Velocities as a probe of dark sector interactions

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Dark energy in General Relativity is typically non-interacting with other matter. However, it is possible that the dark energy interacts with the dark matter, and in this case, the dark matter can violate the universality of free fall (the weak equivalence principle). We show that some forms of the dark sector interaction do not violate weak equivalence. For those interactions that do violate weak equivalence, there are no available laboratory experiments to probe this violation for dark matter. But cosmology provides a test for violations of the equivalence principle between dark matter and baryons – via a test for consistency of the observed galaxy velocities with the Euler equation.

I. INTRODUCTION

Dark matter is currently only detected via its gravitational effects, and there is an unavoidable degeneracy between dark matter and dark energy within General Relativity. There could be a hidden non-gravitational coupling between dark matter and dark energy, and thus it is interesting to develop ways of testing for such an interaction (see [1, 2, 3] for earlier attempts).

One signal of a dark sector interaction could be a violation of the weak equivalence principle (universality of free fall) by dark matter, under the non-gravitational drag due to coupled dark energy. Since Galileo shattered the myth that heavier objects fall faster, the universality of free fall has been established as a fundamental principle of gravity. Laboratory tests have been made to show the independence of the acceleration of objects from their masses and chemical composition. However, these tests apply to baryonic matter, and no direct probe of dark matter is available.

If the interacting dark sector couples non-gravitationally to baryonic matter, then existing laboratory tests provide constraints on the dark sector interaction [4]. Here we assume that there is zero (or negligible) non-gravitational coupling between the dark sector and standard-model fields. A difference in the acceleration between dark matter and baryons could show up in the stellar distribution in tidal trails of satellite galaxies [5]. This same difference should also show up as an inconsistency when interpreting the relation between galaxy peculiar velocities and overdensities, as we explain below.

We assume that gravity on all scales is described by General Relativity. Thus there is no gravitational mechanism to violate the weak equivalence principle. Note that this is also true of scalar-tensor theories, since the gravitational scalar degree of freedom couples equally to all types of matter. Indeed, most metric theories of modified gravity also respect the weak equivalence principle (see, e.g., [6]). Various tests have been developed to discriminate between metric theories of modified gravity, and non-interacting dark energy models in General Relativity (see, e.g., [7]). But these tests do not in general apply to dark energy that interacts with dark matter, since a dark sector interaction can introduce new degeneracies [8]. We confine ourselves to the question of how galaxy peculiar velocities can be used to detect dark sector interactions within General Relativity.

II. INTERACTING DARK ENERGY

We briefly review the necessary background on perturbations of interacting dark energy models in General Relativity. (For recent work with further references, see, e.g., [11].)

A general dark sector coupling may be described in the background by the energy balance equations of cold dark matter (c) and dark energy (x),

$$\rho_c' = -3\mathcal{H}\rho_c + aQ_c, \tag{1}$$
$$\rho_x' = -3\mathcal{H}(1 + w_x)\rho_x + aQ_x, \quad Q_x = -Q_c, \tag{2}$$

where $w_x = P_x/\rho_x$, $\mathcal{H} = d\ln a/d\tau$ and τ is conformal time, with $ds^2 = a^2(-d\tau^2 + d\vec{x}^2)$. Here Q_c, Q_x are the rates of energy density transfer to dark matter and energy respectively. In order to avoid stringent "fifth-force" constraints, we assume that baryons (b), photons (γ) and neutrinos (ν) are not coupled to dark energy and are separately conserved.

In the Newtonian gauge the perturbed metric is given by

$$ds^2 = a^2\left[-(1 + 2\Psi)d\tau^2 + (1 - 2\Psi)d\vec{x}^2 \right], \tag{3}$$

where we have neglected anisotropic stress since we are interested in the late universe. The total (energy-frame) four-velocity is

$$u^\mu = a^{-1}\left(1 - \Psi, \partial^i v\right), \tag{4}$$

where the velocity potential v is defined by

$$(\rho + P)v = \sum(\rho_A + P_A)v_A, \tag{5}$$

and $A = c, x, b, \gamma, \nu$. The A-fluid four-velocity is

$$u_A^\mu = a^{-1}\left(1 - \Psi, \partial^i v_A\right). \tag{6}$$

The covariant form of energy-momentum transfer is

$$\nabla_\nu T_A^{\mu\nu} = Q_A^\mu \,, \tag{7}$$

where $Q_A^\mu = 0$ for $A = b, \gamma, \nu$ in the late universe, while $Q_c^\mu = -Q_x^\mu \neq 0$. The energy-momentum transfer four-vector can be split relative to the total four-velocity as

