DARK ENERGY AND MODIFIED GRAVITY

Ruth Durrer[1] and Roy Maartens[2]

[1]*Départment de Physique Théorique, Université de Genève, 1211 Genève 4, Switzerland*
[2]*Institute of Cosmology & Gravitation, University of Portsmouth, Portsmouth PO1 2EG, UK*

Explanations of the late-time cosmic acceleration within the framework of general relativity are plagued by difficulties. General relativistic models are mostly based on a dark energy field with fine-tuned, unnatural properties. There is a great variety of models, but all share one feature in common – an inability to account for the gravitational properties of the vacuum energy, and a failure to solve the so-called coincidence problem. Two broad alternatives to dark energy have emerged as candidate models: these typically address only the coincidence problem and not the vacuum energy problem. The first is based on general relativity and attempts to describe the acceleration as an effect of inhomogeneity in the universe. If this alternative could be shown to work, then it would provide a dramatic resolution of the coincidence problem; however, a convincing demonstration of viability has not yet emerged. The second alternative is based on infra-red modifications to general relativity, leading to a weakening of gravity on the largest scales and thus to acceleration. Most examples investigated so far are scalar-tensor or brane-world models, and we focus on the simplest candidates of each type: $f(R)$ models and DGP models respectively. Both of these provide a new angle on the problem, but they also face serious difficulties. However, investigation of these models does lead to valuable insights into the properties of gravity and structure formation, and it also leads to new strategies for testing the validity of General Relativity itself on cosmological scales.

I. INTRODUCTION

The current "standard model" of cosmology is the inflationary cold dark matter model with cosmological constant Λ, usually called LCDM, which is based on general relativity and particle physics (i.e., the Standard Model and its minimal supersymmetric extensions). This model provides an excellent fit to the wealth of high-precision observational data, on the basis of a remarkably small number of cosmological parameters [1]. In particular, independent data sets from cosmic microwave background (CMB) anisotropies, galaxy surveys and supernova luminosities, lead to a consistent set of best-fit model parameters (see Fig. 1) – which represents a triumph for LCDM.

The standard model is remarkably successful, but we know that its theoretical foundation, general relativity, breaks down at high enough energies, usually taken to be at the Planck scale,

$$E \gtrsim M_p \sim 10^{16}\,\text{TeV}\,. \tag{1}$$

The LCDM model can only provide limited insight into the very early universe. Indeed, the crucial role played by inflation belies the fact that inflation remains an effective theory without yet a basis in fundamental theory. A quantum gravity theory will be able to probe higher energies and earlier times, and should provide a consistent basis for inflation, or an alternative that replaces inflation within the standard cosmological model.

An even bigger theoretical problem than inflation is that of the late-time acceleration in the expansion of the universe [2, 3]. In terms of the fundamental energy density parameters, the data indicates that the present cosmic energy budget is given by (see Fig. 1)

$$\Omega_\Lambda \equiv \frac{\Lambda}{3H_0^2} \approx 0.75\,, \quad \Omega_m \equiv \frac{8\pi G\rho_{m0}}{3H_0^2} \approx 0.25\,, \quad \Omega_K \equiv \frac{-K}{a_0^2 H_0^2} \approx 0\,, \quad \Omega_r \equiv \frac{8\pi G\rho_{r0}}{3H_0^2} \approx 8 \times 10^{-5}\,. \tag{2}$$

Here $H_0 = 100 h\,\text{km(s Mpc)}^{-1}$ is the present value of the Hubble parameter, Λ is the cosmological constant, K is spatial curvature, ρ_{m0} is the present matter density and ρ_{r0} is the present radiation density. Newton's constant is related to the Planck mass by $G = M_p^{-2}$ (we use units where the speed of light, $c = 1$ and Planck's constant $\hbar = 1$).

The Friedmann equation governs the evolution of the scale factor $a(t)$. It is given by

$$\begin{aligned}
\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 &= \frac{8\pi G}{3}(\rho_m + \rho_r) + \frac{\Lambda}{3} - \frac{K}{a^2} \\
&= H_0^2 \left[\Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_\Lambda + \Omega_K(1+z)^2\right],
\end{aligned} \tag{3}$$

The scale factor, which is related to the cosmological redshift by $z = a^{-1} - 1$. (We normalize the present scale factor to $a_0 = 1$.) Together with the energy conservation equation this implies

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho_m + 2\rho_r\right) + \frac{\Lambda}{3}. \tag{4}$$

The observations, which together with Eq. (3) lead to the values given in Eq. (2), produce via Eq. (4) the dramatic conclusion that the universe is currently accelerating,

$$\ddot{a}_0 = H_0^2\left(\Omega_\Lambda - \frac{1}{2}\Omega_m - \Omega_r\right) > 0. \tag{5}$$

This conclusion holds only if the universe is (nearly) homogeneous and isotropic, i.e., a Friedmann-Lemaître model. In this case the distance to a given redshift z, and the time elapsed since that redshift, are tightly related via the only free function of this geometry, $a(t)$. If the universe instead is isotropic around us but not homogeneous, i.e., if it resembles a Tolman-Bondi–Lemaître solution with our galaxy cluster at the centre, then this tight relation between distance and time for a given redshift would be lost and present data would not necessarily imply acceleration – or the data may imply acceleration without dark energy. This remains a controversial and unresolved issue (see e.g. [5]).

Of course isotropy without homogeneity violates the Copernican Principle as it puts us in the centre of the Universe. However, it has to be stressed that up to now observations of homogeneity are very limited, unlike isotropy, which is firmly established. Homogeneity is usually inferred from isotropy together with the Copernican principle. With future data, it will in principle be possible to distinguish observationally an isotropic but inhomogeneous universe from an isotropic and homogeneous universe (see e.g. [6]). Testing the Copernican Principle is a crucial aspect of testing the standard cosmological model. But in the following, we will assume that the Copernican Principle applies.

The data also indicate that the universe is currently (nearly) spatially flat,

$$|\Omega_K| \ll 1. \tag{6}$$

It is common to assume that this implies $K = 0$ and to use inflation as a motivation. However, inflation does *not* imply $K = 0$, but only $\Omega_K \to 0$. Even if this distinction may be negligible in the present universe, a nonzero curvature can have significant implications for the onset of inflation(see e.g. [7]). In fact, if the present curvature is small but non-vanishing, neglecting it in the analysis of Supernova data can sometimes induce surprisingly large errors [8].

The simplest way to explain acceleration is probably a cosmological constant, i.e., the LCDM model. Even though the cosmological constant can be considered as simply an additional gravitational constant (in addition to Newton's constant), it enters the Einstein equations in exactly the same way as a contribution from the vacuum energy, i.e., via a Lorentz-invariant energy-momentum tensor $T_{\mu\nu}^{\mathrm{vac}} = -(\Lambda/8\pi G)g_{\mu\nu}$. The only observable signature of both a cosmological constant and vacuum energy is their effect on spacetime – and so a vacuum energy and a classical cosmological constant cannot be distinguished by observation. Therefore the 'classical' notion of the cosmological constant is effectively physically indistinguishable from quantum vacuum energy.

Even though the absolute value of vacuum energy cannot be calculated within quantum field theory, *changes* in the vacuum energy (e.g. during a phase transition) can be calculated, and they do have a physical effect – for example, on the energy levels of atoms (Lamb shift), which is well known and well measured. Furthermore, differences of vacuum energy in different locations, e.g., between or on one side of two large metallic plates, have been calculated and their effect, the Casimir force, is well measured [9]. Hence, there is no doubt about the reality of vacuum energy. For a field theory with cutoff energy scale E, the vacuum energy density scales with the cutoff as $\rho_{\mathrm{vac}} \sim E^4$, corresponding to a cosmological constant $\Lambda_{\mathrm{vac}} = 8\pi G\rho_{\mathrm{vac}}$. If $E = M_p$, this yields a naïve contribution to the 'cosmological constant' of about $\Lambda_{\mathrm{vac}} \sim 10^{38}\mathrm{GeV}^2$, whereas the measured effective cosmological constant is the sum of the 'bare' cosmological constant and the contribution from the cutoff scale,

$$\Lambda_{\mathrm{eff}} = \Lambda_{\mathrm{vac}} + \Lambda \simeq 10^{-83}\,\mathrm{GeV}^2\,. \tag{7}$$

Hence a cancellation of about 120 orders of magnitude is required. This is called the *fine-tuning* or *size* problem of dark energy: a cancellation is needed to arrive at a result which is many orders of
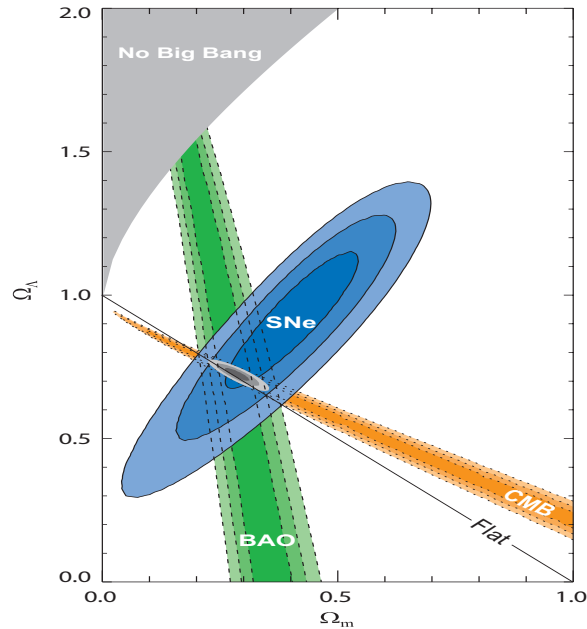
FIG. 1: Observational constraints in the $(\Omega_m, \Omega_\Lambda)$ plane: joint constraints from supernovae (SNe), baryon acoustic oscillations (BAO) and CMB (from [4]).

magnitude smaller than each of the terms.[1] It is possible that the quantum vacuum energy is much smaller than the Planck scale. But even if we set it to the lowest possible SUSY scale, $E_{\rm susy} \sim 1{\rm TeV}$, arguing that at higher energies vacuum energy exactly cancels due to supersymmetry, the required cancellation is still about 60 orders of magnitude.

A reasonable attitude towards this open problem is the hope that quantum gravity will explain this cancellation. But then it is much more likely that we shall obtain directly $\Lambda_{\rm vac} + \Lambda = 0$ and not $\Lambda_{\rm vac} + \Lambda \simeq 24\pi G\rho_m(t_0)$. This unexpected observational result leads to a second problem, *the coincidence problem*: given that

$$\rho_\Lambda = \frac{\Lambda_{\rm eff}}{8\pi G} = \text{constant}, \quad \text{while} \quad \rho_m \propto (1+z)^3, \tag{8}$$

why is ρ_Λ of the order of the *present* matter density $\rho_m(t_0)$? It was completely negligible in most of the past and will entirely dominate in the future.

These problems prompted cosmologists to look for other explanation of the observed accelerated expansion. Instead of a cosmological constant, one may introduce a scalar field or some other contribution to the energy-momentum tensor which has an equation of state $w < -1/3$. Such a component is called 'dark energy'. So far, no consistent model of dark energy has been proposed which can yield a convincing or natural explanation of either of these problems (see, e.g. [10]).

Alternatively, it is possible that there is no dark energy field, but instead the late-time acceleration is a signal of a *gravitational* effect. Within the framework of general relativity, this requires that the impact of inhomogeneities somehow acts to produce acceleration, or the appearance of acceleration (within a Friedman-Lemaître interpretation). A non-Copernican possibility is the Tolman-Bondi–Lemaître model [5]. Another (Copernican) possibility is that the 'backreaction' of inhomogeneities on the background, treated via nonlinear averaging, produces effective acceleration [11].

A more radical version is the 'dark gravity' approach, the idea that gravity itself is weakened on large-scales, i.e., that there is an "infrared" modification to general relativity that accounts for

---

[1] In quantum field theory we actually have to add to the cut-off term $\Lambda_{\rm vac} \simeq E_c^4/M_{pl}^2$ the unmeasurable 'bare' cosmological constant. In this sense, the cosmological constant problem is a fine tuning between the unobservable 'bare' cosmological constant and the term coming from the cut-off scale.

the late-time acceleration. The classes of modified gravity models which have been most widely investigated are scalar-tensor models [12] and brane-world models [13].

Schematically, we are modifying the geometric side of the field equations,

$$G_{\mu\nu} + G_{\mu\nu}^{\text{dark}} = 8\pi G T_{\mu\nu} \,, \tag{9}$$

rather than the matter side,

$$G_{\mu\nu} = 8\pi G \left(T_{\mu\nu} + T_{\mu\nu}^{\text{dark}} \right) \,, \tag{10}$$

as in the general relativity approach. Modified gravity represents an intriguing possibility for resolving the theoretical crisis posed by late-time acceleration. However, it turns out to be extremely difficult to modify general relativity at low energies in cosmology, without violating observational constraints – from cosmological and solar system data, or without introducing ghosts and other instabilities into the theory. Up to now, there is no convincing alternative to the general relativity dark energy models – which themselves are not convincing.

The plan of the remainder of this chapter is as follows. In Section 2 we discuss constraints which one may formulate for a dark energy or modified gravity theory from basic theoretical requirements. In Section 3 we briefly discuss models that address the dark energy problem within general relativity. In Section 4 we present modified gravity models. In Section 5 we conclude. This article is based on a previous review published in [14].

II. CONSTRAINING EFFECTIVE THEORIES

Theories of both dark matter and dark energy often have very unusual Lagrangians that cannot be quantized in the usual way, e.g. because they have non-standard kinetic terms. We then simply call them 'effective low energy theories' of some unspecified high energy theory which we do not elaborate. In this section, we want to point out a few properties which we nevertheless can require of low energy effective theories. We first enumerate the properties which we can require from a good basic physical theory at the classical and at the quantum level. We then discuss which of these requirements are inherited by low energy effective descriptions.

