</td>
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<td>−0.0113</td>
<td>0.602</td>
<td></td>
</tr>
<tr>
<td>Portsmouth</td>
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<td></td>
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<tr>
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<td>0.663</td>
<td></td>
</tr>
<tr>
<td>Reims</td>
<td>92.2</td>
<td>−0.0243</td>
<td>0.471</td>
<td></td>
</tr>
</tbody>
</table>
4.4 Summary

In summary, this chapter has presented the outcome of an extensive experimental study of rain radar data spanning the whole of Western Europe collected by the UK Meteorological Office and provided by the UK British Atmospheric Data Centre (BADC). Five complete years of radar data from 2005 to 2009 have been analysed for modelling the rain. Four key characteristics of rainfall rate have been studied in detail and empirical computations obtained from a Europe-wide rain radar network have been presented. The experimental results, which include: (i) first order statistical distribution, (ii) spatial correlation function of rain rate, (iii) temporal correlation function of rain rate, and (iv) probability of rain/no-rain; have been calculated for a range of spatial and temporal integration intervals. With a focus on their relevance to satellite and terrestrial microwave network design, the assessment of the impact of varying spatial and temporal integration lengths on these quantities has been carried out and the results presented and discussed.

It has been found that all the key characteristics of rain are strongly dependent on the spatial integration length \(L\) and temporal integration length \(T\). The results have shown that the integration length has significant impact on these studied quantities owing to the high variability that rainfall intensity exhibits in space. In addition, databases and equations that can be used to estimate the key first and second order parameters describing rainfall rate at European locations have been created. The main contribution of the next chapter is a new software prediction tool that allows a user to estimate the key statistics for modelling rainfall fields for any Western European location. The method is capable of producing estimates at spatial integration lengths from \(L = 5\ km\) to \(L = 75\ km\) and temporal integration lengths from \(T = 15\ mins\) to \(T = 1440\ mins\). It is very promising that the proposed model can give
a reasonable estimate in high agreement with the computed data at any integration length for all studied rain characteristics. This is critical for future simulation studies and will be highly applicable to satellite network research. The standard Chi-squared goodness of fit test was applied to the log-Normal rain rate distributions, but it was shown to present serious limitations when the application is radio engineering. The spatial and temporal correlation functions of rain rate derived from the 1 km UK NIMROD grid data and the 5 km EU NIMROD data have been shown to be consistent.

The probability of rain occurrence has been shown to exhibit spatial variation over distances as short as 5 km. So far no physical equation has been found that can combine these three factors \((L, T, \text{ and } S)\) together to reasonably provide \(P_0\) estimates. The numerical model proposed in this chapter can accurately estimate the \(P_0\) value throughout the whole range of integration lengths ranging from \(L = 5 \text{ km}\) to \(L = 75 \text{ km}\) and between \(T = 15 \text{ mins}\) and \(T = 1440 \text{ mins}\) (one day). Thus, the \(P_0\) value at smaller integration length can therefore be deduced using this model. However, the proposed model is only valid for cases where either one of the spatial integration length or temporal integration length is constant, whilst the other one would be changing, and for a fixed map size. In short, at least two factors should be constant when utilising the proposed model to estimate \(P_0\) values, and the combination is either \(\{L,S\}\) or \(\{T,S\}\).

Finally, the results presented in this chapter indicate that the rain characteristics are not constant over large areas, as Bell assumed in his works [38][100] but varies from location to location over a few tens of kilometres, although the field is assumed to be homogeneous and isotropic. The model proposed in this thesis tends to be more
accurate as rain characteristics have been studied at multiple locations. The differences amongst those studied locations are self-evident (see Table 4.2-4.7 and/or Fig. 4.1-4.7). Therefore, the accuracy of the rain field simulation (simulator developed in [38]) can be significantly improved and the estimates will be closer to reality if such differences are taken into consideration.
Chapter Five: Parametric Database and Maps for Space-Time Statistics of European Rain Fields

The prediction of rainfall rate characteristics at small space-time scales is currently an important topic, particularly within the context of the planning and design of satellite network systems. A new comprehensive interpolation approach is presented in this chapter to deal with such an issue. There are three novelties in the proposed approach: 1) the proposed interpolation approach is not directly applied to measured rain precipitation (either radar or raingauge-derived data) but focuses on the coefficients of the fitted statistical distributions in previous chapter and/or computed rain characteristics at each location; 2) the parameter databases are provided and the contour maps of coefficients spanning Western Europe have been created. It conveniently and efficiently provides the rain parameter for any location within the studied map. More speculatively, the 3D space-time interpolation approach can extrapolate to rain parameters at space-time resolutions shorter than those in the NIMROD databases.
5.1 Introduction

The spatial and temporal variation of point one-minute rainfall rates is important for the detailed planning and performance prediction for satellite and terrestrial networks (a group of links) [65]. It is increasingly evident that new approaches are needed in order to predict rainfall rate variation at smaller space-time scales than currently available from wide area coverage measured rainfall rate databases.

The first and second order statistics presented in Chapter 4 provide the empirical data and fitted curves for rain characteristics over a wide range of integration scales. However, there are some limitations that affect the application of the traditional models (including the stochastic model [38], Markov chain model [35]), e.g. the resolution of model output is poor. Such problems seriously hinder the application of traditional rain models. An interpolation/extrapolation approach, may help to address this limitation.

The interpolation/extrapolation algorithm proposed is used to estimate the parameters at small scales based on the computed rain field characteristics from radar/raingauge measurements. Traditional interpolation techniques normally use the measured rainfall rate as inputs to derive the rainfall precipitation rate at smaller scales, such as in [20][100]. In contrast, the approach proposed in this chapter does not focus on rainfall intensity but the coefficients of the fitted distributions or the key rain characteristics analysed at different available space-time resolution combinations. A detailed multi-resolution parameters database and contour maps that cover the whole of Western Europe is provided. This significantly reduces the effort required to estimate these statistics as the rain characteristics at any Western European location can be conveniently obtained from the database. Furthermore,
this new approach can estimate the rain characteristics over a range of 3D space-time resolutions.

5.2 Contour Map of Rain Characteristics

Chapter 4 presented a statistical model, which can provide estimates of key rain characteristics (including the first order statistics of rain, the spatial and temporal correlation of rain, as well as the probability of rain/no rain) in two dimensions. Considerable computation is required to extract these summarising statistics from the NIMROD databases. However, this project has produced a multi-resolution database of parameters and contour maps that cover the whole of Western Europe. With the help of this database, the user can easily obtain the characteristics of rain (or the distribution coefficients) at any location within the studied area.

Example contour maps of the log-Normal rain rate distribution parameters \(\{\mu, \sigma\}\) are presented in Fig. 5.1. Fig. 5.1(a) is the map of \(\mu\) values cross the Western Europe and Fig. 5.1(b) is \(\sigma\) values. The background is the map of the Western European coastline, and the calculated parameter at each individual location (here the spatial integration length is 5 km and the temporal integration length is 15 mins) is superimposed on the map. It shows that the contour map can provide the parameter value at any location within the range from \(-9.7370°\) to \(19.8364°\) in longitude and between \(43.1938°\) to \(59.4306°\) in latitude. The calculated values are stored in a database from which \(\{\mu, \sigma\}\) can be easily obtained by simply inputting the longitude and latitude information for any desired location (see details in Section 6.5). This is very convenient as almost no computation time is needed. Similar results of other rain characteristics have also been produced and stored in the database.
Figure 5.1: Contour maps of rain distribution coefficients with spatial integration length of 5 km and temporal integration length 15 min: (a) a plot of $\mu$ values and (b) a plot of $\sigma$ values.
In addition, the rain characteristics at other integration length combinations between \( \{5 \text{ km}, 15 \text{ mins}\} \) and \( \{75 \text{ km}, 120 \text{ mins}\} \) have been computed and stored in the database. Given this database, the prediction of the rain characteristics at some finer space-time resolutions can be estimated by interpolation.