$$Q_A^\mu = Q_A u^\mu + F_A^\mu \,, \quad Q_A = \bar{Q}_A + \delta Q_A \,, \quad u_\mu F_A^\mu = 0 \,, \tag{8}$$

where Q_A is the energy density transfer rate and F_A^μ is the momentum density transfer rate, relative to u^μ. Then it follows that $F_A^\mu = a^{-1}(0, \partial^i f_A)$, where f_A is a momentum transfer potential, and

$$Q_0^A = -a\left[Q_A(1+\Psi) + \delta Q_A\right], \tag{9}$$
$$Q_i^A = a\partial_i\left(f_A + Q_A v\right). \tag{10}$$

In the background, the energy-momentum transfer four-vectors have the form $Q_c^\mu = a^{-1}(Q_c, \vec{0}) = -Q_x^\mu$, so that there is no momentum transfer.

The evolution equations for the dimensionless density perturbation $\delta_A = \delta\rho_A/\rho_A$ and for the velocity perturbation are:

$$\delta_A' + 3\mathcal{H}(c_{sA}^2 - w_A)\delta_A - (1+w_A)k^2 v_A$$
$$- 3\mathcal{H}\left[3\mathcal{H}(1+w_A)(c_{sA}^2 - w_A) + w_A'\right]v_A$$
$$- 3(1+w_A)\Psi' = \frac{a}{\rho_A}\delta Q_A$$
$$+ \frac{aQ_A}{\rho_A}\left[\Psi - \delta_A - 3\mathcal{H}(c_{sA}^2 - w_A)v_A\right], \tag{11}$$

$$v_A' + \mathcal{H}\left(1 - 3c_{sA}^2\right)v_A + \frac{c_{sA}^2}{(1+w_A)}\delta_A + \Psi$$
$$= \frac{a}{(1+w_A)\rho_A}\left\{Q_A\left[v - (1+c_{sA}^2)v_A\right] + f_A\right\}, \tag{12}$$

where $w_c = 0 = c_{sc}^2$ and $c_{sx}^2 = 1$.

For our purposes, we are interested in the behaviour of dark matter in the Newtonian regime on sub-Hubble scales. In this case, the perturbed continuity and Euler equations reduce to

$$\delta_c' - k^2 v_c = \frac{a}{\rho_c}\left(\delta Q_c - Q_c\delta_c\right), \tag{13}$$
$$v_c' + \mathcal{H}v_c + \Psi = \frac{a}{\rho_c}\left[Q_c(v - v_c) + f_c\right], \tag{14}$$

If the right-hand side of the continuity equation (13) is nonzero, then the interaction will lead to a bias in the linear regime between dark matter and baryons [12], since the baryon overdensities obey

$$\delta_b' - k^2 v_b = 0 \,. \tag{15}$$

If the right-hand side of the Euler equation (14) is nonzero, then the dark matter no longer follows geodesics and breaks the weak equivalence principle, unlike baryons, for which

$$v_b' + \mathcal{H}v_b + \Psi = 0 \,. \tag{16}$$

In the Newtonian regime, the Poisson equation becomes

$$k^2\Psi = -4\pi Ga^2\left(\rho_c\delta_c + \rho_b\delta_b\right). \tag{17}$$

Here we neglect dark energy clustering, assuming that the sound velocity of dark energy perturbations is $c_{sx} = 1$. Dark energy perturbations can be important on large scales depending on the strength of interactions but they are not important on sub-horizon scales as long as the sound velocity of dark energy perturbations is positive – since in that case, the gradient term in the evolution equation for δ_x [see Eq. (12)] always dominates over the interaction terms. The evolution equation for δ_c is then given by

$$\delta_c'' + \mathcal{H}\delta_c' - 4\pi Ga^2\rho_c\delta_c - \mathcal{H}\frac{a}{\rho_c}(\delta Q_c - Q_c\delta_c)$$
$$- \left[\frac{a}{\rho_c}(\delta Q_c - Q_c\delta_c)\right]' - \frac{a}{\rho_c}k^2\left[Q_c(v - v_c) + f_c\right] = 0. \tag{18}$$

III. DIFFERENT TYPES OF INTERACTION

There is no fundamental theory that determines the form of the interaction, i.e., of Q_c^μ, so we are forced to use phenomenological models. Here we consider three types of interaction, each illustrated with a particular form: interactions that do not change the continuity or Euler equations; interactions that change only the Euler equation; interactions that change only the continuity equation. The general case, where both equations are modified, can be thought of as a linear superposition of the last two cases.