A. FUNDAMENTAL PHYSICAL THEORIES

Here we give a minimal list of properties which we require from a fundamental physical theory. Of course, all the points enumerated below are open for discussion, but at least we should be aware of what we lose when we let go of them.

In our list we start with very basic requirements which become more strict as we go on. Even though some theorists would be able to live without one or several of the criteria discussed here, we think they are all very well founded. Furthermore, all known current physical theories, including string- and M-theory, do respect them.

1. **A physical theory allows a mathematical description**
   This is the basic idea of theoretical physics.

2. **A fundamental physical theory allows a Lagrangian formulation**
   This requirement is of course much stronger than the previous one. But it has been extremely successful in the past and was the guiding principle for the entire development of quantum field theory and string theory in the 20th century.

   Some 'varying speed of light theories' without Lagrangian formulation leave us more or less free to specify the evolution of the speed of light during the expansion history of the universe. However, if we introduce a Lagrangian formulation, we realize that most of these theories are simply some variant of scalar-tensor theories of gravity, which are well defined and have been studied in great detail.

   If we want to keep deep physical insights like Nöther's theorem, which relates symmetries to conservation laws, we need to require a Lagrangian formulation for a physical theory. A

basic ingredient of a Lagrangian physical theory is that every physical degree of freedom has a kinetic term which consists (usually) of first order time derivatives and may also have a 'potential term' which does not involve derivatives. In the Lagrangian formulation of a fundamental physical theory, we do not allow for external, arbitrarily given functions. Every function has to be a degree of freedom of the theory so that its evolution is determined self-consistently via the Lagrangian equations of motion, which are of first or second order. It is possible that the Lagrangian contains also higher than first order derivatives, but such theories are strongly constrained by the problem of ghosts which we mention below, and by the fact that the corresponding equations of motion are usually described by an unbounded Hamiltonian, i.e. the system is unstable (Ostrogradski's theorem [15, 16]).

3. **Lorentz invariance**
We also want to require that the theory be Lorentz invariant. Note that this requirement is much stronger than demanding simply 'covariance'. It requires that there be no 'absolute element' in the theory apart from true constants. Lorentz covariance can always be achieved by rewriting the equations. As an example, consider a Lagrangian given in flat space by $(\partial_t \phi)^2 - (\partial_x \phi)^2$. This is clearly not Lorentz invariant. However, we can trivially write this term in the covariant form $\alpha^{\mu\nu} \partial_\nu \partial_\mu \phi$, by setting $(\alpha^{\mu\nu}) = \text{diag}(1, -1, 0, 0)$. Something like this should of course not be allowed in a fundamental theory. A term of the form $\alpha^{\mu\nu} \partial_\nu \partial_\mu \phi$ is only allowed if $\alpha^{\mu\nu}$ is itself a dynamical field of the theory. This is what we mean by requiring that the theory is not allowed to contain 'absolute elements', i.e. it is Lorentz invariant and not simply covariant.

4. **Ghosts**
Ghosts are fields whose kinetic term has the wrong sign. Such a field, instead of slowing down when it climbs up a potential, is speeding up. This unstable situation leads to severe problems when we want to quantize it, and it is generally accepted that one cannot make sense of such a theory, at least not at the quantum level. This is not surprising, since quantization usually is understood as defining excitations above some ground state, and a theory with a ghost has no well defined ground state. Its kinetic energy has the wrong sign and the larger $\dot{\phi}^2$ is, the lower is the energy.

5. **Tachyons**
These are degrees of freedom that have a negative mass squared, $m^2 < 0$. Using again the simple scalar field example, this means that the second derivative of the potential about the 'vacuum value' ($\phi = 0$ with $\partial_\phi V(0) = 0$) is negative, $\partial_\phi^2 V(0) < 0$. In general, this need not mean that the theory makes no sense, but rather that $\phi = 0$ is a bad choice for expanding around, since it is a maximum rather than a minimum of the potential and therefore an unstable equilibrium.

This means also that the theory cannot be quantized around the classical solution $\phi = 0$, but it may become a good quantum theory by a simple shift, $\phi \to \phi - \phi_0$, where ϕ_0 is the minimum of the potential. If the potential of a fundamental scalar field has no minimum but only a maximum, the situation is more severe. Then the theory is truly unstable.

The last two problems, together with the Ostrogradski instability that appears in theories with higher derivatives, can be summarized in the requirement that a meaningful theory needs to have an energy functional which is bounded from below.

6. **Superluminal motion and causality**
A fundamental physical theory which does respect Lorentz invariance must not allow for superluminal motions. If this condition is not satisfied, we can construct closed curves along which a signal can propagate [17]. (See Fig. 2.)

At first sight one might think that a Lorentz invariant Lagrangian will automatically forbid superluminal motions. But the situation is not so simple. Generic Lorentz invariant higher spin theories, $s \geq 1$, lead to superluminal motion [18]. While the equations are manifestly Lorentz invariant, their characteristics in general do not coincide with the light cone and can very well be spacelike. There are exceptions, among which are Yang Mills theories for spin 1 and the linearized Einstein equations for spin 2.
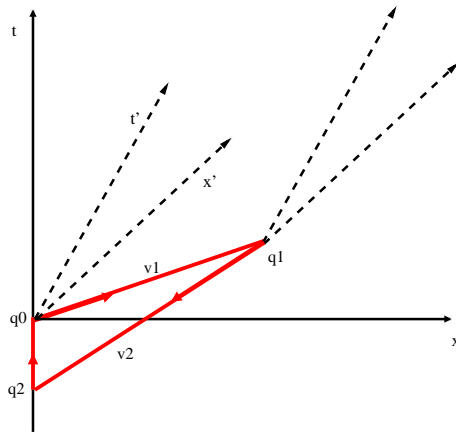
FIG. 2: We assume a signal that can propagate at speeds v_1, $v_2 > 1$. The frame R' with coordinates (t', x') moves with speed $v < 1$ in the x-direction. The speed v is chosen such that both, v_1, $v_2 > 1/v$. A signal is sent with velocity v_1 from q_0 to q_1 in the frame R. Since $v_1 > 1/v$, this signal travels backward in time with respect to frame R'. Then a signal is sent with speed v_2 from q_1 to q_2. Since $|v_2| > 1/v$, this signal, which is sent forward in time in frame R', travels backward in time with respect to R and can arrive at an event q_2 with $t_2 < 0$. The loop generated in this way is not 'causal' since both the trajectory from q_0 to q_1 and the one from q_1 to q_2 are spacelike. So we cannot speak of the formation of closed causal loops, but it is nevertheless a closed loop along which a signal can propagate and which therefore enables the construction of a time machine, leading to the usual problems with causality and entropy. (From [17].)

One may object to this restriction, on the grounds that general relativity, which is certainly a theory that is acceptable (at least at the classical level), can lead to closed causal curves, even though it does not admit superluminal motion [19, 20, 21, 22].

The situation is somewhat different if superluminal motion is only possible in a background which breaks Lorentz-invariance. Then one has in principle a preferred frame and one can specify that perturbations should always propagate with the Green's function that corresponds to the retarded Green's function in this frame [23]. Nevertheless, one has to accept that there will be boosted frames relative to which the Cauchy problem for the superluminal modes is not well defined. The physics experienced by an observer in such a frame is most unusual (to say the least).

Causality of a theory is intimately related to the analyticity properties of the S-matrix of scattering, without which perturbative quantum theory does not make sense. Furthermore, we require the S matrix to be unitary. Important consequences of these basic requirements are the Kramers Kronig dispersion relations, which are a result of the analyticity properties and hence of causality, and the optical theorem, which is a result of unitarity. The analyticity properties have many further important consequences, such as the Froissart bound, which implies that the total cross section converges at high energy [24].

B. LOW ENERGY EFFECTIVE THEORIES

The concept of low energy effective theories is extremely useful in physics. As one of the most prominent examples, consider superconductivity. It would be impossible to describe this phenomenon by using full quantum electrodynamics with a typical energy scale of MeV, where the energy scale of superconductivity is milli-eV and less. However, many aspects of superconductivity can be successfully described with the Ginzburg-Landau theory of a complex scalar field. Microscopically, this scalar field is to be identified with a Cooper pair of two electrons, but this is irrelevant for many aspects of superconductivity.

Another example is weak interaction and four-Fermi theory. The latter is a good approximation to weak interactions at energy scales far below the Z-boson mass. Most physicists also regard the standard model of particle physics as a low energy effective theory which is valid below some high energy scale beyond which new degrees of freedom become relevant, be this supersymmetry, GUT

or string theory.

We now want to investigate which of the properties in the previous subsection may be lost if we 'integrate out' high energy excitations and consider only processes which take place at energies below some cutoff scale E_c. We cannot completely ignore all particles with masses above E_c, since in the low energy quantum theory they can still be produced 'virtually', i.e., for a time shorter than $1/E_c$. This is not relevant for the initial and final states of a scattering process, but plays a role in the interaction.

Coming back to our list in the previous subsection, we certainly want to keep the first point – a mathematical description. But the Lagrangian formulation will also survive if we proceed in a consistent way by simply integrating out the high energy degrees of freedom.

What about higher order derivatives in the Lagrangian? The problem is that, in general, there is no Hamiltonian that is bounded from below if the Lagrangian contains higher derivatives, i.e. the system is unstable [16]. Of course it is possible to find well behaved solutions of this system, since for a given solution energy is conserved. But as soon as the system is interacting, with other degrees of freedom, it will lower its energy and produce more and more modes of these other degrees of freedom. This is especially serious when one quantizes the system. The vacuum is exponentially unstable to simultaneous production of modes of positive and negative energy. Of course one cannot simply 'cut away' the negative energy solutions without violating unitarity. And even if the theory under consideration is only a low energy effective theory, it should at least be 'unitary at low energy'. Introducing even higher derivatives only worsens the situation, since the Hamiltonian acquires more unstable directions.

For this argument, it does not matter whether the degrees of freedom we are discussing are fundamental or only low energy effective degrees of freedom. Even if we modify the Hamiltonian at high energies, the instability, which is a *low energy problem*, will not disappear. There are only two ways out of the Ostrogradski instability: Firstly, if the necessary condition that the lagrangian be non-degenerate is not satisfied. The second possibility is via constraints, whereby one might be able to eliminate the unstable directions. In practice, this has to be studied on a case by case basis. An important example for the dark energy problem, which avoids the Ostrogradski instability via constraints, are modified gravity Lagrangians of the form $f(R)$, discussed below.

If the Ostradgradski theorem does not apply, we have still no guarantee that the theory has no ghosts or that the potential energy is bounded from below (no 'serious' tachyon). The limitation from the Ostragradski theorem, but also the ghost and tachyon problem, can be cast in the requirement that the theory needs to have an energy functional which is bounded from below. This condition can certainly not disappear in a consistent low energy version of a fundamental theory which satisfies it.

The high energy cut-off will be given by some mass scale, i.e. some Lorentz invariant energy scale of the theory, and therefore the effective low energy theory should also admit a Lorentz invariant Lagrangian. Lorentz invariance is not a high energy phenomenon which can simply be lost at low energies.

What about superluminal motion and causality? We do not want to require certain properties of the S matrix of the low energy theory, since the latter may not have a meaningful perturbative quantum theory; like the 4-Fermi theory, it may not be renormalizable. Furthermore, one can argue that in cosmology we do have a preferred frame, the cosmological frame, hence Lorentz-invariance is broken and we can simply demand that all superluminal modes of a field propagate forward in cosmic time. Then no closed signal curves are possible.

But this last argument is very dangerous. Clearly, most solutions of a Lagrangian theory do break several or most of the symmetries of the Lagrangian spontaneously. But when applying a Lorentz transformation to a solution, we produce a new solution that, from the point of view of the Lagrangian, has the same right of existence. If some modes of a field propagate with superluminal speed, this means that their characteristics are spacelike. The condition that the mode has to travel forward in time with respect to a certain frame implies that one has to use the retarded Green's function in this frame. Since spacelike distances have no frame-independent chronology, for spacelike characteristics this is a frame-dependent statement. Depending on the frame of reference, a given mode can represent a normal propagating degree of freedom, or it can satisfy an elliptic equation, a constraint.

Furthermore, to make sure that the mode propagates forward with respect to one fixed reference frame, one would have to use sometimes the retarded, sometimes the advanced and sometimes a mixture of both functions, depending on the frame of reference. In a cosmological setting this can

be done in a consistent way, but it is far from clear that such a prescription can be unambiguously implemented for generic low energy solutions. Indeed in Ref. [25] a solution is sketched that would not allow this, so that closed signal curves are again possible.

Therefore, we feel that Lorentz invariant low energy effective Lagrangians which allow for superluminal propagation of certain modes, have to be rejected. Nevertheless, this case is not as clear-cut and there are opposing opinions in the literature, e.g. [23].

With the advent of the 'landscape' [26], physicists have begun to consider anthropic arguments to justify their theory, whenever it fits the data. Even though the existence of life on earth is an experimental fact, we consider this argument weak, nearly tantamount to giving up physics: 'Things are like they are since otherwise we would not be here'. We nevertheless find it important to inquire also from a purely theoretical point of view, whether really 'anything goes' for effective theories. In the following sections we shall come back to the basic requirements which we have outlined in this section.

III. GENERAL RELATIVISTIC APPROACHES

We give a very brief overview of models for the late-time acceleration within general relativity, before moving on to the main topic of modified gravity.

The "standard" general relativistic interpretation of dark energy is based on the cosmological constant as vacuum energy:

$$G_{\mu\nu} = 8\pi G \left[T_{\mu\nu} + T_{\mu\nu}^{\text{vac}} \right], \quad T_{\mu\nu}^{\text{vac}} = -\frac{\Lambda_{\text{eff}}}{8\pi G} \, g_{\mu\nu} \,, \tag{11}$$

where the vacuum energy-momentum tensor is Lorentz invariant. This approach faces the problem of accounting for the incredibly small and highly fine-tuned value of the vacuum energy, as summarized in Eq. (7).