### 5.3 Predictions of Rain Characteristics in Space-Time

The existing NIMROD radar maps have been integrated to some integration length combinations from \( \{5 \text{ km}, 15 \text{ mins}\} \) to \( \{75 \text{ km}, 120 \text{ mins}\} \). The key characteristics of rain were then analysed to see how they vary with integration length.

Table 5.1 gives an example of the probability of rain \( P_0 \) with a range of integration length combinations, at Portsmouth (UK). It shows that the \( P_0 \) value changes with increasing spatial-temporal integration length. Similar results can be found for other studied parameters. These data allow the prediction of parameters at other space-time resolutions. The top-left hand corner of the table is the computed value with the shortest available spatial-temporal integration length (\( \{5 \text{ km}, 15 \text{ mins}\} \)) derived from EU NIMROD radar, and the right-hand bottom corner is the coarsest one (\( \{75 \text{ km}, 120 \text{ mins}\} \)) after integration. As discussed in Section 5.1, the rain rate at shorter integration lengths is more important for radio propagation studies than the longer ones. From Table 5.1, one can see that the characteristics of rain change systematically with increasing integration length. Given this finding the predictions at finer resolution can be estimated by interpolation.
Table 5.1: Probability of rain occurrence for increasing spatial-temporal integration lengths ranging from $5\ km$ to $75\ km$ and $15\ mins$ to $120\ mins$ at Portsmouth.

<table>
<thead>
<tr>
<th>$L$</th>
<th>15 mins</th>
<th>30 mins</th>
<th>45 mins</th>
<th>60 mins</th>
<th>75 mins</th>
<th>90 mins</th>
<th>105 mins</th>
<th>120 mins</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 km</td>
<td>15.0</td>
<td>22.9</td>
<td>25.9</td>
<td>28.5</td>
<td>30.5</td>
<td>32.6</td>
<td>34.3</td>
<td>36.1</td>
</tr>
<tr>
<td>10 km</td>
<td>23.4</td>
<td>28.61</td>
<td>32.3</td>
<td>35.3</td>
<td>37.7</td>
<td>40.1</td>
<td>41.9</td>
<td>43.8</td>
</tr>
<tr>
<td>15 km</td>
<td>28.6</td>
<td>33.7</td>
<td>37.5</td>
<td>40.3</td>
<td>42.9</td>
<td>44.9</td>
<td>46.7</td>
<td>48.6</td>
</tr>
<tr>
<td>20 km</td>
<td>32.6</td>
<td>37.9</td>
<td>41.6</td>
<td>44.7</td>
<td>47.1</td>
<td>49.4</td>
<td>51.4</td>
<td>53.1</td>
</tr>
<tr>
<td>25 km</td>
<td>35.2</td>
<td>40.6</td>
<td>44.1</td>
<td>47.2</td>
<td>49.5</td>
<td>51.7</td>
<td>53.7</td>
<td>55.2</td>
</tr>
<tr>
<td>35 km</td>
<td>42.5</td>
<td>48.3</td>
<td>52.2</td>
<td>55.4</td>
<td>57.9</td>
<td>60.1</td>
<td>61.2</td>
<td>63.1</td>
</tr>
<tr>
<td>40 km</td>
<td>42.9</td>
<td>49.4</td>
<td>53.2</td>
<td>56.1</td>
<td>57.6</td>
<td>59.6</td>
<td>61.4</td>
<td>62.9</td>
</tr>
<tr>
<td>45 km</td>
<td>48.8</td>
<td>55.6</td>
<td>59.5</td>
<td>62.2</td>
<td>64.7</td>
<td>66.6</td>
<td>68.2</td>
<td>69.6</td>
</tr>
<tr>
<td>50 km</td>
<td>49.9</td>
<td>56.2</td>
<td>60.2</td>
<td>62.9</td>
<td>65.6</td>
<td>67.2</td>
<td>68.5</td>
<td>71.8</td>
</tr>
<tr>
<td>55 km</td>
<td>56.5</td>
<td>62.2</td>
<td>65.8</td>
<td>68.5</td>
<td>70.7</td>
<td>72.2</td>
<td>73.5</td>
<td>75.3</td>
</tr>
<tr>
<td>65 km</td>
<td>60.9</td>
<td>65.5</td>
<td>68.5</td>
<td>70.6</td>
<td>72.3</td>
<td>74.2</td>
<td>75.1</td>
<td>76.5</td>
</tr>
<tr>
<td>75 km</td>
<td>63.9</td>
<td>69.1</td>
<td>72.4</td>
<td>74.4</td>
<td>76.3</td>
<td>77.8</td>
<td>78.7</td>
<td>79.9</td>
</tr>
</tbody>
</table>

In this study, the cubic spline interpolation algorithm has been chosen to implement this task. The cubic spline is a function that is constructed by piecing together cubic polynomial on different intervals [142]. It has the form

\[
S(x) = \begin{cases} 
  s_1(x) & \text{if } x_1 \leq x < x_2 \\
  s_2(x) & \text{if } x_2 \leq x < x_3 \\
  \vdots & \\
  s_{n-1}(x) & \text{if } x_{n-1} \leq x < x_n 
\end{cases} \tag{5.1}
\]

Where $s_i$ is a third degree polynomial defined by:
Cubic spline is often used for 1D interpolation. The data in each row and column of the database (see the example in Table 5.1) can be treated as samples in one dimension. It enables the use of cubic spline interpolation to estimate parameter values at other scales, based on the measured parameters. This is similar to the model proposed in Chapter 4 (see the example in Fig. 4.23). The first step is to extract the multi-scale parameters for a desired location from the database. Cubic spline interpolation is then used to interpolate to a different spatial or temporal integration sizes. In this study, the “bicubic” interpolation algorithm in MATLAB was used. Mathematically, the bicubic interpolation, which is an extension of 1D cubic interpolation, is used to interpolate data points on a two dimensional regular grid. It can be accomplished using cubic spline algorithm.