A. Continuity and Euler equations unchanged

A general class of interactions may be defined by requiring that there is no momentum exchange in dark matter rest frame,

$$Q_c^\mu = Q_c u_c^\mu \,, \tag{19}$$

where Q_c remains to be specified. For this class, we find from Eqs. (9) and (10) that, for any Q_c, we have $f_c = Q_c(v_c - v)$. Thus Eq. (14) becomes

$$v_c' + \mathcal{H}v_c + \Psi = 0 \,. \tag{20}$$

This is the same Euler equation as the non-interacting case, so that the dark matter velocity is not directly affected by the interaction and there is no violation of weak equivalence. The dark matter continues to follow geodesics, and feels no direct drag force from the dark energy.

An example in the form of Eq. (19) is [9, 10, 11, 13]

$$Q_c^\mu = -\Gamma\rho_c u_c^\mu \,, \tag{21}$$

where Γ is a constant interaction rate. In this case $Q_c = -\Gamma\rho_c(1+\delta_c)$ and Eq. (13) becomes

$$\delta_c' - k^2 v_c = 0\,. \qquad (22)$$

The continuity equation is therefore the same as in the non-interacting case.

Thus for this form of interaction, there is no violation of the weak equivalence principle by dark matter, and no bias is induced by the interaction. In fact, in the Newtonian regime, the only signal of the dark sector interaction in structure formation to linear order is via the modification of the background expansion history. The evolution equation (18) for δ_c becomes

$$\delta_c'' + \mathcal{H}\delta_c' - 4\pi Ga^2(\rho_c\delta_c + \rho_b\delta_b) = 0, \qquad (23)$$

which is the same as in the uncoupled case. Thus the only imprint of the dark sector interaction on δ_c is via the different background evolution of $\mathcal{H}$ and ρ_c.

B. Continuity equation modified

If we keep Eq. (19) but generalize Eq. (21) to [14, 15]

$$Q_c^\mu = -(\Gamma_c\rho_c + \Gamma_x\rho_x)\,u_c^\mu\,, \qquad (24)$$

then $\delta Q_c - Q_c\delta_c = \Gamma_x\rho_x(\delta_c - \delta_x)$. Since dark energy does not cluster on sub-Hubble scales, we can neglect the δ_x term, and we have

$$\delta_c' - k^2 v_c = a\Gamma_x\frac{\rho_x}{\rho_c}\delta_c\,. \qquad (25)$$

For this interaction, the dark matter continues to follow geodesics by virtue of Eq. (20), but the continuity equation (25 is modified. As a consequence, there will be a bias induced by the interaction.

The evolution equation (18) for δ_c becomes

$$\delta_c'' + \left(\mathcal{H} - a\Gamma_x\frac{\rho_x}{\rho_c}\right)\delta_c' = 4\pi Ga^2\rho_b\delta_b$$
$$+ \left[4\pi Ga^2\rho_c + 2a\mathcal{H}\Gamma_x\frac{\rho_x}{\rho_c} + a\Gamma_x\left(\frac{\rho_x}{\rho_c}\right)'\right]\delta_c\,. \quad (26)$$

(This generalizes [15], where only the case $\Gamma_c = 0$ is considered.)

The modification of the standard evolution for δ_c occurs in 3 ways: firstly via the modified expansion history in the background $\mathcal{H}$ and ρ_c; secondly by the modified Hubble friction term $\mathcal{H} \to \mathcal{H}[1 - a\Gamma_x\rho_x/\mathcal{H}\rho_c]$; and thirdly by the modified effective gravitational coupling for dark matter – dark matter particle interactions,

$$G_{\rm eff} = G\left[1 + \frac{\mathcal{H}\rho_x}{2\pi Ga\rho_c^2} + \frac{\Gamma_x}{4\pi Ga\rho_c}\left(\frac{\rho_x}{\rho_c}\right)'\right]. \qquad (27)$$

C. Euler equation modified

A second general class of interactions has no momentum exchange in the dark energy frame,

$$Q_c^\mu = Q_c u_x^\mu\,. \qquad (28)$$

It follows that $f_c = Q_c(v_x - v)$, and hence

$$v_c' + \mathcal{H}v_c + \Psi = \frac{a}{\rho_c}Q_c(v_x - v_c)\,. \qquad (29)$$

In this case, there is an explicit deviation of the dark matter velocity relative to the non-interacting case. The dark matter no longer follows geodesics in general. Note that, even though dark energy does not cluster on sub-Hubble scales, we cannot in general neglect the dark energy velocity v_x relative to the dark matter velocity v_c in Eq. (29).