String theory provides a tantalising possibility in the form of the "landscape" of vacua [26]. There appears to be a vast number of vacua admitted by string theory, with a broad range of vacuum energies above and below zero. The idea is that our observable region of the universe corresponds to a particular small positive vacuum energy, whereas other regions with greatly different vacuum energies will look entirely different. This multitude of regions forms in some sense a "multiverse". This is an interesting idea, but it is highly speculative, and it is not clear how much of it will survive the further development of string theory and cosmology.

An alternative view of LCDM is the interpretation of Λ as a classical geometric constant [27], on a par with Newton's constant G. Thus the field equations are interpreted in the geometrical way,

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \,. \tag{12}$$

In this approach, the small and fine-tuned value of Λ is no more of a mystery than the host of other fine-tunings in the constants of nature. For example, more than a 2% change in the strength of the strong interaction means that no atoms beyond hydrogen can form, so that stars and galaxies would not emerge. But it is not evident whether this distinction between Λ and ρ_{vac} is really a physical statement, or a purely theoretical statement that cannot be tested by any experiments. Furthermore, this classical approach to Λ does not evade the vacuum energy problem – it simply shifts that problem to "why does the vacuum not gravitate?" The idea is that particle physics and quantum gravity will somehow discover a cancellation or symmetry mechanism to explain why $\rho_{\text{vac}} = 0$. This would be a simpler solution than that indicated by the string landscape approach, and would evade the disturbing anthropic aspects of that approach.

Within general relativity, various alternatives to LCDM have been investigated, in attempt to address the coincidence problem.

A. DYNAMICAL DARK ENERGY: QUINTESSENCE

Here we replace the constant $\Lambda/8\pi G$ by the energy density of a scalar field φ, with Lagrangian

$$L_\varphi = -\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) \,, \tag{13}$$

so that in a cosmological setting,

$$\rho_\varphi = \frac{1}{2}\dot{\varphi}^2 + V(\varphi)\,, \quad p_\varphi = \frac{1}{2}\dot{\varphi}^2 - V(\varphi)\,, \tag{14}$$

$$\ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) = 0\,, \tag{15}$$

$$H^2 + \frac{K}{a^2} = \frac{8\pi G}{3}\left(\rho_r + \rho_m + \rho_\varphi\right)\,. \tag{16}$$

The field rolls down its potential and the dark energy density varies through the history of the universe. "Tracker" potentials have been found for which the field energy density follows that of the dominant matter component. This offers the possibility of solving or alleviating the fine tuning problem of the resulting cosmological constant. Although these models are insensitive to initial conditions, they do require a strong fine-tuning of the *parameters of the Lagrangian* to secure recent dominance of the field, and hence do not evade the coincidence problem. An attempt to address the coincidence problem is proposed in [28], where the transition from the tracker behavior to dark energy domination is tied to the neutrino mass.

More generally, the quintessence potential, somewhat like the inflaton potential, remains arbitrary, until and unless fundamental physics selects a potential. There is currently no natural choice of potential.

In conclusion, there is no compelling reason as yet to choose quintessence above the LCDM model of dark energy. Quintessence models do not seem more natural, better motivated or less contrived than LCDM. Nevertheless, they are a viable possibility and computations are straightforward. Therefore, they remain an interesting target for observations to shoot at [10].

B. DYNAMICAL DARK ENERGY: MORE GENERAL MODELS

It is possible to couple quintessence to cold dark matter without violating current constraints from fifth force experiments. This could lead to a new approach to the coincidence problem, since a coupling may provide a less unnatural way to explain why acceleration kicks in when $\rho_m \sim \rho_{de}$. In the presence of coupling, the energy conservation equations in the background become

$$\dot{\varphi}\left[\ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi)\right] = Q\,, \tag{17}$$

$$\dot{\rho}_{\rm dm} + 3H\rho_{\rm dm} = -Q\,, \tag{18}$$

where Q is the rate of energy exchange. It is relatively simple to match the geometric data on the background expansion history [29]. The perturbations show that there is a momentum transfer as well as an energy transfer. Analysis of the perturbations typically leads to more stringent constraints, with some forms of coupling being ruled out by instabilities [30].

Another possibility is a scalar field with non-standard kinetic term in the Lagrangian, for example,

$$L_\varphi = F(\varphi, X) - V(\varphi) \text{ where } X \equiv -\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\,. \tag{19}$$

The standard Lagrangian has $F(\varphi, X) = X$. Some of the non-standard F models may be ruled out on theoretical grounds. An example is provided by "phantom" fields, with negative kinetic energy density (ghosts), $F(\varphi, X) = -X$. They have $w < -1$, so that their energy density *grows* with expansion. This bizarre behaviour is reflected in the instability of the quantum vacuum for phantom fields.

Another example is "k-essence" fields [31], which have $F(\varphi, X) = \varphi^{-2}f(X)$. These theories have no ghosts, and they can produce late-time acceleration. The sound speed of the field fluctuations for the Lagrangian in Eq. (19) is

$$c_s^2 = \frac{F_{,X}}{F_{,X} + 2XF_{,XX}}\,. \tag{20}$$

For a standard Lagrangian, $c_s^2 = 1$. But for the class of F that produce accelerating k-essence models, it turns out that there is always an epoch during which $c_s^2 > 1$, so that these models may be ruled out according to our causality requirement. They violate standard causality [32].

For models not ruled out on theoretical grounds, there is the same general problem as with quintessence, i.e. that no model is better motivated than LCDM, none is selected by fundamental physics and any choice of model is more or less arbitrary. Quintessence then appears to at least have the advantage of simplicity – although LCDM has the same advantage over quintessence.

When investigating generic dark energy models we always have to keep in mind that since both dark energy and dark matter are only detected gravitationally, we can only measure the total energy momentum tensor of the dark component,

$$T_{\mu\nu}^{\mathrm{dark}} = T_{\mu\nu}^{\mathrm{de}} + T_{\mu\nu}^{\mathrm{dm}} \,. \tag{21}$$

Hence, if we have no information on the equation of state of dark energy, there is a degeneracy between the dark energy equation of state $w(t)$ and Ω_{dm}. Without additional assumptions, we cannot measure either of them by purely gravitational observations [33]. This degeneracy becomes even worse if we allow for interactions between dark matter and dark energy.

C. DARK ENERGY AS A NONLINEAR EFFECT FROM STRUCTURE

As structure forms and the matter density perturbation becomes nonlinear, there are two questions that are posed: (1) what is the back-reaction effect of this nonlinear process on the background cosmology? (2) how do we perform a covariant and gauge-invariant averaging over the inhomogeneous universe to arrive at the correct FRW background? The simplistic answers to these questions are: (1) the effect is negligible since it occurs on scales too small to be cosmologically relevant; (2) in light of this, the background is independent of structure formation, i.e., it is the same as in the linear regime. A quantitative analysis is needed to fully resolve both issues. However, this is very complicated because it involves the nonlinear features of general relativity in an essential way.

There have been claims that these simplistic answers are wrong, and that, on the contrary, the effects are large enough to mimic an accelerating universe. This would indeed be a dramatic and satisfying resolution of the coincidence problem, without the need for any dark energy field. This issue is discussed in [11]. Of course, the problem of why the vacuum does not gravitate would remain.

However, these claims have been disputed, and it is fair to say that there is as yet no convincing demonstration that acceleration could emerge naturally from nonlinear effects of structure formation [34]. We should however note that backreaction/averaging effects could significantly affect our estimations of cosmological parameters, even if they do not lead to acceleration [35].

It is in principle also possible that the universe around us resembles more a spherically symmetric but inhomogeneous solution of Einstein's equation, a Tolman-Bondi-Lemaître universe, than a Friedmann-Lemaître universe. In this case, what appears as cosmic acceleration to us could perhaps be explained within simple matter models which only contain dust [5]. However, this would imply that we are situated very close to the centre of a huge (nearly) spherical structure. Apart from violating the Copernican principle, this poses another fine tuning problem, and it also not clear to us whether these models are consistent with all observations – not just supernova, but baryon acoustic oscillations, CMB anisotropies, and weak lensing.

IV. THE MODIFIED GRAVITY APPROACH: DARK GRAVITY

Late-time acceleration from nonlinear effects of structure formation is an attempt, within general relativity, to solve the coincidence problem without a dark energy field. The modified gravity approach shares the assumption that there is no dark energy field, but generates the acceleration via "dark gravity", i.e. a weakening of gravity on the largest scales, due to a modification of general relativity itself.

Could the late-time acceleration of the universe be a gravitational effect? (Note that in general also this does not remove the problem of why vacuum energy does not gravitate or is very small.) A historical precedent is provided by attempts to explain the anomalous precession of Mercury's perihelion by a "dark planet, named Vulcan. In the end, it was discovered that a modification to Newtonian gravity was needed.

As we have argued in Section II, a consistent modification of general relativity requires a covariant formulation of the field equations in the general case, i.e., including inhomogeneities and anisotropies. It is not sufficient to propose ad hoc modifications of the Friedman equation, of the form

$$f(H^2) = \frac{8\pi G}{3}\rho \ \text{ or } \ H^2 = \frac{8\pi G}{3}g(\rho) \,, \tag{22}$$

for some functions f or g. Apart from the fundamental problems outlined in Section II, such a relation allows us to compute the supernova distance/ redshift relation using this equation – but we *cannot* compute the density perturbations without knowing the covariant parent theory that leads to such a modified Friedman equation. And we also cannot compute the solar system predictions.

It is very difficult to produce infrared corrections to general relativity that meet all the minimum requirements:

- Theoretical consistency in the sense discussed in Section II.

- Late-time acceleration consistent with supernova luminosity distances, baryon acoustic oscillations and other data that constrain the expansion history.

- A matter-dominated era with an evolution of the scale factor $a(t)$ that is consistent with the requirements of structure formation.

- Density perturbations that are consistent with the observed growth factor, matter power spectrum, peculiar velocities, CMB anisotropies and weak lensing power spectrum.

- Stable static spherical solutions for stars, and consistency with terrestrial and solar system observational constraints.

- Consistency with binary pulsar period data.

One of the major challenges is to compute the cosmological perturbations for structure formation in a modified gravity theory. In general relativity, the perturbations are well understood. The perturbed metric in Newtonian gauge is

$$ds^2 = -(1 + 2\Psi)dt^2 + a^2(1 + 2\Phi)d\vec{x}^{\,2}, \tag{23}$$

and the metric potentials define two important combinations:

$$\Phi_+ = \frac{1}{2}(\Phi + \Psi) \,, \quad \Phi_- = \frac{1}{2}(\Phi - \Psi) \,. \tag{24}$$

In the Newtonian limit $\Psi = -\Phi = -\Phi_-$ is the ordinary Newtonian potential and $\Phi_+ = 0$. The potential Φ_+ is sourced by anisotropic stresses. It vanishes if the gravitational field is entirely due to non-relativistic matter or a perfect fluid. The (comoving) matter density perturbation $\Delta = \delta - 3aHv$ obeys the Poisson and evolution equations on sub-Hubble scales:

$$k^2\Phi = 4\pi G a^2 \rho\Delta \,, \tag{25}$$

$$\ddot{\Delta} + 2H\dot{\Delta} - 4\pi G\rho\Delta = 0 \,. \tag{26}$$

These equations are exact on all scales, if perturbations are purely matter ($w = 0$) and there are no anisotropic stresses. On super-Hubble scales (and for adiabatic perturbations, but in the presence of anisotropic stresses), the evolution of the perturbations is entirely determined by the background [36] (and the anisotropic stresses which relate the potentials Ψ and Φ)

$$\Phi'' - \Psi'' - \frac{H''}{H'}\Phi' - \left(\frac{H'}{H} - \frac{H''}{H'}\right)\Psi = 0 \,, \tag{27}$$

where a prime denotes $d/d\ln a$.

The large-angle anisotropies in the CMB temperature encode a signature of the formation of structure. They are determined by the propagation of photons along the geodesics of the perturbed

geometry. For adiabatic perturbations one obtains on large scales the following expression [37] for the temperature fluctuation in direction $\mathbf{n}$:

$$\frac{\delta T}{T}(\mathbf{n})\bigg|_{SW} = \left[\frac{1}{3}\Psi + \frac{2}{3}H\dot{\Phi} + \frac{1}{4}\Delta_r + \mathbf{V}_b \cdot \mathbf{n}\right](\mathbf{x}_{\text{dec}}, t_{\text{dec}}) - 2\int a\dot{\Phi}_-(\mathbf{x}(v), t(v))\, dv\,. \tag{28}$$

The integral is along the (unperturbed) trajectory of the light ray with affine parameter v, from last scattering to today. The position at decoupling, $\mathbf{x}_{\text{dec}}$, depends on $\mathbf{n}$. The same integral also determines the weak lensing signal, since the deflection angle is given by (see e.g. [37])

$$\vec{\alpha} = 2\int \vec{\nabla}_\perp \Phi_-\, dv\,, \tag{29}$$

where $\nabla_\perp$ is the gradient operator in the plane normal to $\mathbf{n}$.

The first term in the square brackets of Eq. (28) is called the ordinary Sachs Wolfe effect (OSW), the second term is usually small since at the time of decoupling the Universe is matter dominated and this term vanishes in a purely matter dominated Universe. The third term is responsible for the acoustic peaks in the CMB anisotropy spectrum and the fourth term is the Doppler term, due to the motion of the emitting electrons, $\mathbf{V}_b$ is the baryon velocity field. The integral is the integrated Sachs Wolfe effect (ISW). It comes from the fact that the photons are blue shifted when they fall into a gravitational potential and redshifted when they climb out of it. Hence if the potential varies during this time, they acquire a net energy shift.