The software proposed in this work uses the database of parameters produced in Chapter 4. It contains the fitted rain parameters for a range of integration lengths between \{5 \text{ km}, 15 \text{ mins}\} and \{75 \text{ km}, 120 \text{ mins}\} for the whole of the studied area (Western Europe). The software extracts the rain characteristics with all available integration lengths at the location of interest. Taking the extracted data as input values, the interpolation algorithm then processes the data and gives the prediction at other space-time resolutions. Note that this is true only for the locations for which radar measurements data is available (the black area in Fig. 3.1).
Fig. 5.2 presents the example of predicted probability of rain occurrence at other spatial-temporal integration lengths, along with the measured data in Table 5.1, for Portsmouth. It is clear that the outcome of the 3D interpolation is a surface constructed from many 2D curves both in space and time domains. The dots are the measured values at a range of spatial-temporal integration lengths that are multiples of the data resolution, whilst the surface is produced by the interpolation algorithms to be consistent with these data. The multi-scale data are regularly spaced, which reduces the complexity of the interpolation algorithm. Interestingly, the results show that \( P_0 \) values increase systematically with increasing spatial-temporal integration length. In addition, by interpolation the values at resolutions smaller than \( \{5 \text{ km}, 15 \text{ mins}\} \) can also be predicted. The interpolation can be constrained by the assumption that \( P_0 \to 0 \) as either \( \lambda \to 0 \) or \( \varphi \to 0 \). This enables the predictions to be plotted smoothly to form a 3D surface. The resolution of the studied key characteristics of rain offers significant improvements over previous methods (e.g. [39]) and it is these that are important for rainfall field simulation studies in future. The validity of the interpolated parameters needs to be tested, and this is limited by the availability of data at small spatial and temporal integration volumes. One test that can be performed is to use \( \{5 \text{ km}, 15 \text{ mins}\} \) EU Nimrod data to predict the distribution and correlation functions of \( \{1 \text{ km}, 5 \text{ mins}\} \) UK NIMROD data.
Figure 5.2: An example of 3D space-time interpolation of $P_0$ at Portsmouth.

5.4 Validation

The absence of measured data at the smaller space-time scales causes great difficulties in validating the proposed method. However, the $\{1 \text{ km, } 5 \text{ mins}\}$ NIMROD radar measurements can be used to address this issue to some extent. In this thesis, the key rain characteristics at Portsmouth have been estimated at scales of $\{1 \text{ km, } 5 \text{ mins}\}$ and these were compared with interpolations from the EU NIMROD data.

Fig. 5.3-5.5 present comparisons of rainfall rate characteristics estimated by interpolation from $5 \text{ km}$ EU NIMROD to $1 \text{ km}$ data and estimated directly from $1 \text{ km}$ data.
UK NIMROD data. The predicted \( \{\mu, \sigma\} \) values are \(-3.75, 2.12\) and the computed values are \(-3.85, 2.04\). The predicted probability of rain occurrence \( (P_0) \) and measured one are 12.5\% and 12.1\%, respectively. Although the \( \{\mu, \sigma, P_0\} \) values of both are marginally different \( (2.7\%, 3.8\%, 2.7\% \text{ differences for } \mu, \sigma, P_0, \text{ respectively}) \), the associated 0.1\%, 0.01\% and 0.001\% exceeded rain rates are similar, this can be seen in Fig. 5.3. Fig. 5.4 shows that the spatial correlation using the predicted values is in agreement with the computed values. There is a small difference between the temporal correlation functions of rain rate using predicted data and measured data at short time lags up to roughly 150 mins, see Fig. 5.5. However, the result is still acceptable as the trend is similar, especially for large time lags. This shows that the approach proposed in this thesis has potential and requires considerable less computational effort than the direct estimation of these distributions from the data. However, the rain characteristics at scales finer than \( \{1 \text{ km, 5 mins}\} \) cannot be validated due to lack of radar/raingauge data.
Figure 5.3: A comparison exceedance distribution of rainfall rate estimated by interpolation from 5 km data to 1 km data and estimated directly for 1 km data.

Figure 5.4: A comparison of spatial correlation function of rainfall rate estimated by interpolation from 5 km data to 1 km data and estimated directly for 1 km data.
**Figure 5.5:** A comparison of spatial correlation function of rainfall rate estimated by interpolation from 5 km data to 1 km data and estimated directly for 1 km data.

Fig. 5.6 shows the map of 0.01% exceeded rain rates across Western Europe predicted by interpolation from the 5 km EU NIMROD to 1 km. The results are plausible for most areas. However, it shows that radars’ accuracy is affected in the Grand Massive alpine area of France. Fig. 5.7 presents the map of 0.01% exceeded rain rates across the British Isles given by the 1 km UK NIROMD. Note that the rain rate with 0.01% exceedance in both figures tends to reduce towards the edge of the radar region and this is almost certainly an artefact. It could be due to how the contour function deals with NaN (caused by data unavailable); or something to do with the data at the edge of the radar network. The contour map of 0.01% exceeded rain rate of the average year given by ITU-R P 837-6 is presented in Fig. 5.8. These three figures (Fig. 5.6 - 5.8), illustrate that the results of the statistics in Fig. 5.6 and Fig. 5.7 are very similar. This indicates that the proposed model can give a
reasonable estimation of rain parameters that can be used to produce rain rates with 0.01% exceedance. However, the rain rate statistics given by ITU-R P 837-6 seems quite larger compared with the results from EU NIMROD data interpolate from 5 km to 1 km and estimated directly for 1 km data. This suggests that the ITU. Rec tends to over-estimates rain. It is because ITU. Rec estimation is based on Numerical Weather Prediction (NWP) data with averaging areas of 200 km across and integration times of 6 hours. This is why the ITU. Rec recommends users to use their own data in order to produce better results.

Figure 5.6: Contour map of 0.01% exceeded rain rates predicted by interpolation from 5 km EU NIMROD to 1 km.
**Figure 5.7:** Contour map of 0.01% exceeded rain rates measured from 1 km UK NIMROD.

**Figure 5.8:** Contour map of 0.01% exceeded rain rates of the average year given by ITU-R P 837-6.
The differences between EU contour map, UK contour map and ITU contour map have been studied to show how accurate the proposed approach is. Fig. 5.9 presents the contour map of the difference of rain rates with 0.01% exceedance based on the EU data minus UK data. It shows that the proposed approach tends to overestimate the rain rates over land (see the example in middle area of Fig. 5.9), but under-estimates over the ocean/sea areas (see the left-bottom area of Fig. 5.9). However, the difference is acceptable as it is in the range $2 - 5\ mm/h$ for most areas. For some areas, the difference can up to $10\ mm/h$ or higher, but this is rare.

**Figure 5.9:** Contour map of 0.01% exceeded rain rates difference between the prediction from proposed approach and the measurements from 1 km UK NIMROD.
Fig. 5.10 presents the difference between the prediction from the proposed approach and ITU-R P 837-6 (EU predicted rain rates minus ITU predicted rain rates). The contour map shows that the ITU-R P 837-6 tends to over-estimate rain rate compared to the proposed approach for most areas. The difference can up to 40 mm/h for some regions. This indicates that the proposed approach gives more plausible estimates than ITU-R P 837-6, although it is restricted to Western Europe. However, it is necessary to highlight that for the Grand Massive alpine area of France, the proposed approach gives larger rain rates exceedance than ITU-R P 837-6. This indicates that it is hard to give accurate rainfall rate measurements or prediction over mountain area due to the difficulties associated with obtaining accurate rain radar readings [117].