An example of the form of Eq. (28) is [16]

$$Q_c^\mu = -\alpha\rho_c\nabla^\mu\varphi\,, \qquad (30)$$

where φ is the scalar field that describes dark energy and α is a coupling constant. Note that $\nabla^\mu\varphi$ is parallel to the dark energy four-velocity u_x^μ:

$$u_x^\mu = \frac{1}{a}\left(1 - \Psi, -\frac{\partial^i\delta\varphi}{\varphi'}\right), \quad v_x = -\frac{\delta\varphi}{\varphi'}\,. \qquad (31)$$

In this case, $Q_c = a^{-1}\alpha(\rho_c\varphi' + \delta\rho_c\varphi' + \rho_c\delta\varphi' - \rho_c\varphi'\Psi)$. The perturbed Klein-Gordon equation is [17]

$$\delta\varphi'' + 2\mathcal{H}\delta\varphi' + (k^2 + a^2 V_{\varphi\varphi})\delta\varphi$$
$$= 2\varphi'(\Psi' + \mathcal{H}\Psi) + 2\varphi''\Psi - \alpha a^2\rho_c\delta_c\,, \quad (32)$$

where $V(\varphi)$ is the quintessence potential. In the Newtonian regime, the last term on the right dominates over the other terms, while the k^2 term dominates on the left, leading to

$$k^2\delta\varphi = -\alpha a^2\rho_c\delta_c\,. \qquad (33)$$

It follows from Eqs. (29), (30) and (31) that

$$v_c' + \mathcal{H}v_c + \Psi = -\alpha\varphi'\left(v_c + \frac{\delta\varphi}{\varphi'}\right), \qquad (34)$$

confirming the violation of weak equivalence. For the perturbed continuity equation (13), the right-hand side becomes $-\alpha\delta\varphi'$. By Eq. (33), this term is suppressed by k^{-2} relative to the δ_c' term on the left-hand side, and therefore to a good approximation we have

$$\delta_c' - k^2 v_c = 0\,. \qquad (35)$$

Using (33) and (35), the evolution equation (18) for δ_c becomes

$$\delta_c'' + (\mathcal{H} + \alpha\varphi')\delta_c' = 4\pi Ga^2\rho_b\delta_b$$
$$+ 4\pi Ga^2\left(1 + \frac{\alpha^2}{4\pi G}\right)\rho_c\delta_c\,. \qquad (36)$$

As in the case of Eq. (26), the modification of the standard evolution for δ_c occurs in 3 ways [12]: firstly via the modified expansion history in the background $\mathcal{H}$ and ρ_c; secondly by the modified Hubble friction term $\mathcal{H} \to \mathcal{H}[1 + \alpha\varphi'/\mathcal{H}]$; and thirdly by the modified effective gravitational coupling for dark matter – dark matter particle interactions,

$$G_{\text{eff}} = G\Big(1 + \frac{\alpha^2}{4\pi G}\Big). \qquad (37)$$

These effects are incorporated in the modified N-body simulations for this form of interacting dark energy [18].

IV. TESTING FOR DARK SECTOR INTERACTIONS

In this section, we discuss several possible ways to use observations to constrain the dark sector interactions discussed in the previous section.

A. Continuity and Euler equations unchanged

We first consider the case where there is no modification to the dynamics of perturbations in the Newtonian regime. The difference comes purely from the modified background evolution. If dark matter interacts with dark energy, the dark matter density no longer decays like a^{-3}. This affects the distance measures in the Universe and thus changes the measurements of CMB, SNe and Baryon Acoustic Oscillations. By combining these observations, we can measure today's matter density and then determine the matter energy density at the last scattering surface. However, the distance is determined by integrating over the expansion history and we cannot directly check the deviation at each redshift from the standard behaviour, $\rho_c \propto a^{-3}$.