In a modified gravity theory, which we assume to be a metric theory obeying energy-momentum conservation, Eq. (23) still holds, and so does the super-Hubble evolution equation (27), and the SW and lensing relations (28) and (29). But in general

$$\Phi_+ \neq 0\,, \tag{30}$$

even in the absence of matter anisotropic stress – the modified-gravity effects produce a "dark" anisotropic stress. In addition, the Poisson equation and the evolution of density perturbations will be modified.

A. $f(R)$ AND SCALAR-TENSOR THEORIES

General relativity has a unique status as a theory where gravity is mediated by a massless spin-2 particle, and the field equations are second order. Consider modifications to the Einstein-Hilbert action of the general form

$$-\int d^4x\, \sqrt{-g}\, R \;\rightarrow\; -\int d^4x\, \sqrt{-g}\, f(R, R_{\mu\nu}R^{\mu\nu}, C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta})\,, \tag{31}$$

where $R_{\mu\nu}$ is the Ricci tensor, $C_{\mu\nu\alpha\beta}$ is the Weyl tensor and $f(x_1, x_2, x_3)$ is an arbitrary (at least three times differentiable) function. Since the curvature tensors contain second derivatives of the metric, the resulting equations of motion will in general be fourth order, and gravity is carried also by massless spin-0 and spin-1 fields. However Ostrogradski's theorem applies: The usual Hamiltonian formulation of general relativity leads to six independent metric components g_{ij} which all acquire higher derivative terms. There is actually only one way out, which is the case $\partial_2 f = \partial_3 f = 0$, i.e., f may only depend on the Ricci scalar.[2] The reason is that in the Ricci scalar R, only a single component of the metric contains second derivatives. In this case, the consequent new degree of freedom can be fixed completely by the g_{00} constraint, so that the only instability in $f(R)$ theories is the usual one associated with gravitational collapse [16].

---

[2] Another possibility is the addition of a Gauss Bonnet term, $\sqrt{-g}f(L_{GB})$, where $L_{GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho}$. In four dimensions $\sqrt{-g}L_{GB}$ contributes only a surface term and does not enter the equations of motion. However, $\sqrt{-g}f(L_{GB})$ is non-trivial. Such a term also becomes interesting in scalar-tensor theories of gravity where one may consider a contribution of the form $\sqrt{-g}\phi L_{GB}$ to the Lagrangian.
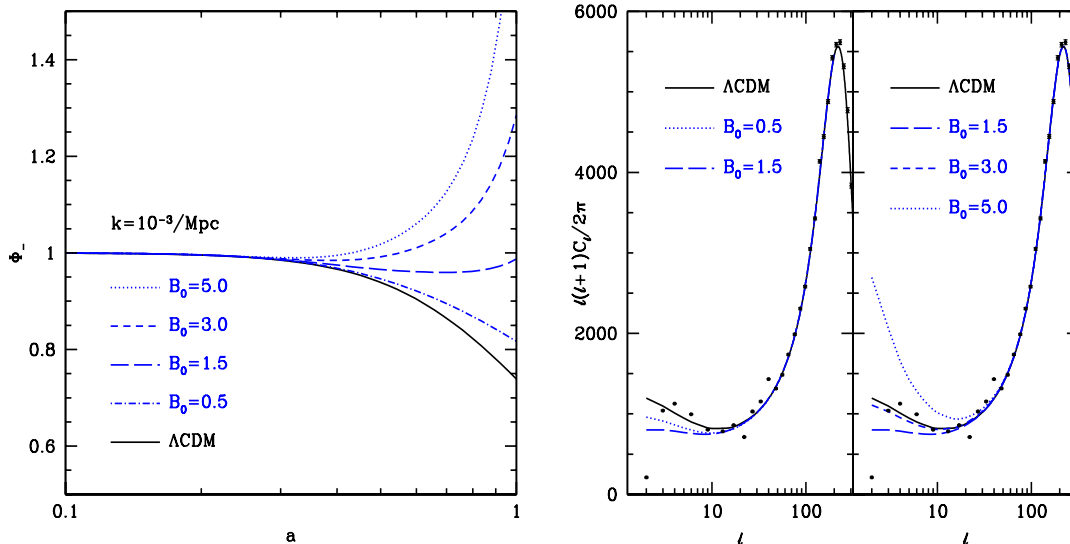
FIG. 3: *Left:* The ISW potential, $(\Phi - \Psi)/2$, for $f(R)$ models, where the parameter B_0 indicates the strength of deviation from general relativity [see Eq. (43)].
Right: The large-angle CMB anisotropies for the models of the left figure. (For more details see [43], where this figure is taken from.)

Therefore, the only acceptable low-energy generalizations of the Einstein-Hilbert action of general relativity are $f(R)$ theories, with $f''(R) \neq 0$. The field equations are

$$f'(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - \left[\nabla_\mu\nabla_\nu - g_{\mu\nu}\nabla^\alpha\nabla_\alpha\right]f'(R) = 8\pi G T_{\mu\nu}\,, \tag{32}$$

and standard energy-momentum conservation holds:

$$\nabla_\nu T^{\mu\nu} = 0\,. \tag{33}$$

The trace of the field equations is a wave-like equation for f', with source term $T = T_\mu{}^\mu$:

$$3\nabla^\alpha\nabla_\alpha f'(R) + Rf'(R) - 2f(R) = 8\pi G T\,. \tag{34}$$

This equation is important for investigating issues of stability in the theory, and it also implies that Birkhoff's theorem does not hold.

There has been a revival of interest in $f(R)$ theories due to their ability to produce late-time acceleration [38]. However, it turns out to be extremely difficult for this simplified class of modified theories to pass the observational and theoretical tests. A simple example of an $f(R)$ model is [39]

$$f(R) = R - \frac{\mu}{R}\,. \tag{35}$$

For $|\mu| \sim H_0^4$, this model successfully achieves late-time acceleration as the μ/R term starts to dominate. But the model strongly violates solar system constraints, can have a strongly non-standard matter era before the late-time acceleration, and suffers from nonlinear matter instabilities [41].

In $f(R)$ theories, the additional degree of freedom can be interpreted as a scalar field, and in this sense, $f(R)$ theories are mathematically equivalent to scalar-tensor theories via

$$\psi \equiv f'(R)\,, \quad U(\psi) \equiv -\psi R(\psi) + f(R(\psi))\,, \tag{36}$$

$$L = -\frac{1}{16\pi G}\sqrt{-g}\left[\psi R + U(\psi)\right]\,. \tag{37}$$

This Lagrangian is the Jordan-frame representation of $f(R)$. It can be conformally transformed to the Einstein frame, via the transformation

$$\tilde{g}_{\mu\nu} = \psi g_{\mu\nu}\,, \quad \varphi = \sqrt{\frac{3}{4\pi G}}\ln\psi\,. \tag{38}$$

In terms of $\tilde{g}_{\mu\nu}$ and φ the Lagrangian then becomes a standard scalar field Lagrangian,

$$L = \frac{-1}{16\pi G} \sqrt{-\tilde{g}} \left[\tilde{R} + \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + V(\varphi) \right] , \tag{39}$$

where

$$V(\varphi) = \frac{U(\psi(\varphi))}{\psi(\varphi)^2} . \tag{40}$$

This example shows that modifying gravity (dark gravity) or modifying the energy momentum tensor (dark energy) can be seen as a different description of the same physics. Only the coupling of the scalar field φ to ordinary matter, shows that this theory originates from a scalar-tensor theory of gravity – and this non-standard coupling reflects the fact that gravity is also mediated by a spin-0 degree of freedom, in contrast to general relativity with a standard scalar field.

The spin-0 field is precisely the cause of the problem with solar system constraints in most $f(R)$ models, since the requirement of late-time acceleration leads to a very light mass for the scalar. The modification to the growth of large-scale structure due to this light scalar may be kept within observational limits. But on solar system scales, the coupling of the light scalar to the sun and planets, induces strong deviations from the weak-field Newtonian limit of general relativity, in obvious violation of observations. In terms of the Lagrangian (39) this scalar has an associated Brans-Dicke parameter that vanishes, $\omega_{BD} = 0$, whereas solar system and binary pulsar data currently require $\omega_{BD} > 40000$.

The only way to evade this problem is to increase the mass of the scalar near massive objects like the sun, so that the Newtonian limit can be recovered, while preserving the ultralight mass on cosmological scales. This "chameleon" mechanism can be used to construct models that evade solar system/ binary pulsar constraints [42]. However the price to pay is that additional parameters must be introduced, and the chosen $f(R)$ tends to look unnatural and strongly fine-tuned. An example is

$$f(R) = R + \lambda R_0 \left[\left(1 + \frac{R^2}{R_0^2} \right)^{-n} - 1 \right], \tag{41}$$

where λ, R_0, n are positive parameters.

Cosmological perturbations in $f(R)$ theory are well understood [40]. The modification to general relativity produces a dark anisotropic stress

$$\Phi_+ \propto \frac{f''(R)}{f'(R)} , \tag{42}$$

and deviations from general relativity are conveniently characterized by the dimensionless parameter

$$B = \frac{dR/d\ln a}{d\ln H/d\ln a} \frac{f''(R)}{f'(R)} . \tag{43}$$

If we invoke a chameleon mechanism, then it is possible for these models to match the observed large-angle CMB anisotropies (see Fig. 3) and linear matter power spectrum [43]. However, there may also be fatal problems with singularities in the strong gravity regime, which would be incompatible with the existence of neutron stars [44]. These problems appear to arise in the successful chameleon models, and they are another unintended, and unexpected, consequence of the scalar degree of freedom, this time at high energies.

It is possible that an ultraviolet completion of the theory will cure the high-energy singularity problem. If we assume this to be the case, then $f(R)$ models that pass the solar system and late-time acceleration tests are valuable working models for probing the features of modified gravity theories and for developing tests of general relativity itself. In order to pursue this programme, one needs to compute not only the linear cosmological perturbations and their signature in the growth factor, the matter power spectrum and the CMB anisotropies – but also the weak lensing signal. For this, we need the additional step of understanding the transition from the linear to the nonlinear regime. Scalar-tensor behaviour on cosmological scales relevant to structure formation in

the linear regime, must evolve to Newtonian-like behaviour on small scales in the nonlinear regime – otherwise we cannot recover the general relativistic limit in the solar system. This means that the standard fitting functions in general relativity cannot be applied, and we require the development of N-body codes in $f(R)$ theories [46].

More general scalar-tensor theories [45], which may also be motivated via low-energy string theory, have an action of the form

$$ -\int d^4x \sqrt{-g} \left[F(\psi)R + \frac{1}{2}g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi + U(\psi) \right], \tag{44} $$

where ψ is the spin-0 field supplementing the spin-2 graviton. In the context of late-time acceleration, these models are also known as "extended quintessence". Scalar-tensor theories contain two functions, F and U. This additional freedom allows for greater flexibility in meeting the observational and theoretical constraints. However, the price we pay is additional complexity – and arbitrariness. The $f(R)$ theories have one arbitrary function, and here there are two, $F(\psi)$ and $U(\psi)$. There is no preferred choice of these functions from fundamental theory.

Modifications of the Einstein-Hilbert action, which lead to fourth-order field equations, either struggle to meet the minimum requirements in the simplest cases, or contain more complexity and arbitrary choices than quintessence models in general relativity. Therefore, none of these models appears to be a serious competitor to quintessence in general relativity.

B. BRANE-WORLD MODELS

Modifications to general relativity within the framework of quantum gravity are typically ultra-violet corrections that must arise at high energies in the very early universe or during collapse to a black hole. The leading candidate for a quantum gravity theory, string theory, is able to remove the infinities of quantum field theory and unify the fundamental interactions, including gravity. But there is a price – the theory is only consistent in 9 space dimensions. Branes are extended objects of higher dimension than strings, and play a fundamental role in the theory, especially D-branes, on which open strings can end. Roughly speaking, the endpoints of open strings, which describe the standard model particles like fermions and gauge bosons, are attached to branes, while the closed strings of the gravitational sector can move freely in the higher-dimensional "bulk" space-time. Classically, this is realised via the localization of matter and radiation fields on the brane, with gravity propagating in the bulk (see Fig. 4).
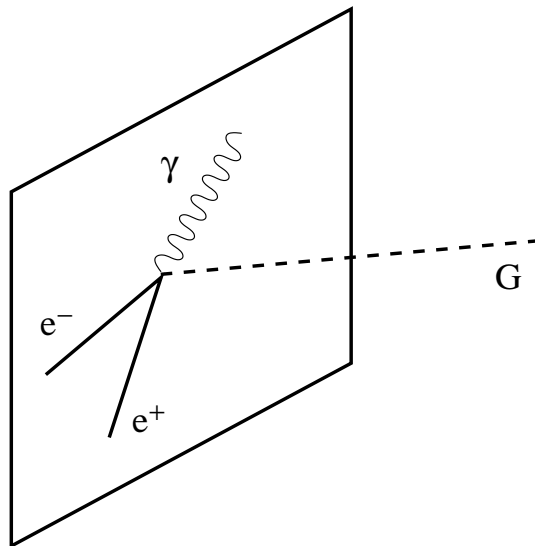

FIG. 4: The confinement of matter to the brane, while gravity propagates in the bulk (from [47]).

The implementation of string theory in cosmology is extremely difficult, given the complexity of the theory. This motivates the development of phenomenological models, as an intermediary between observations and fundamental theory. Brane-world cosmological models inherit some aspects of string theory, but do not attempt to impose the full machinery of the theory. Instead, simplifications are introduced in order to be able to construct cosmological models that can be used to compute observational predictions (see [48] for reviews in this spirit). Cosmological data can then be used to constrain the brane-world models, and hopefully provide constraints on string theory, as well as pointers for the further development of string theory.

It turns out that even the simplest (5D, we effectively assume that 5 of the extra dimensions in the "parent" string theory may be ignored at low energies) brane-world models are remarkably rich – and the computation of their cosmological perturbations is complicated, and in many cases still incomplete. A key reason for this is that the higher-dimensional graviton produces a tower of 4-dimensional massive spin-0, spin-1 and spin-2 modes on the brane, in addition to the standard massless spin-2 mode on the brane (or in some cases, instead of the massless spin-2 mode). In the case of some brane models, there are in addition a massless gravi-scalar and gravi-vector which modify the dynamics.