**Figure 5.10:** Contour map of 0.01% exceeded rain rates difference between the prediction from proposed approach and ITU-R P 837-6.
Fig. 5.9 and Fig. 5.10 present the visual comparison of 0.01% exceeded rain rates difference between the prediction from proposed approach and the measurements from 1 km UK NIMROD and ITU-R P 837-6. However, the error function can give more information to the model performance validation. According to [143][144], the error function can be defined as:

$$Error = \ln \left( \frac{R_{measured}}{R_{predicted}} \right)$$  \hspace{1cm} (5.1)

Where $R_{measured}$ and $R_{predicted}$ are the measured and predicted rainfall rate with 0.01% exceedance, respectively. The error at each individual location therefore can be calculated by Eq. (5.1). Fig. 5.11 shows the error contour maps at 0.01% exceeded rain rate over the UK for both the proposed approach and ITU-Rec model. Theoretically, the smaller the error value, the more accurate the model prediction will be. Fig. 5.11(a) shows that the error of the proposed approach is between 0.02 and 0.15. It indicates that the approach proposed in this thesis can produce reasonable prediction. However, the error from ITU-R model can up to nearly 2, Fig. 5.11(b). Such high error value suggests that it is better to use the local rain radar measurements for the model development if the data is available.

The mean error $\overline{Error}$ is calculated by:

$$\overline{Error} = \frac{1}{n} \sum_{i=1}^{n} Error_i$$  \hspace{1cm} (5.2)

where $Error_i$ is the error for individual location and $n$ is the location index. The $\overline{Error}$ for ITU-R model is 0.62. It is roughly 7 times than the proposed model for which the $Error$ is as low as 0.09.
Figure 5.11: Contour map of error at 0.01% exceeded rain rate: (a) error distribution of proposed model, and (b) error distribution of ITU-R P 837-6.
5.5 Graphical User Interface

The statistical model and interpolation/extrapolation approach have been proposed in previous section to generate the key rain characteristics for rain field simulation. As to its implementation, the transformation between coordinate (latitude and longitude values) at a location of interest in a map and its corresponding coordinate (row and column numbers) in the NIMROD radar-derived grid (with existing radar resolution of \( L = 5 \text{ km}, \) and \( T = 15 \text{ mins} \)) is important. For this reason 30 locations were selected to investigate the relationship between latitude/longitude and row/column (details are presented in Appendix B and C). Linear equations for achieving such transformation are given as follows:

\[
\begin{align*}
\gamma_{(\text{latitude})} &= -0.0409x + 59.4306 \quad (5.1) \\
\gamma_{(\text{longitude})} &= 0.0658x - 19.8364 \quad (5.2)
\end{align*}
\]

Here \( x \) denotes either row number or column number of the NIMROD data grid with spatial integration length of \( 5 \text{ km} \), and \( y \) is the corresponding coordinate value (latitude or longitude). For the model implementation, the row and column number can be easily deduced from Eq. (5.1) and Eq. (5.2) when the latitude and longitude value of the location of interest is given.
Figure 5.12: GUI interface for obtaining the rain characteristic parameters and location information.

The graphical user interface (GUI), has been created and will be described in this section. The GUI allows users to interacting with proposed model and easily obtain the key rain characteristics at any chosen location(s) through graphical icons, visual indicator and message box. The basic interface is shown by Fig. 5.12 in which the map of the studied European area and three function buttons are provided in the left and right of the interface, respectively. Four message boxes presented in Fig. 5.13, including longitude, latitude, space resolution and time resolution information are presented as pop-ups when the “Search” button is clicked. The detailed results are given in a new message box as illustrated in Fig. 5.14 and the location of interested is marked on the map.
Figure 5.13: Message boxes appeared after clicking the search button.

Fig. 5.14(a) gives the detailed information of key rain characteristics and coordinate information at a chosen location. In this example, the coordinate of the chosen location is $56.3950^\circ$ in latitude and $-4.4308^\circ$ in longitude. Combined with Eq. (5.1) and Eq. (5.2), the software automatically calculates the rain characteristics at the chosen location. The user, therefore, can easily obtain the characteristics of rain over a range of integration lengths at any location within the studied European area. In addition, if invalid data has been input (for example, the latitude or longitude value is beyond the studied area), an error warning will be generated, see Fig. 5.14(b).
Figure 5.14: (a) example results of the rain characteristics at the location of interested given by GUI, and (b) error message box.

5.6 Summary

A simple but efficient interpolation/extrapolation approach has been presented. Instead of the radar-/raingauge-derived rainfall rate data, the analysed rain characteristics and fitted coefficients are used to predict rain at many space-time resolutions. The statistical model proposed in Chapter 4 can reasonably estimate rain parameters with different integration lengths. Databases with estimated parameter values, and maps for Europe, have been created to allow users to access the key rain characteristics at any location within the study area. This provides great
assistance to users as the rain characteristics can be easily obtained without long computation. In particular, an approach to interpolate the fitted coefficients and/or rain characteristics in space-time domain with arbitrary integration length has been proposed. Although parameters can be estimated at any combination of spatial and temporal integration lengths by interpolation or/and extrapolation, the results have only been tested down to 1 km spatial. The predictions have been validated through comparing with the measurements from UK NIMROD data. The results show that there is a reasonable agreement between the predicted and computed values. However, the predictions with resolution finer than \{1 km, 5 mins\} cannot be validated due to lack of radar/raingauge data. In particular, the contour map of 0.01% exceeded rain rates cross Western Europe and the British Isles have been generated and compared using the data interpolated from 5 km to 1 km and estimated directly from 1 km data. The results are also compared with ITU-R P 837-6 estimations. It shows that the proposed model can give reasonable prediction which is better than the one given by ITU-R P 837-6.

Finally, a GUI has been developed and provided that allows users to interact with the proposed model. The user can easily obtain the information of the key rain characteristics at any space-time scale by simply inputting the longitude, latitude, space resolution and time resolution of the location of interest. The detailed results are then automatically calculated and displayed in the message box using the provided software. This will be great convenience for researchers conducting rain rate studies.
Chapter Six: Conclusions and Future Work

This chapter brings together all the research work presented in this thesis. In particular, it summarised the major results, identifies the author’s key contributions and presents future work.

6.1 Introduction

A comprehensive study of rain radar data spanning the whole of Western Europe has been presented in this thesis. In this chapter, the proposed space-time statistical rain model and interpolation approach are analysed in detail to show their advantages and disadvantages. This serves to help the reader to gain a better understanding of the study carried out. This chapter also gives the reasons why the statistical model is combined with the proposed interpolation approach. The details are given in the following section.

6.2 Contributions of This Study

6.2.1 Contributions of the Proposed Space-Time Statistical Model

The proposed space-time statistical rain model in this thesis is based on studies of the area- and/or time- average rain data with different spatial and temporal integration lengths. The absence of high-resolution rain data at desired space and time scales over large areas was the main motivation for this study to investigate rain characteristics at finer space and time scales. Improving the understanding of spatial and temporal structure of rainfall fields is important to the design of reliable rain model. As a result, rain fields with spatial integration lengths between \( L = 5 \, km \) and \( L = 75 \, km \) and, temporal integration lengths ranging from \( T = 15 \, mins \) to \( T = \)
1440 mins (one day) have been studied in detail. With a focus on their relevance to satellite and terrestrial microwave radio network design, this model comprehensively highlights the impact of varying spatial and temporal integration lengths on the key rain characteristics. In particular, how changes to map sizes affect the studied quantities was first analysed in the proposed statistical model. The results show that some rain characteristics will attain stable values and remain relative constant when the map size, $S$, is larger than a certain value(such as $\{\mu, \sigma\}$), and some of them is irregular (such as $P_0$). Such novelties can provide more accurate understanding of rain. Empirical equations that can provide estimates of the key rain characteristics throughout a whole studied spatial and temporal integration lengths have been proposed. The equations are given as:

For correlation of rainfall rate:

$$\rho = \frac{a}{a+x^q} \quad \text{(6.1)}$$

For probability of rain occurrence:

$$P_0(x) = 100 - b \exp(cx^e) \quad \text{(6.2)}$$

With the help of these mathematical equations one can easily obtain accurate rain characteristic parameter estimates without any computation.