There is an independent way to measure the dark matter density using structure formation. From the Poisson equation, the dark matter density can be written as

$$\omega_m(a) \equiv \Omega_m(a)h^2 = -\frac{2\Psi(k,a)}{3\delta_c(k,a)}\left(\frac{kh}{aH_0}\right)^2, \qquad (38)$$

where we neglected the baryon contribution for simplicity (we are only illustrating the principle, rather than making quantitative predictions). One way to measure δ_c is to reconstruct δ_c from peculiar velocities using the continuity equation Eq. (22) because in this case there is no modification to the continuity equation and no violation of weak equivalence principle. On the other hand, weak lensing measures directly Ψ without bias. Thus we can use Eq. (38) to predict the background evolution of matter density from structure formation.

In Fig. 1, we plot $\omega_m/\omega_m^{\text{eff}}$, where ω_m is the true matter density measured by weak lensing, and ω_m^{eff} is derived
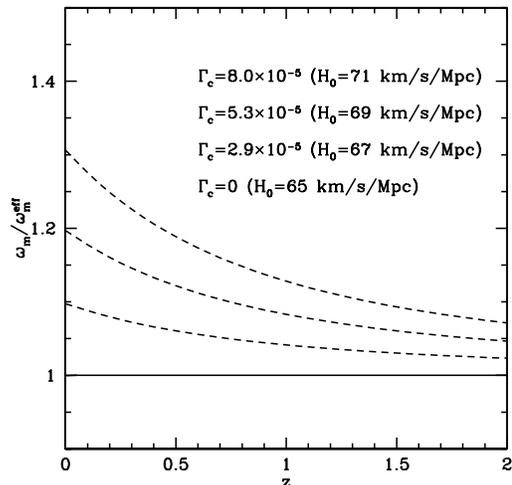

FIG. 1: The ratio between the true matter density obtained from structure formation and the density estimated from geometrical tests assuming the non-interacting adiabatic behaviour $\rho_m \propto a^{-3}$.

from the background measurement of ω_m at the last scattering surface, assuming $\rho_m \propto a^{-3}$. At late times when the interaction becomes important, this ratio deviates from 1 due to the non-adiabatic decay of the dark matter density. In this way, we can check the modification to the behaviour of the matter density at each redshift, using tomographic measurements of Ψ from weak lensing.

B. Test of the continuity equation

In the case where the interaction changes only the continuity equation, there is no difference between the peculiar velocities of baryons and dark matter. We assume that galaxies can be treated as test particles that are made of baryons and whose peculiar velocities, v_g, are determined by baryon peculiar velocities. Although there is an indication that this assumption is valid [19], this should be tested by N-body simulations carefully in the presence of interaction. We leave this for a future work.

With this assumption, we can determine peculiar velocities of baryons, v_b, from peculiar velocities of galaxies, v_g. The latter can be measured for example by redshift-space distortions (see [20, 21] for recent work). Then it is possible to determine dark matter peculiar velocities because $v_c = v_b$.

On the other hand, density perturbations can be measured from the galaxy distribution with a knowledge of bias. One possibility to measure bias is to use weak lensing. Weak lensing measures Ψ without bias and δ_c can be derived from the Poisson equation (17). Note that in order to measure Ψ from δ_c, it is necessary to measure the true evolution of ρ_c, which is modified by interactions. However, we found that the modification to the
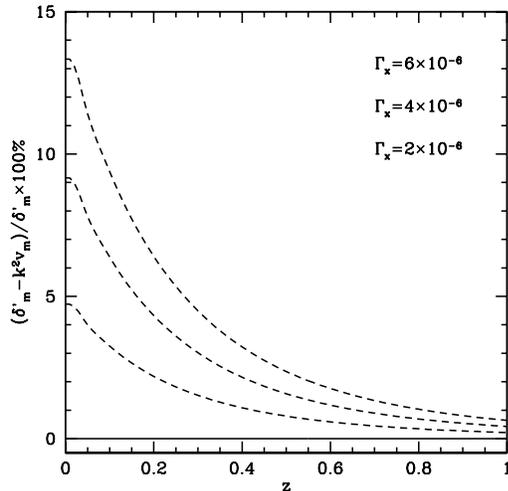
FIG. 2: The breakdown of the continuity equation by an interacting dark energy model. In this model, $v_b = v_c$ and $\delta'_c - k^2 v_c = a\Gamma_x(\rho_x/\rho_c)\delta_c$.

continuity equation has significant effects even in weak interactions cases where the effect of interactions on ρ_c is negligible. Thus in the following we only consider the case where we can neglect the effect of interactions on the evolution of ρ_c. Another possibility is to use the peculiar velocity measurements. In the case that we consider here, the Euler equation is not modified [see Eq. (20)] and it is possible to reconstruct Ψ from v_c. Then again using the Poisson equation, δ_c can be derived [22, 23, 24]. In this way we can test whether the continuity equation is modified.