Most brane-world models modify general relativity at high energies. The main examples are those of Randall-Sundrum (RS) type [51], where a FRW brane is embedded in a 5D anti de Sitter bulk, with curvature radius ℓ. At low energies $H\ell \ll 1$, the zero-mode of the graviton dominates on the brane, and general relativity is recovered to a good approximation. At high energies, $H\ell \gg 1$, the massive modes of the graviton dominate over the zero-mode, and gravity on the brane behaves increasingly five-dimensional. On the brane, the standard conservation equation holds,

$$\dot{\rho} + 3H(\rho + p) = 0 \,, \tag{45}$$

but the Friedmann equation is modified by an ultraviolet correction:

$$H^2 + \frac{K}{a^2} = \frac{8\pi G}{3}\rho\left(1 + \frac{2\pi G\ell^2}{3}\rho\right) + \frac{\Lambda}{3}\,. \tag{46}$$

The ρ^2 term is the ultraviolet correction. At low energies, this term is negligible, and we recover $H^2 + K/a^2 \propto \rho + \Lambda/8\pi G$. At high energies, gravity "leaks" off the brane and $H^2 \propto \rho^2$. This 5D behaviour means that a given energy density produces a greater rate of expansion than it would in general relativity. As a consequence, inflation in the early universe is modified in interesting ways [48].

By contrast, the brane-world model of Dvali-Gabadadze-Porrati [49] (DGP), which was generalized to cosmology by Deffayet [50], modifies general relativity at *low* energies. This model produces 'self-acceleration' of the late-time universe due to a weakening of gravity at low energies. Like the RS model, the DGP model is a 5D model with infinite extra dimension.

The action is given by

$$\frac{-1}{16\pi G}\left[\frac{1}{r_c}\int_{\text{bulk}} d^5x\,\sqrt{-g^{(5)}}\,R^{(5)} + \int_{\text{brane}} d^4x\,\sqrt{-g}\,R\right]\,. \tag{47}$$

The bulk is assumed to be 5D Minkowski spacetime. Unlike the AdS bulk of the RS model, the Minkowski bulk has infinite volume. Consequently, there is no normalizable zero-mode of the 4D graviton in the DGP brane-world. Gravity leaks off the 4D brane into the bulk at large scales, $r \gg r_c$, where the first term in the sum (47) dominates. On small scales, gravity is effectively bound to the brane and 4D dynamics is recovered to a good approximation, as the second term dominates. The transition from 4D to 5D behaviour is governed by the crossover scale r_c. For a Minkowski brane, the weak-field gravitational potential behaves as

$$\Psi \propto \begin{cases} r^{-1} & \text{for } r \ll r_c \\ r^{-2} & \text{for } r \gg r_c \end{cases} \tag{48}$$

On a Friedmann brane, gravity leakage at late times in the cosmological evolution can initiate acceleration – not due to any negative pressure field, but due to the weakening of gravity on the brane. 4D gravity is recovered at high energy via the lightest massive modes of the 5D graviton, effectively via an ultra-light metastable graviton.
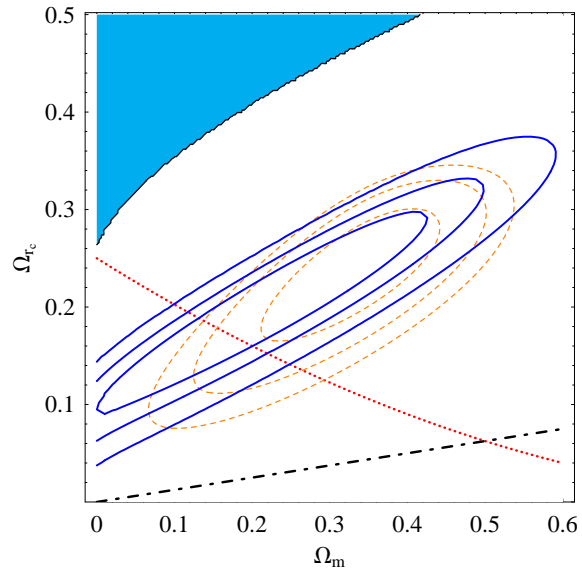
FIG. 5: The confidence contours for supernova data in the DGP density parameter plane. The blue (solid) contours are for SNLS data, and the brown (dashed) contours are for the Gold data. The red (dotted) curve defines the flat models, the black (dot-dashed) curve defines zero acceleration today, and the shaded region contains models without a big bang. (From [52].)
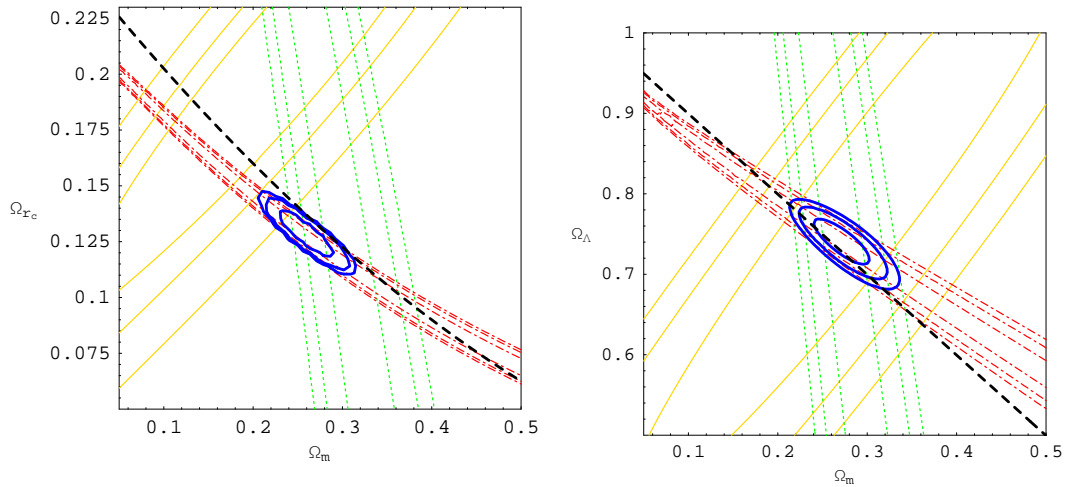

FIG. 6: Joint constraints [solid thick (blue)] from the SNLS data [solid thin (yellow)], the BAO peak at $z = 0.35$ [dotted (green)] and the CMB shift parameter from WMAP3 [dot-dashed (red)]. The left plot show DGP models , the right plot shows LCDM. The thick dashed (black) line represents the flat models, $\Omega_K = 0$. (From [52].)

The energy conservation equation remains the same as in general relativity, but the Friedmann equation is modified:

$$\dot{\rho} + 3H(\rho + p) \;=\; 0 \,, \tag{49}$$

$$H^2 + \frac{K}{a^2} - \frac{1}{r_c}\sqrt{H^2 + \frac{K}{a^2}} \;=\; \frac{8\pi G}{3}\rho \,. \tag{50}$$

To arrive at Eq. (50) we have to take a square root which implies a choice of sign. As we shall see,

the above choice has the advantage of leading to acceleration but the disadvantage of the presence of a 'ghost' in this background. It is not clear whether these facts are related. We shall discuss the 'normal' DGP model, where the opposite sign of the square root is chosen in the next section.

From Eq. (50) we infer that at early times, i.e., $Hr_c \gg 1$, the general relativistic Friedman equation is recovered. By contrast, at late times in an expanding CDM universe, with $\rho \propto a^{-3} \to 0$, we have

$$H \to H_\infty = \frac{1}{r_c}, \tag{51}$$

so that expansion accelerates and is asymptotically de Sitter. The above equations imply

$$\dot{H} - \frac{K}{a^2} = -4\pi G\rho \left[1 + \frac{1}{\sqrt{1 + 32\pi Gr_c^2\rho/3}}\right]. \tag{52}$$

In order to achieve self-acceleration at late times, we require

$$r_c \gtrsim H_0^{-1}, \tag{53}$$

since $H_0 \lesssim H_\infty$. This is confirmed by fitting supernova observations, as shown in Fig. 5. The dimensionless cross-over parameter is defined as

$$\Omega_{r_c} = \frac{1}{4(H_0 r_c)^2}, \tag{54}$$

and the LCDM relation,

$$\Omega_m + \Omega_\Lambda + \Omega_K = 1, \tag{55}$$

is modified to

$$\Omega_m + 2\sqrt{\Omega_{r_c}}\sqrt{1 - \Omega_K} + \Omega_K = 1. \tag{56}$$

LCDM and DGP can both account for the supernova observations, with the fine-tuned values $\Lambda \sim H_0^2$ and $r_c \sim H_0^{-1}$ respectively. When we add further constraints on the expansion history from the baryon acoustic oscillation peak at $z = 0.35$ and the CMB shift parameter, the DGP flat models are in strong tension with data, whereas LCDM models provide a consistent fit. This is evident in Fig. 6. The open DGP models provide a somewhat better fit to the geometric data – essentially because the lower value of Ω_m favoured by supernovae reduces the distance to last scattering and an open geometry is able to extend that distance. For a combination of SNe, CMB shift and Hubble Key Project data, the best-fit open DGP also performs better than the flat DGP [53], as shown in Fig. 7.

Observations based on structure formation provide further evidence of the difference between DGP and LCDM, since the two models suppress the growth of density perturbations in different ways [54]. The distance-based observations draw only upon the background 4D Friedman equation (50) in DGP models – and therefore there are quintessence models in general relativity that can produce precisely the same supernova distances as DGP. By contrast, structure formation observations require the 5D perturbations in DGP, and one cannot find equivalent quintessence models [56]. One can find 4D general relativity models, with dark energy anisotropic stress and variable sound speed, that can in principle mimic DGP [58]. However, these models are highly unphysical and can be discounted on grounds of theoretical consistency.

For LCDM, the analysis of density perturbations is well understood. For DGP the perturbations are much more subtle and complicated [13]. Although matter is confined to the 4D brane, gravity is fundamentally 5D, and the 5D bulk gravitational field responds to and back-reacts on 4D density perturbations. The evolution of density perturbations requires an analysis based on the 5D nature of gravity. In particular, the 5D gravitational field produces an effective "dark" anisotropic stress on the 4D universe. If one neglects this stress and other 5D effects, and simply treats the perturbations as 4D perturbations with a modified background Hubble rate – then as a consequence, the 4D Bianchi identity on the brane is violated, i.e., $\nabla^\nu G_{\mu\nu} \neq 0$, and the results are inconsistent. When the 5D effects are incorporated [56, 57], the 4D Bianchi identity is automatically satisfied. (See Fig 8.)

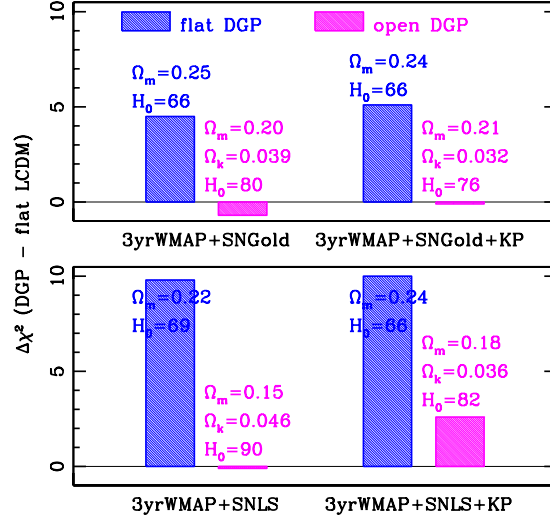There are three regimes governing structure formation in DGP models:

FIG. 7: The difference in χ^2 between best-fit DGP (flat and open) and best-fit (flat) LCDM, using SNe, CMB shift and H_0 Key Project data. (From [53].)

- On small scales, below the so-called Vainshtein radius (which for cosmological purposes is roughly the scale of clusters), the spin-0 scalar degree of freedom becomes strongly coupled, so that the general relativistic limit is recovered [59].

- On scales relevant for structure formation, i.e. between cluster scales and the Hubble radius, the spin-0 scalar degree of freedom produces a scalar-tensor behaviour. A quasi-static approximation to the 5D perturbations shows that DGP gravity is like a Brans-Dicke theory with parameter [56]

$$\omega_{BD} = \frac{3}{2}(\beta - 1),\tag{57}$$

where

$$\beta = 1 + 2H^2 r_c \left(H^2 + \frac{K}{a^2}\right)^{-1/2}\left[1 + \frac{\dot{H}}{3H^2} + \frac{2K}{3a^2 H^2}\right].\tag{58}$$

At late times in an expanding universe, when $Hr_c \gtrsim 1$, it follows that $\beta < 1$, so that $\omega_{BD} < 0$. (This signals a pathology in DGP which is discussed below.)

- Although the quasi-static approximation allows us to analytically solve the 5D wave equation for the bulk degree of freedom, this approximation breaks down near and beyond the Hubble radius. On super-horizon scales, 5D gravity effects are dominant, and we need to solve numerically the partial differential equation governing the 5D bulk variable [57].