The main findings using the space-time statistical model can be summarised as follows:

1) The rain characteristics are not constant but change with varying spatial and temporal integration lengths. The results show that the spatial/temporal correlation function of rain rate and $P_0$ value increase with increasing integration
length both in space and time domains. In addition, the lognormal distribution parameter \( \mu \) increases while \( \sigma \) value steadily drops with increasing integration length. The 0.1\%, 0.01\% and 0.001\% exceeded rain rates, which is important in radio science, have been studied. Good results have been achieved and validated using measurement from rain radar data at a finer resolution.

2) A study of how rain characteristic parameter values change with map size was carried out. The results show that the statistics of rain will attain stable values and remain relative constant when the map size, \( S \), is larger than a certain value (400 km \( \times \) 400 km). The results also show that the probability of rain occurrence is very irregular and difficult to model as it does not only change with integration length, both in the space and time domains, but is also variable over a range of area sizes. Further study showed that characterising \( P_0 \) on a point-by-point basis is a good starting point as it can avoid the issue of map size (\( S \)) and also simplify the work;

3) Empirical equations that can be used to estimate the key characteristics of rain from 5 km to 75 km in space and 15 mins to 120 mins in time integration lengths have been proposed. The estimates of the proposed model are in reasonable agreement with measurements. With the help of the proposed equations, one can easily obtain accurate rain characteristic parameter values without any computation. This is the main advantage of the proposed statistical model; and

4) The key characteristics of rain at different spatial and temporal integration lengths were analysed to show their space and time variability. Detailed multi-resolution databases and software for predicting the statistical/dynamic parameters of Western Europe’s precipitation rate fields have been developed and provided.
This forms the basis for ongoing further system studies and the results can be used for the planning of satellite network systems.

6.2.2 Contributions of the Proposed Interpolation Approach

Although the accuracy of the proposed statistical model is very high, it has a number of limitations, which will affect its application, such as:

1) Only locations for which radar data is available have been studied. The proposed model gives excellent estimates but is only applicable to the locations that are scanned by the NIMROD radar. Parameter values are not available for location beyond the radar scan range;

2) Long computation time. It takes a long time to compute the characteristics of rain. The calculation of the fitted coefficients of the proposed empirical equations is time-consuming but this time can be significantly shortened by using a faster computer, e.g. super computer; and

3) Resolution issue. The application of the proposed statistical model is limited by the resolution of NIMROAD data. There is no guarantee of the estimates at finer scales due to the fact that there was no data that could be used to evaluate its performance at higher resolution than 1 km in space and 5 mins in time.

Therefore, to address the above limitations of the statistical model, an interpolation technique was proposed and implemented. The main novelties using the proposed interpolation approach are summarised as follows:

1) the interpolation technique is not directly applied to the measured rain precipitation data (either radar-derived or raingauge-derived data) but to the fitted coefficients of the statistical model and/or computed rain characteristics. This is
novel and different from other published works that mainly focus on the measured rainfall rate;

2) the proposed interpolation approach is combined with the statistical model in such a way that the advantages of both the technique and the model can be fully utilised and the limitations of applying each, individually, mutually compensated. The statistical model cannot provide 3D space-time estimates but the interpolation approach can solve this problem. However, the interpolation approach needs to use the output of the statistical model as inputs. Because of this, it is better to combine the proposed model and the interpolation technique;

3) the databases of rain characteristic parameters have been developed and the contour maps of the values of coefficients of the proposed equations and the characteristics of rain over Western Europe have been created. These will significantly improve rain studies as the rain characteristic parameter values at any location within the studied map can be efficiently obtained;

4) the 3D space-time interpolation approach that can predict rain characteristics has been presented. It can interpolate rain parameters at space-time resolutions shorter than those in the NIMROD databases although exhaustive validation could not be carried out for lack of data at finer resolutions; and

5) a GUI has been provided that allows users to interact with the proposed model and obtain the rain characteristic parameter values easily.

6.3 Conclusion

In conclusion, a new space-time statistical rain model and a new interpolation approach have been proposed. Key rain characteristics were studied from a statistical perspective to better understand the mechanics of rainfall processes and to determine how models can be developed and used to facilitate rain prediction for
radio network planning. A detailed database of the key rain characteristics has been proposed and developed for further study and application in European satellite networks.

According to [51] one can note that no individual model can fully predict all rain characteristics as each method has its own strengths and weaknesses. The proposed statistical model can accurately estimate rain characteristics but has integration length and data availability limitations. This problem is efficiently resolved using an interpolation approach. Meanwhile, it has been noted that the input to the proposed interpolation approach is the estimates of the proposed statistical model. This implies that the proposed interpolation approach cannot be used independent of the statistical model.

Results show that the combination of the two proposed models presents a more powerful tool than when implemented individually. This is because they mutually compensate their limitations.

The objectives at the start of this research project were to

- Study the first order statistics of rainfall field in Western Europe using radar measurement;
- Develop a space-time statistical rain model;
- Study and develop an interpolation technique that can be used to predict rain at finer space and time resolution.

The summary presented in this chapter and results in previous chapters show that all the objectives set out at the start of this research project have been achieved.
6.4 Future Work

It is true that no scientific research can be exhaustively investigated due to many uncontrollable constrains such as time, data availability, etc. In this section, future work is briefly proposed based on the limitation of the models proposed in this thesis.

6.4.1 Evaluation of the Model in Other Climates

From the model evaluation, one can note that the accuracy of the proposed space-time statistical model can be improved if there is no time limitation or fast computers can be used to reduce the computation time. The key characteristics of rain at more locations (worldwide) are required to ascertain the variability of these quantities over different climatic regimes. It is logical to expect that if more characteristics are studies, a greater understanding would be gained and perhaps more parameters would be available as inputs for the interpolation method.

6.4.2 Further Modification of the Proposed Statistical Model

The statistical model proposed in this study can give accurate estimates but is not suitable for situations where the spatial and temporal integration lengths are changing simultaneously. A majority of existing models have the same problem and this is a big gap in rain modelling. A future study can investigate how the key characteristics of rain changes in space and time at the same time. A good example has been given by Bell in [131].

6.4.3 Further Study of Interpolation Technique

In this study, the author chose three existing interpolation techniques. The results are acceptable but the accuracy could be improved. According to [145], various interpolation techniques are available and more detailed study may result in may be more suitable algorithm been found or developed. Perhaps, a 3D space-time
interpolation model can be developed that is independent of any other models without sacrificing accuracy.
References:


Appendices:

Appendix A: Approach for Fast Data Processing

The computation of the NIMROD radar data is time-consuming. The fast processing approach is considered in order to improve the efficiency. The general procedure can be summarised as follows:

1). Download and decompress the data

The function of download and decompress can be written in the software program but the processing takes a long time as it depends on many factors, e.g. internet speed, performance of the computer, etc. In order to speed up the computation, the data downloading and decompression can be done manually. In this way the efficiency can be significantly improved.