Fig. 2 demonstrates the breakdown of the standard continuity equation by an interacting dark energy model. We used a model where $\Gamma_x \neq 0$ and $\Gamma_c = 0$.

C. Test of weak equivalence principle

The weak equivalence principle is broken when the Euler equation for dark matter is modified. In this case, there is a difference between the peculiar velocities of dark matter and baryons. With the assumption that galaxies trace baryon peculiar velocities, we measure baryon peculiar velocities say from red-shift distortions. Unlike the previous case, dark matter peculiar velocities are different. However, without knowing that there is an interaction between dark matter and dark energy, we *estimate* dark matter peculiar velocity as

$$v_c^{\text{est}} = v_b = v_g \,. \tag{39}$$

The estimated peculiar velocity is different from true peculiar velocity of dark matter: $v_c \neq v_c^{\text{est}}$.

If the continuity equation is not modified, as happens for the model of Eq. (30), then the true peculiar velocity satisfies the same continuity equation as the uncoupled

case. Thus, if we use the estimated peculiar velocity, the continuity equation is *apparently* broken

$$\delta'_c - k^2 v_c^{\text{est}} \neq 0. \tag{40}$$

In this case, the continuity equation is not broken but $v_c \neq v_b$. Then we can apply the same analysis as in the previous section. We can measure δ_c from weak lensing. Then it is possible to prove the breakdown of the weak equivalence principle through the apparent breakdown of the continuity equation. Fig. 3 demonstrates this idea.

V. CONCLUSIONS

An interaction between dark matter and dark energy could exist in various ways which are not detectable by any direct probe. We investigated how the Euler equation and the continuity equation for dark matter could be modified by such an interaction, taking care to provide a covariant analysis of momentum transfer. Modification of the Euler equation indicates a deviation of the dark matter motion from geodesic, under the drag force of dark energy – and a consequent breaking of the weak equivalence principle for dark matter. Using three different forms of interaction as examples, we considered interacting models in which:
(A) neither the Euler nor continuity equations are modified, so that the effect of the interaction in the Newtonian regime is purely via the different background evolution;
(B) the Euler equation is unchanged but the continuity equation is modified (and consequently a new bias is introduced by the interaction);
(C) the continuity equation is unchanged but the Euler equation is modified, leading to violation of weak equivalence.

We discussed how in principle observations could be used to detect these different forms of interacting dark energy. In case (A), we used the fact that the continuity and Euler equations are unchanged to devise a test based on the non-adiabatic redshifting of the dark matter. This test uses independent measurements of the Newtonian potential and the density perturbation via the Poisson equation, to compute the true matter density and show that it deviates from the non-interacting case.

In cases (B) and (C), the effects of the violation of the continuity equation or Euler equation are stronger than the non-standard redshifting of the background matter density. We showed how, given a knowledge of bias from weak lensing, tests could be constructed for the breakdown of the continuity or the Euler equation.

A further issue raised by our investigations is how to distinguish interacting dark energy from modified gravity. This is left for future work.
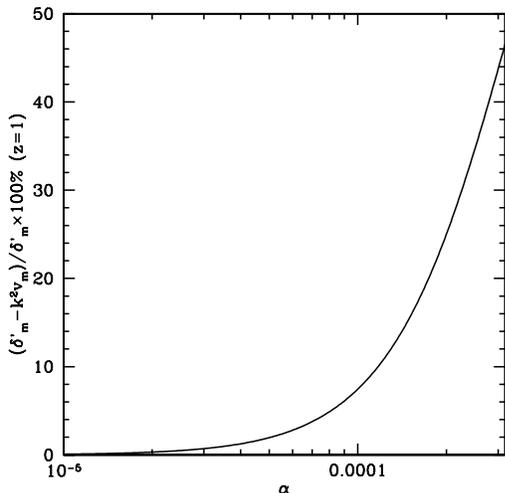
FIG. 3: The breakdown of the weak equivalence principle for dark matter. In this model, the continuity equation is not broken but $v_c \neq v_b$.

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