On sub-horizon scales relevant for linear structure formation, 5D effects produce a difference between Φ an $-\Psi$:

$$k^2\Phi = 4\pi Ga^2\left(1 - \frac{1}{3\beta}\right)\rho\Delta\,,\tag{59}$$

$$k^2\Psi = -4\pi Ga^2\left(1 + \frac{1}{3\beta}\right)\rho\Delta\,,\tag{60}$$

so that there is an effective dark anisotropic stress on the brane:

$$k^2(\Phi + \Psi) = -\frac{8\pi Ga^2}{3\beta^2}\rho\Delta\,.\tag{61}$$
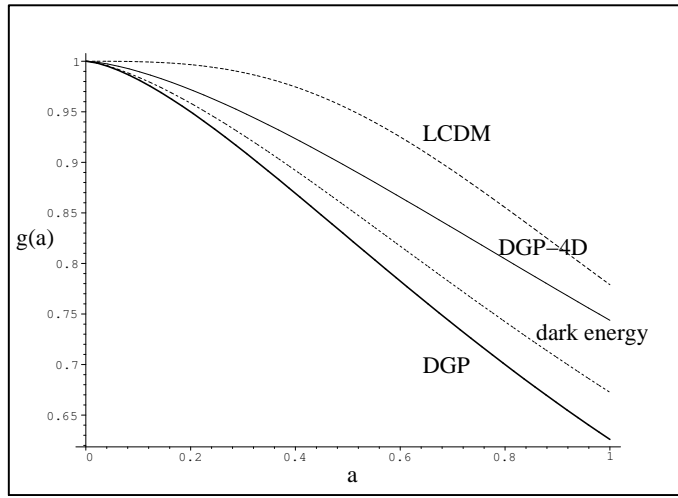
FIG. 8: The growth factor $g(a) = \Delta(a)/a$ for LCDM (long dashed) and DGP (solid, thick), as well as for a dark energy model with the same expansion history as DGP (short dashed). DGP-4D (solid, thin) shows the incorrect result in which the 5D effects are set to zero. (From [56].)

The density perturbations evolve as

$$\ddot{\Delta} + 2H\dot{\Delta} - 4\pi G \left(1 - \frac{1}{3\beta} \right) \rho\Delta = 0 \,. \tag{62}$$

The linear growth factor, $g(a) = \Delta(a)/a$ (i.e., normalized to the flat CDM case, $\Delta \propto a$), is shown in Fig. 8. This shows the dramatic suppression of growth in DGP relative to LCDM – from both the background expansion and the metric perturbations. If we parameterize the growth factor in the usual way, we can quantify the deviation from general relativity with smooth dark energy [55]:

$$f := \frac{d\ln\Delta}{d\ln a} = \Omega_m(a)^{\gamma}, \quad \gamma \approx \begin{cases} 0.55 + 0.05[1 + w(z = 1)] & \text{GR, smooth DE} \\ 0.68 & \text{DGP} \end{cases} \tag{63}$$

Observational data on the growth factor [60] are not yet precise enough to provide meaningful constraints on the DGP model. Instead, we can look at the large-angle anisotropies of the CMB, i.e. the ISW effect. This requires a treatment of perturbations near and beyond the horizon scale. The full numerical solution has been given by [57], and is illustrated in Fig. 9. The CMB anisotropies are also shown in Fig. 9, as computed in [62] using a scaling approximation to the super-Hubble modes [61] (the accuracy of the scaling ansatz is verified by the numerical results [57]).

It is evident from Fig. 9 that the DGP model which provides a best fit to the geometric data (see Fig. 7), is in serious tension with the WMAP5 data on large scales. The problem arises form the large deviation of $\Phi_- = (\Phi - \Psi)/2$ in the DGP model from the LCDM model. This deviation, i.e. a stronger decay of Φ_-, leads to an over-strong ISW effect [see Eq. (28)], in tension with WMAP5 observations.

As a result of the combined observations of background expansion history and large-angle CMB anisotropies, the DGP model provides a worse fit to the data than LCDM at about the 5σ level [62]. Effectively, the DGP model is ruled out by observations in comparison with the LCDM model.

In addition to the severe problems posed by cosmological observations, a problem of theoretical consistency is posed by the fact that the late-time asymptotic de Sitter solution in DGP cosmological models has a ghost. The ghost is signaled by the negative Brans-Dicke parameter in the effective theory that approximates the DGP on cosmological sub-horizon scales:

$$\omega_{BD} < 0 \,. \tag{64}$$

The existence of the ghost is confirmed by detailed analysis of the 5D perturbations in the de Sitter limit [63, 64]. The DGP ghost is a ghost mode in the scalar sector of the gravitational field – which is more serious than the ghost in a phantom scalar field. It effectively rules out the DGP, since it is hard to see how an ultraviolet completion of the DGP can cure the *infrared* ghost problem..
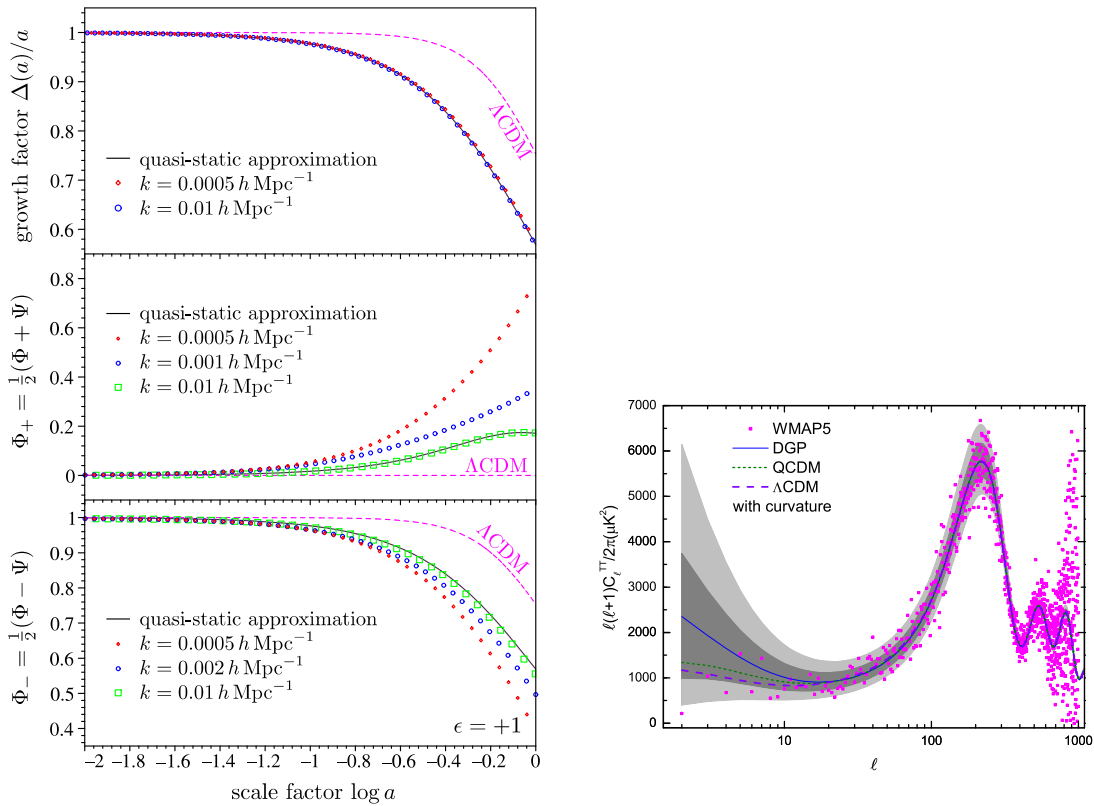
FIG. 9: *Left:* Numerical solutions for DGP density and metric perturbations, showing also the quasistatic solution, which is an increasingly poor approximation as the scale is increased. (From [57].)
Right: Constraints on DGP (the open model that provides a best fit to geometric data) from CMB anisotropies (WMAP5). The DGP model is the solid curve, QCDM (short-dashed curve) is the quintessence model with the same background expansion history as the DGP model, and LCDM is the dashed curve (a slightly closed model that gives the best fit to WMAP5, HST and SNLS data). (From [62].)

C. DEGRAVITATION AND NORMAL DGP

The self-accelerating DGP is effectively ruled out as a cosmological model by observations and by the problem of the ghost in the gravitational sector. Indeed, it may be the case that self-acceleration comes with the price of ghost states. An alternative idea is that massive-graviton theories (like the DGP) may lead to *degravitation* [65], i.e., the feature that the vacuum energy (cosmological constant), does not gravitate at the level expected [as in Eq. (7)], and possibly not at all.

To achieve a reduction of gravitation on very large scales, degravitation, Newton's constant is promoted to a 'high-pass filter' and Einstein's equations are modified to

$$G^{-1}(L^2\Box)G_{\mu\nu} = 8\pi T_{\mu\nu} \ . \tag{65}$$

We want $G(L^2\Box)$ to act as a high pass filter: for scales smaller than L it is constant while scales much larger than L are filtered out, degravitated. For this to work, G^{-1} must contain inverse powers of $\Box$, hence it must be non-local. Furthermore, this equation cannot describe a massless spin 2 graviton with only two degrees of freedom, but it leads, at the linear level to massive gravitons with mass $1/L$ or a superposition (spectral density) of massive gravitons. These are known to carry three additional polarizations two of helicity 1 and one helicity 0 state. The latter couples to the trace of the energy momentum tensor and remains present also in the zero-mass limit, the well known van Dam-Veltman-Zakharov discontinuity of massive gravity [66]. This problem might be solved on small scales, where the extra polarizations become strongly coupled due to non-linear self interactions [67]. One can show that in regions where the curvature exceeds L^{-2}, the extra
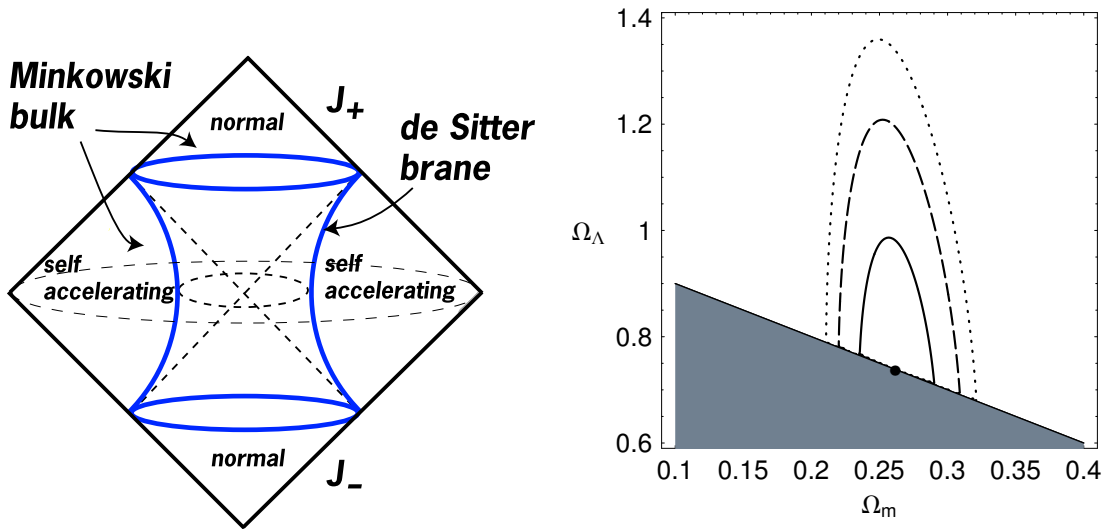
FIG. 10: *Left:* The embedding of the self-accelerating and normal branches of the DGP brane in a Minkowski bulk. (From [64].)
Right: Joint constraints on normal DGP (flat, $K = 0$) from SNLS, CMB shift (WMAP3) and BAO ($z = 0.35$) data. The best-fit is the solid point, and is indistinguishable from the LCDM limit. The shaded region is unphysical and its upper boundary represents flat LCDM models. (From [70].)

polarizations are suppressed by powers of L and we recover ordinary spin-2 gravity.

Contrary to the models discussed so far, these theories can in principle address the cosmological constant problem: the cosmological constant is not necessarily small, but we cannot see it in gravitational experiments since it is (nearly) degravitated. On the other hand, the problem of the present cosmological acceleration is not addressed.

Apart from a simple massive graviton, the simplest example of degravitation is provided by the so-called "normal" (i.e., non-self-accelerating and ghost free) branch of the DGP [68], which arises from a different embedding of the DGP brane in the Minkowski bulk (see Fig. 10). In the background dynamics, this amounts to a replacement $r_c \rightarrow -r_c$ in Eq. (50) – and there is no longer late-time self-acceleration. It is therefore necessary to include a Λ term in order to accelerate the late universe:

$$H^2 + \frac{K}{a^2} + \frac{1}{r_c}\sqrt{H^2 + \frac{K}{a^2}} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} \,. \tag{66}$$

(Normal DGP models with a quintessence field have also been investigated [69].) Using the dimensionless crossover parameter defined in Eq. (54), the densities are related at the present time by

$$\sqrt{1 - \Omega_K} = -\sqrt{\Omega_{r_c}} + \sqrt{\Omega_{r_c} + \Omega_m + \Omega_\Lambda} \,, \tag{67}$$

which can be compared with the self-accelerating DGP relation (56).

The degravitation feature of normal DGP is that Λ is effectively screened by 5D gravity effects. This follows from rewriting the modified Friedmann equation (66) in standard general relativistic form, with

$$\Lambda_{\text{eff}} = \Lambda - \frac{3}{r_c}\sqrt{H^2 + \frac{K}{a^2}} < \Lambda \,. \tag{68}$$

Thus 5D gravity in normal DGP can in principle reduce the bare vacuum energy significantly. However, figure 10 shows that best-fit flat models, using geometric data, only admit insignificant screening [70]. The closed models provide a better fit to the data [71], and can allow a bare vacuum energy term with $\Omega_\Lambda > 1$, as shown in Fig. 11. This does not address the fundamental problem
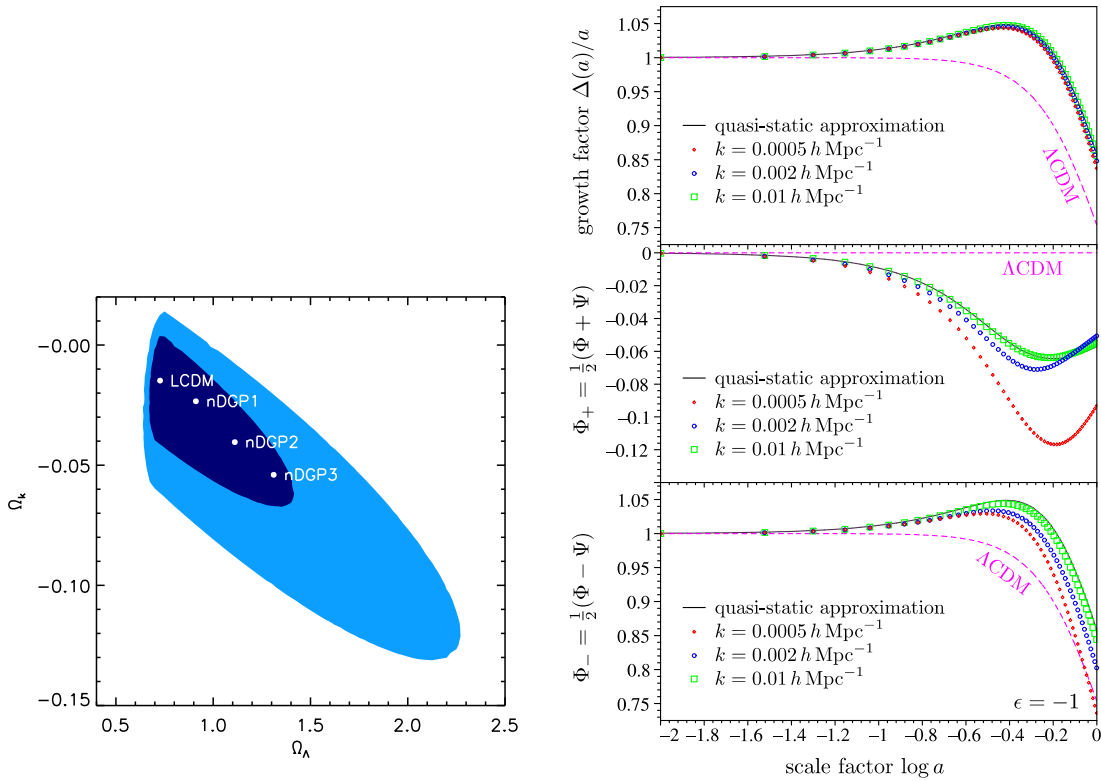
FIG. 11: *Left:* Joint constraints on normal DGP from SNe Gold, CMB shift (WMAP3) and H_0 data in the projected curvature-Λ plane, after marginalizing over other parameters. The best-fits are the solid points, corresponding to different values of Ω_m. (From [71].)
Right: Numerical solutions for the normal DGP density and metric perturbations, showing also the quasistatic solution, which is an increasingly poor approximation as the scale is increased. Compare with the self-accelerating DGP case in Fig. 9. (From [57].)

of the smallness of Ω_Λ, but it is nevertheless an interesting feature. We can define an effective equation of state parameter via

$$\dot{\Lambda}_{\rm eff} + 3H(1 + w_{\rm eff})\Lambda_{\rm eff} = 0 \,. \tag{69}$$

At the present time (setting $K = 0$ for simplicity),

$$w_{\rm eff,0} = -1 - \frac{(\Omega_m + \Omega_\Lambda - 1)\Omega_m}{(1 - \Omega_m)(\Omega_m + \Omega_\Lambda + 1)} < -1 \,, \tag{70}$$

where the inequality holds since $\Omega_m < 1$. This reveals another important property of the normal DGP model: effective phantom behaviour of the recent expansion history. This is achieved without any pathological phantom field (similar to what can be done in scalar-tensor theories [45]). Furthermore, there is no "big rip" singularity in the future associated with this phantom acceleration, unlike the situation that typically arises with phantom fields. The phantom behaviour in the normal DGP model is also not associated with any ghost problem – indeed, the normal DGP branch is free of the ghost that plagues the self-accelerating DGP [64].

Perturbations in the normal branch have the same structure as those in the self-accelerating branch, with the same regimes – i.e. below the Vainshtein radius (recovering a GR limit), up to the Hubble radius (Brans-Dicke behaviour), and beyond the Hubble radius (strongly 5D behaviour). The quasistatic approximation and the numerical integrations can be simply repeated with the replacement $r_c \to -r_c$ (and the addition of Λ to the background). In the sub-Hubble regime, the effective Brans-Dicke parameter is still given by Eqs. (57) and (58), but now we have $\omega_{BD} > 0$ – and this is consistent with the absence of a ghost. Furthermore, a positive Brans-Dicke parameter

signals an extra positive contribution to structure formation from the scalar degree of freedom, so that there is *less* suppression of structure formation than in LCDM – the reverse of what happens in the self-accelerating DGP. This is confirmed by computations, as illustrated in Fig. 11.

The closed normal DGP models fit the background expansion data reasonably well, as shown in Fig. 11. The key remaining question is how well do these models fit the large-angle CMB anisotropies, which is yet to be computed at the time of writing. The derivative of the ISW potential $\dot{\Phi}_-$ can be seen in Fig. 11, and it is evident that the ISW contribution is negative relative to LCDM at high redshifts, and goes through zero at some redshift before becoming positive. This distinctive behaviour may be contrasted with the behaviour in $f(R)$ models (see Fig. 3): both types of model lead to less suppression of structure than LCDM, but they produce different ISW effects. However, in the limit $r_r \to \infty$, normal DGP tends to ordinary LCDM, hence observations which fit LCDM will always just provide a lower limit for r_c.

V.   CONCLUSION

The evidence for a late-time acceleration of the universe continues to mount, as the number of experiments and the quality of data grow. This revolutionary discovery by observational cosmology, confronts theoretical cosmology with a major crisis – how to explain the origin of the acceleration. The core of this problem may be "handed over" to particle physics, since we require at the most fundamental level, an explanation for why the vacuum energy either has an incredibly small and fine-tuned value, or is exactly zero. Both options violently disagree with naive estimates of the vacuum energy.

If one accepts that the vacuum energy is indeed nonzero, then the dark energy is described by Λ, and the LCDM model is the best current model. The cosmological model requires completion via developments in particle physics that will explain the value of the vacuum energy. In many ways, this is the best that we can do currently, since the alternatives to LCDM, within and beyond general relativity, do not resolve the vacuum energy crisis, and furthermore have no convincing theoretical motivation. None of the contenders so far appears any better than LCDM, and it is fair to say that at the theoretical level, there is as yet no serious challenger to LCDM. One consequence of this is the need to develop better observational tests of LCDM, which could in principle rule it out, e.g. by showing, to some acceptable level of statistical confidence, that $w \neq -1$. However, observations are still quite far from the necessary precision for this.

It remains necessary and worthwhile to continue investigating alternative dark energy and dark gravity models, in order better to understand the space of possibilities, the variety of cosmological properties, and the observational strategies needed to distinguish them. The lack of any consistent and compelling theoretical model means that we need to keep exploring alternatives – and also to keep challenging the validity of general relativity itself on cosmological scales.

We have focused in this chapter on two of the simplest infrared-modified gravity models: the $f(R)$ models (the simplest scalar-tensor models), and the DGP models (the simplest brane-world models). In both types of model, the new scalar degree of freedom introduces severe difficulties at theoretical and observational levels. Strictly speaking, the $f(R)$ models are probably ruled out by the presence of singularities that exclude neutron stars (even if they can match all cosmological observations, including weak lensing). And the DGP models are likely ruled out by the appearance of a ghost in the asymptotic de Sitter state – as well as by a combination of geometric and structure-formation data.

Nevertheless, the intensive investigation of $f(R)$ and DGP models has left an important legacy – in a deeper understanding of

- the interplay between gravity and expansion history and structure;

- the relation between cosmological and local observational constraints;

- the special properties of general relativity itself;

- the techniques needed to distinguish different candidate models, and the limitations and degeneracies within those techniques;

- the development of tests that can probe the validity of general relativity itself on cosmological scales, independent of any particular alternative model.

The last point is one of the most important by-products of the investigation of modified gravity models. It involves a careful analysis of the web of consistency relations that link the background expansion to the evolution of perturbations [72], and opens up the real prospect of testing general general relativity well beyond the solar system and its neighbourhood.

[1] J. Dunkley *et al.* [WMAP Collaboration], arXiv:0803.0586 [astro-ph].
[2] See, e.g., E. J. Copeland, M. Sami and S. Tsujikawa, Int. J. Mod. Phys. D **15**, 1753 (2006) [arXiv:hep-th/0603057];
L. Perivolaropoulos, arXiv:astro-ph/0601014;
S. Nojiri and S. D. Odintsov, Int. J. Geom. Meth. Math. Phys. **4**, 115 (2007) [arXiv:hep-th/0601213];
T. Padmanabhan, AIP Conf. Proc. **861**, 179 (2006) [arXiv:astro-ph/0603114];
N. Straumann, Mod. Phys. Lett. A **21**, 1083 (2006) [arXiv:hep-ph/0604231];
S. Bludman, arXiv:astro-ph/0605198;
J. P. Uzan, arXiv:astro-ph/0605313;
D. Polarski, AIP Conf. Proc. **861**, 1013 (2006) [arXiv:astro-ph/0605532];
P. Ruiz-Lapuente, Class. Quant. Grav. **24**, R91 (2007) [arXiv:0704.1058].
[3] G.F.R. Ellis, H. Nicolai, R. Durrer and R. Maartens (eds.), *Special issue on dark energy*, Gen. Rel. Grav. **40**, Nos. 2–3 (2008).
[4] M. Kowalski *et al.*, arXiv:0804.4142 [astro-ph].
[5] K. Enqvist in [3];
R. A. Vanderveld, E. E. Flanagan and I. Wasserman, Phys. Rev. D **74**, 023506 (2006) [arXiv:astro-ph/0602476];
A. Paranjape and T. P. Singh, Class. Quant. Grav. **23**, 6955 (2006) [arXiv:astro-ph/0605195];
R. A. Vanderveld, E. E. Flanagan and I. Wasserman, Phys. Rev. D **78**, 083511 (2008) [arXiv:0808.1080 [astro-ph]];
J. P. Zibin, A. Moss and D. Scott, arXiv:0809.3761 [astro-ph];
J. Garcia-Bellido and T. Haugboelle, arXiv:0810.4939 [astro-ph].
[6] J. Goodman, Phys. Rev. **D52**, 1821 (1995) [arXiv:astro-ph/9506068];
C. Clarkson, B. Bassett and T. C. Lu, Phys. Rev. Lett. **101**, 011301 (2008) [arXiv:0712.3457 [astro-ph]].;
J. P. Uzan, C. Clarkson and G. F. R. Ellis, Phys. Rev. Lett. **100**, 191303 (2008) [arXiv:0801.0068 [astro-ph]];
T. Clifton, P. G. Ferreira and K. Land, Phys. Rev. Lett. **101**, 131302 (2008) [arXiv:0807.1443 [astro-ph]].
[7] G. F. R. Ellis and R. Maartens, Class. Quant. Grav. **21**, 223 (2004) [arXiv:gr-qc/0211082];
A. Lasenby and C. Doran, Phys. Rev. D **71**, 063502 (2005) [arXiv:astro-ph/0307311].
[8] C. Clarkson, M. Cortes and B. A. Bassett, JCAP **0708**, 011 (2007) [arXiv:astro-ph/0702670].
[9] G. Bressi, G. Carugno, R. Onofrio, and G. Ruoso, Phys. Rev. Lett. **88**, 041804 (2002);
M. Bordag, U. Mohideen, and V. M. Mostepanenko, Phys. Rept. **353**, 1 (2001) [arXiv:quant-ph/0106045].
[10] E. Linder in [3].
[11] T. Buchert in [3].
[12] S. Capozziello and M. Francaviglia in [3].
[13] K. Koyama in [3].
[14] R. Durrer and R. Maartens in [3].
[15] M. Ostrogradski, Memoire Academie St. Petersbourg, Ser. VI **4**, 385 (1850).
[16] R. P. Woodard, arXiv:astro-ph/0601672.
[17] C. Bonvin, C. Caprini and R. Durrer, arXiv:0706.1538 [astro-ph].
[18] G. Velo and D. Zwanzinger, Phys. Rev. **186**, 1337 (1969);
G. Velo and D. Zwanzinger, Phys. Rev. **188**, 2218 (1969).
[19] M. S. Morris, K. S. Thorne and U. Yurtsever, Phys. Rev. Lett. **61**, 1446 (1988).
[20] J. R. Gott, Phys. Rev. Lett. **66**, 1126 (1991).
[21] A. Ori, Phys. Rev. **D76**, 044002 (2007) [arXiv:gr-qc/0701024].

[22] W. B. Bonnor B. R. Steadman, Gen. Rel. Grav. **37**, 1833 (2005).

[23] E. Babichev, V. Mukhanov and A. Vikman, JHEP **0802**, 101 (2008) [arXiv:0708.0561 [hep-th]].

[24] M. Froissart, Phys. Rev. **123**, 1053 (1961);
For an overview see, C. Itzykson and J. B. Zuber, *Quantum Field Theory*, Chapter 5, McGraw Hill, New York (1980).

[25] A. Adams, N. Arkani-Hamed, S. Dubovsky, A. Nicolis, and R. Rattazzi, JHEP **0610**, 014 (2006) [arXiv:hep-th/0602178].

[26] J. Polchinski, arXiv:hep-th/0603249;
R. Bousso, arXiv:hep-th/0610211;
R. Bousso in [3].

[27] T. Padmanabhan, Int. J. Mod. Phys. D **15**, 2029 (2006) [arXiv:gr-qc/0609012].

[28] C. Wetterich, Phys. Lett. **B655**, 201 (2007) [arXiv:0706.4477[hep-ph]].

[29] L. Amendola, G. Camargo Campos and R. Rosenfeld, Phys. Rev. D **75**, 083506 (2007) [arXiv:astro-ph/0610806];
Z.K. Guo, N. Ohta and S. Tsujikawa, Phys. Rev. D **76**, 023508 (2007) [arXiv:astro-ph/0702015];
M. Quartin, M. O. Calvao, S. E. Joras, R. R. R. Reis and I. Waga, JCAP **0805**, 007 (2008) [arXiv:0802.0546 [astro-ph]].

[30] T. Koivisto, Phys. Rev. D **72**, 043516 (2005) [arXiv:astro-ph/0504571];
R. Bean, E. E. Flanagan and M. Trodden, New J. Phys. **10**, 033006 (2008) [arXiv:0709.1124 [astro-ph]];
J. Valiviita, E. Majerotto and R. Maartens, JCAP **0807**, 020 (2008) [arXiv:0804.0232 [astro-ph]].

[31] C. Armendariz-Picon, V. Mukhanov and P.J. Steinhardt, Phys. Rev. Lett. **85** 4438 (2000) [arXiv:astro-ph/0004134].

[32] C. Bonvin, C. Caprini and R. Durrer, "A no-go theorem for k-essence dark energy," Phys. Rev. Lett. **97**, 081303 (2006) [arXiv:astro-ph/0606584];
G. Ellis, R. Maartens and M. A. H. MacCallum, Gen. Rel. Grav. **39**, 1651 (2007) [arXiv:gr-qc/0703121].

[33] M. Kunz, arXiv:astro-ph/0702615.

[34] See, e.g.,
E. W. Kolb, S. Matarrese, A. Notari and A. Riotto, arXiv:hep-th/0503117;
G. Geshnizjani, D. J. H. Chung and N. Afshordi, Phys. Rev. D **72**, 023517 (2005) [arXiv:astro-ph/0503553];
C. M. Hirata and U. Seljak, Phys. Rev. D **72**, 083501 (2005) [arXiv:astro-ph/0503582];
E. E. Flanagan, Phys. Rev. D **71**, 103521 (2005) [arXiv:hep-th/0503202];
S. Rasanen, Class. Quant. Grav. **23**, 1823 (2006) [arXiv:astro-ph/0504005];
A. A. Coley, N. Pelavas and R. M. Zalaletdinov, Phys. Rev. Lett. **95**, 151102 (2005) [arXiv:gr-qc/0504115];
H. Alnes, M. Amarzguioui and O. Gron, JCAP **0701**, 007 (2007) [arXiv:astro-ph/0506449];
M. Giovannini, JCAP **0509**, 009 (2005) [arXiv:astro-ph/0506715];
Y. Nambu and M. Tanimoto, arXiv:gr-qc/0507057;
A. Ishibashi and R. M. Wald, Class. Quant. Grav. **23**, 235 (2006) [arXiv:gr-qc/0509108];
T. Buchert, Class. Quant. Grav. **23**, 817 (2006) [arXiv:gr-qc/0509124];
P. Martineau and R. Brandenberger, arXiv:astro-ph/0510523;
R. Mansouri, arXiv:astro-ph/0601699;
J. W. Moffat, arXiv:astro-ph/0603777;
R. A. Vanderveld, E. E. Flanagan and I. Wasserman, Phys. Rev. D **76**, 083504 (2007) [arXiv:0706.1931 [astro-ph]];
E. Rosenthal and E. E. Flanagan, arXiv:0809.2107 [gr-qc];
D. L. Wiltshire, Phys. Rev. D **78**, 084032 (2008) [arXiv:0809.1183 [gr-qc]].

[35] N. Li and D.J. Schwarz, arXiv:gr-qc/0702043 (2007).

[36] E. Bertschinger, Astrophys. J. **648**, 797 (2006) [arXiv:astro-ph/0604485].

[37] R. Durrer, The Cosmic Microwave Background, Cambridge University Press (2008).

[38] T. P. Sotiriou and V. Faraoni, arXiv:0805.1726 [gr-qc].

[39] S. Capozziello, S. Carloni and A. Troisi, Recent Res. Dev. Astron. Astrophys. **1**, 625 (2003) [arXiv:astro-ph/0303041].

[40] Y. S. Song, W. Hu and I. Sawicki, Phys. Rev. D **75**, 044004 (2007) [arXiv:astro-ph/0610532].

[41] See, e.g., A. D. Dolgov and M. Kawasaki, Phys. Lett. B **573**, 1 (2003) [arXiv:astro-ph/0307285].;
R. Bean, D. Bernat, L. Pogosian, A. Silvestri and M. Trodden, Phys. Rev. D **75**, 064020 (2007) [arXiv:astro-ph/0611321];
T. Chiba, T. L. Smith and A. L. Erickcek, Phys. Rev. **D75**, 124014 (2007) [arXiv:astro-ph/0611867];
L. Amendola, R. Gannouji, D. Polarski and S. Tsujikawa, Phys. Rev. **D75**, 083504 (2007) [arXiv:gr-qc/0612180].

[42] W. Hu and I. Sawicki, Phys. Rev. D **76**, 064004 (2007) [arXiv:0705.1158 [astro-ph]];
A. A. Starobinsky, JETP Lett., **86**, 157 (2007) [arXiv:0706.2041v2];
S. Nojiri and S.D. Odintsov, Phys. Lett. B **657**, 238 (2007) [arXiv:0707.1941]

[43] Y. S. Song, H. Peiris and W. Hu, Phys. Rev. D **76**, 063517 (2007) [arXiv:0706.2399 [astro-ph]].

[44] T. Kobayashi and K. i. Maeda, arXiv:0807.2503 [astro-ph];
A. V. Frolov, Phys. Rev. Lett. **101**, 061103 (2008) [arXiv:0803.2500 [astro-ph]].

[45] B. Boisseau, G. Esposito-Farese, D. Polarski and A. A. Starobinsky, Phys. Rev. Lett. **85**, 2236 (2000) [arXiv:gr-qc/0001066];
A. Riazuelo and J. P. Uzan, Phys. Rev. D **66**, 023525 (2002) [arXiv:astro-ph/0107386];
G. Esposito-Farese, AIP Conf. Proc. **736**, 35 (2004) [arXiv:gr-qc/0409081];
S. Nesseris and L. Perivolaropoulos, Phys. Rev. D **75**, 023517 (2007) [arXiv:astro-ph/0611238].

[46] H. Oyaizu, M. Lima and W. Hu, arXiv:0807.2462 [astro-ph].

[47] M. Cavaglia, Int. J. Mod. Phys. A **18**, 1843 (2003) [arXiv:hep-ph/0210296].

[48] R. Maartens, Living Rev. Rel. **7**, 7 (2004) [arXiv:gr-qc/0312059];
P. Brax, C. van de Bruck and A. C. Davis, Rept. Prog. Phys. **67**, 2183 (2004) [arXiv:hep-th/0404011];
V. Sahni, arXiv:astro-ph/0502032;
R. Durrer, AIP Conf. Proc. **782**, 202 (2005) [arXiv:hep-th/0507006];
D. Langlois, Prog. Theor. Phys. Suppl. **163**, 258 (2006) [arXiv:hep-th/0509231];
A. Lue, Phys. Rept. **423**, 1 (2006) [arXiv:astro-ph/0510068];
D. Wands, arXiv:gr-qc/0601078.

[49] G. R. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. B **485**, 208 (2000) [arXiv:hep-th/0005016].

[50] C. Deffayet, Phys. Lett. B **502**, 199 (2001) [arXiv:hep-th/0010186].

[51] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999) [arXiv:hep-th/9906064];
P. Binetruy, C. Deffayet, U. Ellwanger and D. Langlois, Phys. Lett. B **477**, 285 (2000) [arXiv:hep-th/9910219].

[52] R. Maartens and E. Majerotto, Phys. Rev. D **74**, 023004 (2006) [arXiv:astro-ph/0603353].

[53] Y. S. Song, I. Sawicki and W. Hu, Phys. Rev. D **75**, 064003 (2007) [arXiv:astro-ph/0606286].

[54] A. Lue, R. Scoccimarro and G. D. Starkman, Phys. Rev. D **69**, 124015 (2004) [arXiv:astro-ph/0401515];
A. Lue and G. Starkman, Phys. Rev. D **67**, 064002 (2003) [arXiv:astro-ph/0212083].

[55] E. V. Linder, Phys. Rev. D **72**, 043529 (2005) [arXiv:astro-ph/0507263].

[56] K. Koyama and R. Maartens, JCAP **0610**, 016 (2006) [arXiv:astro-ph/0511634].

[57] A. Cardoso, K. Koyama, S. S. Seahra and F. P. Silva, Phys. Rev. D **77**, 083512 (2008) [arXiv:0711.2563 [astro-ph]].

[58] M. Kunz and D. Sapone, Phys. Rev. Lett. **98**, 121301 (2007) [arXiv:astro-ph/0612452].

[59] K. Koyama and F. P. Silva, Phys. Rev. D **75**, 084040 (2007) [arXiv:hep-th/0702169].

[60] L. Guzzo *et al.*, Nature **451**, 541 (2008) [arXiv:0802.1944 [astro-ph]].

[61] I. Sawicki, Y. S. Song and W. Hu, Phys. Rev. D **75**, 064002 (2007) [arXiv:astro-ph/0606285].

[62] W. Fang, S. Wang, W. Hu, Z. Haiman, L. Hui and M. May, arXiv:0808.2208 [astro-ph].

[63] D. Gorbunov, K. Koyama and S. Sibiryakov, Phys. Rev. **D73**, 044016 (2006) [arXiv:hep-th/0512097];
K. Koyama, Class. Quant. Grav. **24**, R231 (2007) [arXiv:0709.2399 [hep-th]].

[64] C. Charmousis, R. Gregory, N. Kaloper and A. Padilla, JHEP **0610**, 066 (2006) [arXiv:hep-th/0604086].

[65] G. Dvali, S. Hofmann and J. Khoury, Phys. Rev. D **76**, 084006 (2007) [arXiv:hep-th/0703027];
C. de Rham, G. Dvali, S. Hofmann, J. Khoury, O. Pujolas, M. Redi and A. J. Tolley, Phys. Rev. Lett. **100**, 251603 (2008) [arXiv:0711.2072 [hep-th]];
C. de Rham, S. Hofmann, J. Khoury and A. J. Tolley, JCAP **0802**, 011 (2008) [arXiv:0712.2821 [hep-th]].

[66] H. van Dam and M.J.G. Veltman, Nucl. Phys. **B22**, 397 (1970);
V.I. Zakharov, JETP Lett. **12**, 312 (1970).

[67] A.I. Vainshtein, Phys. Lett. **B39**, 393 (1972).

[68] V. Sahni and Y. Shtanov, JCAP **0311**, 014 (2003) [arXiv:astro-ph/0202346];
A. Lue and G. D. Starkman, Phys. Rev. D **70**, 101501 (2004) [arXiv:astro-ph/0408246].

[69] L. P. Chimento, R. Lazkoz, R. Maartens and I. Quiros, JCAP **0609**, 004 (2006) [arXiv:astro-ph/0605450].

[70] R. Lazkoz, R. Maartens and E. Majerotto, Phys. Rev. D **74**, 083510 (2006) [arXiv:astro-ph/0605701].

[71] T. Giannantonio, Y. S. Song and K. Koyama, Phys. Rev. D **78**, 044017 (2008) [arXiv:0803.2238 [astro-ph]].

[72] Y. S. Song, Phys. Rev. D **71**, 024026 (2005) [arXiv:astro-ph/0407489];
M. Ishak, A. Upadhye and D. N. Spergel, Phys. Rev. D **74**, 043513 (2006) [arXiv:astro-ph/0507184];
Y. S. Song, arXiv:astro-ph/0602598;
D. Huterer and E. V. Linder, Phys. Rev. D **75**, 023519 (2007) [arXiv:astro-ph/0608681];
E. V. Linder and R. N. Cahn, Astropart. Phys. **28**, 481 (2007) [arXiv:astro-ph/0701317];
R. Caldwell, A. Cooray and A. Melchiorri, Phys. Rev. D **76**, 023507 (2007) [arXiv:astro-ph/0703375];
P. Zhang, M. Liguori, R. Bean and S. Dodelson, Phys. Rev. Lett. **99**, 141302 (2007) [arXiv:0704.1932 [astro-ph]];
L. Amendola, M. Kunz and D. Sapone, JCAP **0804**, 013 (2008) [arXiv:0704.2421 [astro-ph]];

S. Wang, L. Hui, M. May and Z. Haiman, Phys. Rev. D **76**, 063503 (2007) [arXiv:0705.0165 [astro-ph]];

C. Di Porto and L. Amendola, Phys. Rev. D **77**, 083508 (2008) [arXiv:0707.2686 [astro-ph]];

W. Hu and I. Sawicki, Phys. Rev. D **76**, 104043 (2007) [arXiv:0708.1190 [astro-ph]];

B. Jain and P. Zhang, arXiv:0709.2375 [astro-ph];

S. Nesseris and L. Perivolaropoulos, Phys. Rev. D **77**, 023504 (2008) [arXiv:0710.1092 [astro-ph]];

O. Dore *et al.*, arXiv:0712.1599 [astro-ph];

W. Hu, Phys. Rev. D **77**, 103524 (2008) [arXiv:0801.2433 [astro-ph]];

S. F. Daniel, R. R. Caldwell, A. Cooray and A. Melchiorri, Phys. Rev. D **77**, 103513 (2008) [arXiv:0802.1068 [astro-ph]];

Y. S. Song and K. Koyama, arXiv:0802.3897 [astro-ph];

P. Zhang, R. Bean, M. Liguori and S. Dodelson, arXiv:0809.2836 [astro-ph];

G. B. Zhao, L. Pogosian, A. Silvestri and J. Zylberberg, arXiv:0809.3791 [astro-ph];

M. J. Mortonson, W. Hu and D. Huterer, arXiv:0810.1744 [astro-ph];

S. A. Thomas, F. B. Abdalla and J. Weller, arXiv:0810.4863 [astro-ph].