2). Data extraction in two dimensions

The NIMROD radar-derived rain maps cover a large area with size of $3100 \, km \times 3500 \, km$. However, the radar data is only available for the middle part of the map. Only this part of area is extracted in order to reduce the unnecessary processing. Fig. A(a) is the original radar map that covers a large area. One can see that there is a large part of the map that has no radar data. The outline, which covers the data available area in Fig. A(a) is extracted and forms the Fig. A(b).

The main advantage of this approach is that the extracted database has been converted and documented in a software readable format. In this way the computation can be easier and more focus.
3). Data extraction in one dimension

The software readable 2-dimensional database is then extracted in one dimension for measurements covering five complete years period (2005 to 2009). The advantage is that it allows the computation to be more focused on a location of interest. This is especially useful to the study of the temporal correlation function and probability of rain occurrence. In particular, the $P_0$ at each individual location within the studied area can be accurately achieved in a short time. The efficiency has been significantly improved.

**Figure A**: The 2-dimensional extraction of radar map.
Appendix B: Calibration of NIMROD data

The calibration of NIMROD data is significant for this study. By choosing some samples (normally the more samples that are chosen the more accurate the result will be, here the author use 30 samples), two algebraic equations are used, one is for latitude and the other one is for the longitude. These two numerical equations could allocate the roughly latitude and longitude values for different locations of Western Europe.

For the development of the relative algebraic equations, the general procedures are summarised as following steps:

1. Choosing some radar images from NIMROD data set

The NIMROD radar-derived rain maps are helpful and critical for the calibration therefore some maps should be selected at the initial stage. The maps need to meet the following requirements.

i) There is not too much rain in the selected map, the less the better. Under this circumstance, it could be easier to find some small rainy areas or even single rain point (ideal situation) from the map. In this way the error can be greatly reduced.

ii) The separation of different rainy areas in the same map should be large enough; otherwise, it is easy to make a mistake when trying to find out the corresponding coordinate (row and column) in the grid. Here, the grid is constituted by rain rate, with space resolution of 5 km and time resolution of 15 min.
2. Decompress NIMROD data

The NIMROD radar system continues scans the European area and produce rainfall rate measurements and maps at every 15 mins. The generated data is compressed and documented in a special form.

3. Allocating the selected samples

This piece of work used a map of the Europe (not the NIMROD radar map) that has accurate latitude and longitude information. As to the scale, ideally, is the finer the better. Based on this an accurate result can be achieved. In this study, the finest precision of the European map used to provide the latitude and longitude information is 20 mins. Through comparing the radar images and the used map, the locations of the selected samples can be physically allocated on the map. In addition, both the latitude and longitude values of the location of interest can be read and recorded as it is visible on the map.

The achieved latitude and longitude values of all selected locations can be transformed into degrees by using the following mathematical equation:

\[ \text{Final value} = X + \frac{Y}{60} + \frac{Z}{3600} \]

4. Fitting the line

It is difficult to get the real latitude and longitude value for the location of interest since error is unavoidable. However, by making use of the achieved data from the selected samples, a reasonable line to offset and reduce the error can be proposed.

The final equations are given as follows:
Here $x$ denotes either row or column number of the NIMROD data grid with spatial integration length of 5 km, and $y$ is the corresponding coordinate value in either latitude or longitude.

The fitted lines are shown in following figures:

**Figure B1**: Tendency of latitude changing with distance.
Figure B2: Tendency of longitude changing with distance.

Viewed from the software generated figures (see Fig. B1 and Fig. B2), it is clear that the fitted lines are straight. Fig. B1 shows that the slope for the latitude is negative. The reason is that the origin of the data matrix for rain field image is starts from the top left to bottom right. It means that the smaller the row number (the value of $x$), the higher the latitude value. In other words, the latitude value decreases with the increasing row number. Fig. B2 shows that the slope for the longitude is positive. Noticeably, the larger the column number (the value of $x$), the higher the longitude value. Here, it is important to highlight that the longitude values can be either positive or negative. The reason is that the Prime Meridian goes across the studied map, roughly at column 233. If the column number is smaller than 233, it means the
chosen location is on the left of the Prime Meridian, the longitude value will be negative. On the contrary, if the column number is larger than 233, it means the chosen location is on the right of the Prime Meridian, the longitude value will be positive. Therefore, the longitude values are comprised of both positive and negative values.
Appendix C: Computation for the Locations at the Edge of Radar Scan Range

As aforementioned, radar scan does not cover all area. This, therefore, causes the problem that rain characteristics at the area where data is unavailable cannot be directly obtained from measurements. In addition, due to some of the rain characteristics are based on the computation of data over a large size of area, (e.g. statistics of rain), this introduces limitation to the study of rain at some locations, especially those at the edge of the radar scan range where the radar data is only partial available.

Figure C: An example of a location that located at the edge of radar scan range. The triangle is the location of interested and the outline (big square box) is the covered area for the study of rain characteristics at the location of interest.
Fig. C presents an example of a location that is located at the edge of radar scan range. The triangle is the location of interest and the outline (big square box) is the area in which the radar data is used to study the rain characteristics. For this case, one can clearly see that only part of the outline can be scanned by the NIMROD meteorological radars. Under this circumstance, only the available data within the outline will be taken into consideration for the rain study. The computation, thus, will be affected to some extent.
Appendix D:  List of Publications


Appendix E: Other Publications


CHARACTERIZATION OF RAIN FIELDS FOR UK SATELLITE NETWORKS

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ABSTRACT
High frequency satellite links are significantly affected by rain-induced scintillation in the satellite footprint [1]. The detailed planning and performance prediction of satellite systems networks (a group of links) requires a detailed understanding of the space-time characteristics of rain fields. [1][2][3][4]. This paper presents results of key empirical properties of rainfall rate relevant to developing a UK space-time stochastic model. In particular, we assess the impact of integration time and spatial scales on the space-time correlation of rain fields, the probability of rain occurrence and the point statistics of rainfall rate.

1. Introduction
Rainfall involves the interplay of many complex physical processes in the atmosphere [5]. Our main interest is in any communication system operating at frequencies above 10 GHz that are significantly affected by rain due to large radio propagation losses resulting in network-level or link unavailability situations [6]. In order to cope with rain induced effect using so-called fade mitigation techniques, it is necessary to gain an accurate knowledge of the dynamic statistical characteristics and variability of rainfall rate which is the primary natural cause of microwave frequency attenuation. Many interesting studies on the modelling of rainfall rate or equivalent have already been carried out. For example, in 1981, Maseng and Balken proposed the stochastic-dynamic time-series model for rain attenuation, [1]. Menabde in [8] used a discrete random cascade to generate a field with the desired statistical structure. In [2] Bell proposed an alternative approach achieving the desired spatial and time correlation structure and a lognormally distributed rain field. In this paper, we present experimental results obtained from UK radar data that are compatible with [2].

The main novelty is that we provide detailed empirical results showing how key properties of rain actually scale with varying spatial and temporal resolutions. For this we studied rain fields with space resolutions range between 1 and 250 km and time resolutions ranging from 5 minutes to 640 minutes.

