Understanding Complexity in Multiobjective Optimization

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This special issue of *JMCDA* was inspired by the work of three Dagstuhl seminars aimed at strengthening the links between thescientific communities of Multiple Criteria Decision Making (MCDM) and Evolutionary Multiobjective Optimization (EMO). These three Dagstuhl seminars were devoted to the following topics:

- Hybrid and Robust Approaches to Multiobjective Optimization (January 18-23 2009; http://www.dagstuhl.de/09041);
- Learning in Multiobjective Optimization (January 22-27 2012; http://www.dagstuhl.de/12041);
- Understanding Complexity in Multiobjective Optimization (11-16 January 2015: http://www.dagstuhl.de/15031).

From the above seminars arose clearly the need for better understanding the complexity of multiobjective optimization. As recent work has sometimes shown, there are opposing views regarding how problems scale and grow in difficulty dependent on their features and challenges. On the one hand, we know that multiobjective optimization problems are complex problems by their very nature; optimization problems that are easy to solve in the single objective case are often intractable and highly complex already in the biobjective case. Moreover, recent work has pointed to further fundamental limitations in multiobjective optimization as we scale up to many objectives. On the other hand, a multiobjective perspective can in a sense also help reduce complexity. For example, it often leads to a better understanding of a problem and hence supports the decision making process. Moreover, adding objectives to a problem does not always make it harder, because decomposing it can reduce the presence of local optima. And multiobjective approaches can also be used to support constraint handling, to model robustness criteria, or to approach bilevel optimization problems, simplifying these aspects. Further afield, too, in the machine learning community, we are seeing that the multiobjective optimization perspective is being used to get at the root of ill-posed problems in dimensionality reduction, pattern recognition and classification.
From the MCDM point of view, we observe that there is an intrinsic complexity in the process of understanding the optimization problem and building preferences on the solutions proposed by the multiobjective optimization. At the beginning of the decision process the Decision Maker (DM) has a rather vague idea of the decision problem at hand and, consequently, also the preferences are incomplete, approximate, uncertain or fuzzy.
Thus, better understanding complexity in multiobjective optimization is of central importance for the two communities, MCDM and EMO, and several related disciplines. It would enable us to wield existing methodologies with greater knowledge, control and effect, and should, more importantly, provide the foundations and impetus for the development of new, principled methods, in this area.
This special issue, that was mainly addressed to the participants to the above mentioned Dagstuhl seminars, but that was open to contribution of all researchers interested in the topics, contains six papers that we briefly introduce in the following.

Richard Allmendiger, Michael T. M. Emmerich, Jussi Hakanen, Yaochu Jin and Enrico Rigoni consider surrogate assisted multicriteria optimization. Surrogates are efficient computational models used to approximate the individual objective functions, multiple objectives simultaneously, and even the entire Pareto front. After discussing basic questions such as what to approximate, where to use surrogates, and how to manage the surrogate outcomes of a simulation or physical experiment, the paper explores emerging complexity-related topics in surrogate-assisted multicriteria optimization, proposing several promising future research directions and prospective solutions both from a theoretical and an industrial point of view.

Fritz Bökler, Matthias Ehrgott, Christopher Morris and Petra Mutzel investigate complexity for multiobjective combinatorial optimization problems, taking into consideration output-sensitive complexity of an algorithm for a general enumeration problem, that is the property that its running time is bounded by a polynomial in the input and the output size. The paper shows that output-sensitive complexity is able to separate efficiently solvable from presumably not efficiently solvable problems, proving also that multiobjective s-t-path problems do not admit an output-sensitive algorithm under weak complexity theoretic assumptions as **P** $\neq$ **NP.**

Rodrigo Lankaites Pinheiro, Dario Landa-Silva and Jason Atkin present a technique that supports understanding the relationships between objectives in a multiobjective optimization problem through a visualization and analysis of the local and global relationships between objectives. The advantages of the proposed technique are shown in experiments on three different combinatorial optimization problems (multiobjective multidimensional knapsack problem, multiobjective nurse scheduling problem and multiobjective vehicle routing problem with time windows).

Richard Allmendiger, Matthias Ehrgott, Xavier Gandibleux, Martin Josef Geiger, Kathrin Klamroth and Mariano Luque propose a detailed view of navigation that is the interactive procedure of traversing through a set of points (the navigation set) in the objective space guided by a decision maker, with the ultimate goal of identifying the single most-preferred Pareto optimal solution. The authors describe a general framework to capture a wide range of navigation methods taking also into account real-world problems to which these methods have been applied and highlighting directions of future research.

Kathrin Klamroth, Sanaz Mostaghim, Boris Naujoks, Silvia Poles, Robin Purshouse, GÜnter Rudolph, Stefan Ruzika, Serpil Sayın, Margaret M. Wiecek and Xin Yao consider complex systems composed of strongly interrelated subsystems or subproblems with single or multiple objectives that are usually not sequentially ordered or obviously decomposable. In the literature, these systems are also referred to as “interwoven systems” or “systems of systems”.
Due to the correlation between the components, the overall system performance does not equal the simple sum of their performances, and inclusion of complex synergy may imply possible inaccuracies in the model and prohibitively expensive computations. The authors review recent developments in this field and present a preliminary mathematical model of an interwoven system introducing some approaches to its multiobjective optimization.

José Rui Figueira, Carlos Fonseca, Pascal Halffmann, Kathrin Klamroth, Luís Paquete, Stefan Ruzika, Britta Schulze, Michael Stiglmayr and David Willems start from the consideration that despite the fact that in general multiobjective combinatorial optimization problems are known to be hard problems because very often they are NP-complete and intractable, there are also variants or cases of multiobjective combinatorial optimization problems that are easy. The article focuses on particular cases of multiobjective combinatorial optimization problems, which are polynomially solvable, aiming at categorizing them, explaining their polynomial solvability in terms of general structural properties, and exploring the grey zone between easy and hard multiobjective combinatorial optimization problems.

We believe that these six articles give a valuable contribution to the discussion on complexity of multiobjective optimization, proposing new perspectives both from the theoretical and the applicative point of view, and confirming the wealth of interesting work there is still to do in this domain.