The experimental input data used here are 5 mins radar maps collected by the UK NIMROD radar network. The NIMROD dataset consists of a series of radar-derived rain-rate maps on a 1-km grid. The complete maps have size 2175x1725 km covering the whole UK. In this paper, we only present processing from a 250x250 km area centred at 50.9 degree latitude and -1.7214 degree longitude in the vicinity of Portsmouth, UK. The rest of this paper is organized as follows: in Section 2, we present the background information on the space-time stochastic modelling of rain fields. The experimental results of the properties of rain, including the probability of rain, spatial-temporal correlation of rain as well as the statistics of rain are presented in Section 3. Section 4 gives the conclusion and further work. Further experimental results are given in the appendix.

2. Characterisation of Rain Attenuation Fields
The average rainfall over an area A during a time period T can be expressed as:
\[ R = \frac{1}{A} \int_0^T d\tau \int_A r(x, \tau) dx \]

where \( r(x, \tau) \) denotes the point rainfall rate in mm/h at location \( x \) and time \( \tau \) on a 2D Cartesian grid. The interval \( T \) is the integration time or time resolution while the area \( A = L \times L \) with \( L \) being the spatial resolution. In practice, each pixel of a NIMROD radar map consists of a Cartesian grid of such space-time averaged values. Each new rain field map is produced every \( T \) seconds. In this paper, we analyze rainfall rate radar data collected in 2006. Although, it is well known (e.g., see [6]) that...
longer durations ought to be looked at in order to capture seasonal and yearly cycles, the results presented is still significant especially for spatial characteristics.

The change of time scale, $T$ or spatial scale $L$ is particularly relevant to network system studies. In particular, one might be particularly interested in shorter time scales commensurate with typical network durations (e.g. 1 second). Thus time deconvolution (i.e. time interpolation) is very important since typical radar maps are produced with long integration times (the NIMROD system has a 5 mins sampling period). Spatial scaling is less of an issue since for example the NIMROD data achieve a 1 km resolution already. A priori, this seems more adequate for prediction of rain affected satellite networks with fade mitigation techniques. Following the lead in [2] the following parameters are key to the development a reasonable space-time model of rainfall rate.

The probability $P_T$ of rain occurrence in geographical area $A$, representing the spot beam or footprint of the satellite network is the first main parameter. There is no real physical way of determining such a probability which thus will be measured at different spatial and time scales. We will see in the next section that a good curve fit is:

$$P_T = a - b \exp(bx^c + d)$$  
(2)

where $a$, $b$, $c$ and $d$ are experimental constants and $x$ denotes either $L$ or $T$ if we consider spatial or time scaling of $P_T$, respectively. The second important characteristic is the spatial correlation function (equal to the inverse Fourier transform of the spectrum):

$$\rho = \frac{\text{cov}(R_1, R_2)}{\sigma_1 \sigma_2}$$  
(3)

where $R_1$ and $R_2$ are the point rainfall rates (millimetre/hour) for two locations 1 and 2, respectively, of interest and $\rho$ is the cross-correlation factor between $R_1$ and $R_2$. $\text{cov}(\cdot, \cdot)$ and $\sigma$ are covariance and variance respectively. In our case, we have measured the spatial correlation function of rainfall rate assuming that the field is space homogeneous and isotropic. In such a case, the correlation function of rainfall rate only depends on the separation distance between the selected points. However we want to measure how this changes with different spatial scales.

Rain events are also highly variable in time, so the third characteristic must be the time correlation:

$$\rho = \frac{\text{cov}(R_{t1}, R_{t2})}{\sigma_t \sigma_t}$$  
(4)

where $R_{t1}$ and $R_{t2}$ are the point rainfall rates (millimetre/hour) at two different times 1 and 2, respectively. The correlation level is changing with different scales. Typically, it goes up with the increase of scale because the variance will be compensated and become smaller at larger scales; and then the value of correlation will go up.

A broad agreement is that point rainfall rate (when it is raining, $R > 0$) is well modeled as a lognormal random variable, [1], [2] and [6] with probability density function (PDF):

$$f(R) = \frac{1}{\sqrt{2\pi}\sigma R} \exp\left(-\frac{1}{2} \left(\frac{\ln R - \mu}{\sigma}\right)^2\right)$$  
(5)

We will test the applicability of this statistical model for different space and time scales using the method described in [9]. The details are given in section 3.3.

3. Experimental Results

3.1. Scaling Probability of Rain

The probability of rain occurrence $P_T$ (for which $R > 0$) represents equally well the probability of rain at one location over a long period of time, or, the expected fraction of the rainy area $h_T = A_{rain}/A$ that one can expect in a satellite network. In this paper, we processed one full month of radar data from January 2008. Experimental results showing the impact of spatial and time resolutions are given in Table 1 and Table 2 bearing in mind that the original scales of the experimental data was 1 km and 5 mins.
<table>
<thead>
<tr>
<th>L (km)</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>16</th>
<th>32</th>
<th>64</th>
<th>128</th>
<th>256</th>
</tr>
</thead>
<tbody>
<tr>
<td>P0 (%)</td>
<td>13.15</td>
<td>15.95</td>
<td>20.95</td>
<td>29.51</td>
<td>43.30</td>
<td>60.34</td>
<td>77.06</td>
<td>92.07</td>
<td>96.95</td>
</tr>
</tbody>
</table>

Table 1: Probability of rain for different spatial resolutions L (km) for UK in 2008 (T=5mins)

<table>
<thead>
<tr>
<th>T (mins)</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>40</th>
<th>80</th>
<th>160</th>
<th>320</th>
<th>640</th>
</tr>
</thead>
<tbody>
<tr>
<td>P0 (%)</td>
<td>13.15</td>
<td>15.95</td>
<td>17.46</td>
<td>22.80</td>
<td>29.78</td>
<td>39.64</td>
<td>49.90</td>
<td>52.43</td>
</tr>
</tbody>
</table>

Table 2: Probability of rain for different temporal resolutions T (mins) for UK in 2008 (L=1km)

Fig.1: Scaling of probability of rain occurrence for increasing resolution in UK (Jan 2008); (a) is for different space scales L (km) and (b) is for varying time scales T (mins).

Fig.1 depicts how the probability of rain changes with resolution both in space and time domain. Clearly the probability of rain occurrence increases with increasing scales. The dots represent the data based on NIMRUD measurement and the curves are the fitted lines. It is evident (and logical) that the larger the scale the higher the probability of rain. The mathematical equations for the fitted curves are given as follows.

For space domain (result (a)), with L in km:

$$ P_0 = 100 - 13.15 \exp(-0.042L^{0.85} + 1.9298) \% $$ (6)

For time domain (result (b)), with T in mins:

$$ P_0 = 100 - 13.15 \exp(-0.0250T^{0.85} + 1.9498) \% $$ (7)

The fitted equations give a reasonable estimate of the measured data (see Fig.1).

3.2. Scaling of Correlation of Rainfall Rate

The space-time correlation function is an important element for developing the rain attenuation model. The spatial and temporal correlation of rainfall rate is in particular critical for statistical simulation models of rain probability both in space and time domains. In this paper, one month of rainfall rate radar data (Jan 2008) were processed to study the correlation of rain in the UK at different scales.

Fig 2 (a) is the comparison of spatial correlations of 1-km gridded rain rate. The dots and curve are the measured results and the solid line is the fitted line. For our work, the general empirical equation for the spatial correlation function for the UK is given by:

$$ \rho(r) = \left( \frac{a}{\exp(br^2 + c)} \right)^n $$ (8)

where r stands for the distance in km, a, b, c and n are parameters to be determined from the data. We note that the correlation function falls off very quickly with distance and that the fitted curve is very accurate throughout the whole range of distances.
Fig. 2: The spatial correlation functions of rainfall rate: (a) spatial correlations with 1 km resolution (the curve is the fitted line) and (b) spatial correlations for different space scales.

Fig. 2(b) shows the correlation functions for different spatial resolutions ranging from $L=1$ km to $32$ km. This clearly indicates that the spatial correlation function is dependent on the actual spatial resolution. The fitted values of the parameters in Eq. (3) for each resolution are listed in Table 3. (see also the fitted curves for different resolutions in Appendix A). The results for the time correlation of rainfall rate are depicted in Fig. 3 and Fig. 4, respectively. Fig. 3(a) shows that the correlation of rainfall rate is highly peaked. It drops down quickly over short time lags (labelled $l$), then the correlation drops more gently from value 0.1 down to zero over time lags up to about 10000 mins ($\approx 7$ days). Fig. 3(b) shows the zoomed in short-lag correlation function. The dots and curve are the measured data and fitted line, respectively. The fitted curve based on Eq. (3) agrees quite well with the measured results at small time lags up to 300 minutes. This fact is also true at others time scales (see the fitted curves in Appendix B).

<table>
<thead>
<tr>
<th>L (km)</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>16</th>
<th>32</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0.4</td>
<td>0.8</td>
<td>1.4</td>
<td>1.6</td>
<td>1.9</td>
<td>2.0</td>
</tr>
<tr>
<td>b</td>
<td>1.0</td>
<td>1.1</td>
<td>0.9</td>
<td>0.8</td>
<td>0.7</td>
<td>0.3</td>
</tr>
<tr>
<td>c</td>
<td>0.5</td>
<td>0.5</td>
<td>0.6</td>
<td>0.7</td>
<td>0.8</td>
<td>1.2</td>
</tr>
<tr>
<td>n</td>
<td>4.5</td>
<td>4.3</td>
<td>4.0</td>
<td>3.1</td>
<td>3.0</td>
<td>2.0</td>
</tr>
</tbody>
</table>

Table 3: Experimental values of correlation function parameters for each spatial resolution.

Fig. 3: (a) is the temporal correlation of rain with $T=5$ mins resolution and, (b) is the zoomed-in correlation function for short-time lags.

Fig. 4: Time correlation functions of rainfall rate for different resolutions. (a) different spatial scales and, (b) is for the different temporal scales.
Fig. 4(a) exhibits the interesting fact that the temporal correlation is not greatly affected by the spatial resolution. This might indicate that rainfall rate fields might have a separable space-time correlation function. In Fig. 4(b), we however can find that the time correlation function of rainfall rate changes significantly with time resolution (i.e., integration time) between 5 minutes and 150 minutes. We note that for time lags greater than about 300 mins, irrespective of the time resolution, the time correlation falls off very slowly and is below 0.2. Table 4 lists the values of parameters for different time scales ranging from 5 minutes to 30 minutes.

<table>
<thead>
<tr>
<th>T (mins)</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
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</thead>
<tbody>
<tr>
<td>a</td>
<td>14.5</td>
<td>14.5</td>
<td>14.5</td>
<td>14</td>
<td>14.5</td>
<td>14.5</td>
</tr>
<tr>
<td>b</td>
<td>1.1</td>
<td>1.1</td>
<td>1.2</td>
<td>1.3</td>
<td>1.5</td>
<td>1.8</td>
</tr>
<tr>
<td>c</td>
<td>0.074</td>
<td>0.072</td>
<td>0.07</td>
<td>0.07</td>
<td>0.063</td>
<td>0.063</td>
</tr>
<tr>
<td>n</td>
<td>1.8</td>
<td>1.8</td>
<td>1.8</td>
<td>1.8</td>
<td>1.7</td>
<td>1.7</td>
</tr>
</tbody>
</table>

Table 4: Experimental values of correlation parameters for each temporal resolution.

3.3. Scaling of the Statistics of Rain

The statistics of rainfall rate for 20 locations have been chosen in order to obtain average statistics representative of typical rain conditions for Southern UK regions. Through summing up the histograms for each location in the selected area, an ordered and cumulative distribution of the measured occurrences of different intensities can be generated.

Fig. 6: (a) is the histograms of rainfall rate and, (b) is the test for log-normality of the rainfall rate distribution of UK in January 2008. The time resolution is 5 mins.

Fig. 6(a) gives the histogram of rainfall rate conditioned on actual occurrence of rain for rainfall rate ranging from 1 mm/h to 150 mm/h. Over the period of observation (one month in Jan 2006), it is clear that the rainfall rate is less than 20 mm/h and the probability of heavy rain events is extremely low. The complementary CDF (CCDF) of rainfall rate has then been determined. Using the technique described in [9], the experimental CCDF is then tested for its log-normality. This is shown in Fig. 6(b). If the data is log-normal, then the transformed CCDF shows up as a straight line. This is clearly the case (see additional results in the appendix).

The general formula for the fitted line is given by $y = \frac{x}{\sigma} + \mu / \sigma$, where $\mu$ and $\sigma$ are the mean and variance of the log-normal PDF given in Eq (3). The values of $\mu$ and $\sigma$ for different time scales (i.e., integration times) between 5 minutes and 640 minutes are listed in Table 4. Our results show that the distribution of rainfall rate remains log-normal (see the fitted line in Appendix C).

<table>
<thead>
<tr>
<th>T (mins)</th>
<th>$\mu$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>-4.6210</td>
<td>1.8914</td>
</tr>
<tr>
<td>10</td>
<td>-4.5379</td>
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<td>20</td>
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<td>1.3974</td>
</tr>
<tr>
<td>160</td>
<td>-3.6789</td>
<td>1.3208</td>
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<td>320</td>
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<td>1.1758</td>
</tr>
<tr>
<td>640</td>
<td>-7.5369</td>
<td>2.9343</td>
</tr>
</tbody>
</table>

Table 5: Experimental values of parameters of fitted line for each time resolutions
4. Conclusion and further work
In conclusion, empirical results characterizing the space-time nature of rainfall rate have been presented. We focussed on measuring how the probability of rain occurrence, the spatial and the time correlation functions and the first order log-normal statistics of rainfall rate change with varying spatial and temporal resolutions/scales. We provide empirical equations that can be used in later statistical or simulation studies. This will be applied to satellite network studies. Further work is required to ascertain the variability of these characteristics over large areas spanning different climatic regimes.

5. Acknowledgement
The authors would like to thank the British Atmospheric Data Centre (BADC), which is part of the NERC National Centre for Atmospheric Science (NCAS), and the British MetOffice for providing access to the NIMROD rain radar data set. Partial support from ICT COST action IC0802, "Propagation tools and data for integrated telecommunication, Navigation and earth observation systems" is gratefully acknowledged.

Appendix
A: Fitted Correlation Function for Different Spatial Resolutions (1km to 32km)

B: Fitted Correlation Function for Different Temporal Resolutions (5mins to 30mins)
C: Rainfall Rate Distribution Test for Different Time Scales from 5mins to 640mins

References: