CHARACTERISATION OF ALTERNATIVE MATERIALS FOR HUMAN VIBRATION REPRESENTATION

by

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The biomechanical responses (i.e. apparent mass and transmissibility) to whole-body vibration have been found to be nonlinear - resonance frequency decreases with increasing vibration magnitude. The main mechanism causing the nonlinearity has been recently found to be caused by the soft tissue at the excitation-subject interface (i.e. buttock). Despite the experimental evidence of the cause, mathematical formulation of the dynamic forces at the interface and the prediction of the body movement at different magnitudes of excitation remain absent. The principal objective of the research reported in this thesis was to examine the dynamic behaviour of excitation-subject interface of the whole-body vibration using a scaled rigid mass-soft tissue system. The research was also designed to investigate the single degree of freedom (SDOF) linear and nonlinear models in time domain to predict the responses of the whole-body vibration at different magnitudes.

The preliminary study looked into the time domain modelling of whole-body vibration (WBV) responses using four different SDOF linear and nonlinear viscoelastic models found that all four models failed to predict the responses at different magnitudes.

In the first experiment, the dynamic characterisation of the silicone rubber was accomplished by means of uni-axial cyclic compression test. The experiment was conducted using different excitation frequencies (i.e. 2, 5, 10, 15, 20 and 40 Hz) and magnitudes (i.e. 25, 50, 75 and 100 N). The effect of the excitation frequency and the magnitude on the mechanical properties was found to be significant. The stiffness and elastic modulus was observed to be increased with increasing excitation frequency and the magnitude. The thixotropic or memory dependent behaviour observed in WBV is missing in this experiment with silicone rubber.

In the second experiment, scaled rigid mass-silicone rubber system was studied using base excitation with broadband (2 to 80 Hz) random vibration at four different magnitudes (i.e. 0.5, 1.0, 1.5 and 2.0 ms$^{-2}$ r.m.s.). The silicone rubber specimens with three different thickness (i.e. 10, 15 and 20 mm) and three different diameters (i.e. 50, 75 and 100 mm) were tested with three different sprung masses (i.e. 1.5 kg, 2.5 kg and 5.0 kg) and two different sprung mass contact contour (i.e. flat and hip-borne). A dominant single resonance was observed for the rigid mass-silicone rubber system in the present study and also in most whole-body vibration studies with vertical vibration. Although the frequency range at which the resonance occurred were different: 20 to 33 Hz for current study and below 10 Hz for WBV studies, the mode of sprung mass might be similar in both scenarios. The effect of thickness, diameter, sprung mass and sprung mass contact contour on resonance frequency and stiffness was found to be significant. However the magnitude dependency was observed to be absent in the present study.

In the third experiment, Impulse responses of the scaled rigid mass-soft tissue system were studied for the first time using silicone rubber and the porcine muscle. A linear SDOF viscoelastic model was utilised to extract stiffness and damping from the measured
accelerance and receptance frequency response functions. With a porcine muscle exposed to vertical impact, a repeatable bimodal resonance was observed at 25 Hz and 40 Hz and the similar size silicone rubber specimen showed a single resonance at 20 Hz for 20 mm, 22 Hz for 15 mm and 26 Hz for 20 mm thick specimens - a higher frequency range than those observed in vertical WBV. A repeatable principal resonance is observed for the horizontal impact tests at around 3 Hz in both silicone rubber and porcine muscle specimens – similar to that observed in horizontal WBV. The effect of thickness, diameter, sprung mass and sprung mass contact contour on resonance frequency and stiffness of the silicon specimens was found to be significant in both vertical and horizontal impacts. However the porcine specimen thickness has no clear effect on the parameters extracted.

It is concluded that the thixotropic or memory effect of the human body is missing in the SDOF rigid mass-soft tissue system studied in this thesis.
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## Definitions and Abbreviations

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<tr>
<td>BWM</td>
<td>Bouc-Wen Model</td>
</tr>
<tr>
<td>CSD</td>
<td>Cross Spectral Density</td>
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<tr>
<td>DSP</td>
<td>Digital signal processor</td>
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<tr>
<td>FRF</td>
<td>Frequency response function</td>
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<tr>
<td>PSD</td>
<td>Power Spectral Density</td>
</tr>
<tr>
<td>RF</td>
<td>Restoring Force</td>
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<tr>
<td>r.m.s.</td>
<td>Root-mean-square value</td>
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<tr>
<td>SDOF</td>
<td>Single degree-of-freedom</td>
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<td>WBV</td>
<td>Whole-body vibration</td>
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Declaration of authorship

I, Thirumavalavan Thirulogasingam, declare that the thesis entitled
CHARACTERISATION OF ALTERNATIVE MATERIALS FOR HUMAN VIBRATION REPRESENTATION
and the work presented in the thesis are both my own, and have been generated by me as the result of my own original research. I confirm that:

whilst registered as a candidate for the above degree, I have not been registered for any other research award. The results and conclusions embodied in this thesis are the work of the named candidate and have not been submitted for any other academic award;

where I have consulted the published work of others, this is always clearly attributed;

where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work;

I have acknowledged all main sources of help;

Signed: .........................................................................................

Date: ..............................................................................................
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To my MUM...

All that I am, or
Ever hope to be,

I owe to my angel Mother, Kanakaranjini Thirulogasingam.
CHAPTER 1
Introduction

Modern transports expose human body to whole-body vibration (WBV) with substantially different magnitudes, waveforms and durations. Prediction of human body movement in whole-body vibration environment is a prerequisite for understanding how vibration affects comfort, health and activity. Such understanding helps improve design of vibration mitigation systems. Dynamic forces at the excitation-subject interface govern the prediction of body movement.

Frequency response functions (FRFs) between resultant force and excitation acceleration, i.e. apparent mass, and between resultant motion and excitation motion, i.e. transmissibility, of such biomechanical system were found to be nonlinear. The resonance frequency of the body decreases with increasing vibration magnitude. The main mechanism causing this nonlinearity was associated with soft tissue at the excitation-subject interface, be it the buttocks for a seated, the soles for a standing, or the back for a recumbent person (Huang and Griffin, 2008 a, b and 2009). It is plausible that the soft tissue at the interface is likely to play a crucial role in transmitting force and motion to the human body. There have been various speculations about how changes in the mechanical (i.e. stiffness and damping) or geometric (i.e. thickness and contact area) properties of soft tissue at such interface alter the global biomechanical responses of human body, but no experimental findings have been presented yet.

Various models of biomechanical responses with vertical excitation or fore-and-aft excitation have been developed to interpret experimental findings. Majority of models have been calibrated in frequency domain assuming the human body is a linear system so that the models are effective in predicting the human response within specific testing conditions. However, models those take into account the nonlinearity in resonance and that are able to predict the responses at varying excitation levels are much more complex and need a more comprehensive calibration.

The research undertaken for this PhD thesis was designed to investigate the biomechanical responses of the excitation-subject interface of the human body vibration using scaled model.
The thesis is divided into nine chapters including this introductory chapter.

Chapter 2 reviews and discusses the up to date knowledge of the human biomechanical nonlinearity and mechanical behaviour of biological and artificial soft tissue. The research scope of this thesis is defined at the end of the review.

Chapter 3 describes the main experimental equipment and the methods employed for data analysis.

Chapter 4 investigates the use of single degree of freedom linear and nonlinear models in time domain as a tool for predicting the dynamic responses at the excitation-subject interface with varying excitation levels using WBV data (Huang and Griffin, 2009). This was designed as preliminary study.

Chapter 5 investigates the dynamic mechanical behaviours of silicone rubber using standard uni-axial cyclic compression test.

Chapter 6 investigates the dynamic mechanical behaviours of the base-excited rigid mass-silicon rubber system.

Chapter 7 investigates the dynamic mechanical behaviours of rigid mass- silicone rubber and -porcine muscle system using impact hammer test.

Chapter 8 presents a general discussion of the findings reported in this thesis.

Chapter 9 provides the main conclusions from each study in this thesis, discusses the limitation of this study and provide recommendations for the future work.
CHAPTER 2

Literature review

2.1 Introduction

The nonlinearity is a common aspect in engineering structures and there will be some situations where this introduces a threat to human life, whole-body vibration (WBV) is one of them. For over two decades, this biomechanical nonlinearity has been reported in different postures with single and multi-directions of excitation (e.g. Fairley and Griffin, 1989; Zheng, 2012) and reported to be dependent on different types of variables. These may include: posture, muscle activity (e.g. relaxed and tensed), seating condition (e.g. increased pressure at buttocks and body constraints), body characteristics (e.g. age and gender), vibration waveform (e.g. sinusoids, narrowband random and broadband random) and vibration magnitude etc.

The nonlinear biomechanical responses have been consistently observed with all different stationary sitting, standing and recumbent postures and different directions of excitation and different axes of responses (e.g. Fairley and Griffin, 1989; Matsumoto and Griffin, 1998a, b, 2001, 2002a, b; Mansfield and Griffin, 2000, 2002; Nawayseh and Griffin, 2003, 2005a, b; Huang and Griffin, 2008a, b). The ordinary coherence function associated with the response acceleration and dynamic forces exhibited values above 0.8 to 0.9 in the frequency range of the major resonances. This suggested that the nonlinear magnitude-dependent responses were ‘coherent’ to their excitations – not seen in applications of nonlinear structural dynamic analysis such as conditioned reverse path (CRP) or restoring force surface (RFS) (e.g. Kerschen, 2002).

Through a series of experimental investigation, the primary cause of the nonlinear magnitude dependence was found to be a softening effect in the superficial soft tissue at the excitation-subject interface, i.e. the buttocks of a seated or the soles of a standing or the back of a recumbent person (Huang and Griffin, 2006; Huang and Griffin, 2008a; Huang and Griffin, 2008b; Huang and Griffin, 2009). Previous studies have developed various models to represent the biomechanical responses. This models includes, lumped parameter models (e.g. Wei and Griffin, 1998a; Matsumoto and Griffin, 2001; Qiu, 2007), multi-body models (e.g. Matsumoto and Griffin, 2001;
Zheng et al., 2011), finite element models (e.g. Kitazaki and Griffin, 1997; Siefert et al., 2008a; Zheng, 2012).

This chapter reviews two different areas of approaches, whole-body vibration and biomechanics of soft tissue. The review aimed to find the link between these two studies to develop an experimental method to study the excitation-subject interface of WBV.

Section 2.2 reviews the measures used to represent the biomechanical responses of the human body, the factors that influence the nonlinear biomechanical responses in terms of driving-point apparent mass and transmissibility, the reported major causes of the nonlinear biomechanical responses and the models used to represent the biomechanical responses of the human body. Section 2.3 reviews the biomechanics of soft tissue which includes experimental and theoretical studies of biological and artificial soft tissues. Section 2.4 summarizes the review and reports the findings and research gap. And finally the research scope of this thesis is defined.

2.2 Human biomechanical nonlinearity

2.2.1 Measures of biomechanical response of the human body

*Frequency Response Functions (FRFs)*

Frequency Response Function (FRF) is most widely used method of visualising the input-output properties of the system. FRFs can be used to identify the presence of nonlinearities in a system. Distortions appears in the FRF can be used to provide information about system nonlinearity (Worden and Tomlinson, 2001). The symptoms of nonlinearity include the changes in FRF structure for different excitation levels and response at multiples of the forcing frequency in the case of a pure sinusoidal excitation.

Consider a linear single input/output system as shown in Figure 2.1. The FRF, \( H(f) \), between the Fourier transform of the input \( X(f) \) and the Fourier transform of the output \( Y(f) \) is given by Equation 2.1.

\[
X(f) \xrightarrow{H(f)} Y(f)
\]

*Figure 2.1*  Linear single input/output systems
The FRF is commonly estimated using spectral relations. The calculation of auto-
spectra and cross-spectra, as an intermediate step, allows averaging operations
to be carried out to minimise the random errors. The most commonly used FRF
estimators are known as $H_1$ and $H_2$ estimators.

The standard $H_1$ and $H_2$ estimators, using cross spectral density (CSD) method, are
given by:

$$H_1(f) = \frac{S_{yx}(f)}{S_{xx}(f)}$$

(2.2)

$$H_2(f) = \frac{S_{yy}(f)}{S_{xy}(f)}$$

(2.3)

Where $H_1(f)$ and $H_2(f)$ both measure the amount of output that is linearly
correlated by the input. $H_1(f)$ assumes nonlinearity or noise comes from output
whereas $H_2(f)$ assumes nonlinearity or noise coming from input. $S_{yx}(f)$ is the cross
spectral density (CSD) function between the output and the input, $S_{xx}(f)$ and
$S_{yy}(f)$ are the power spectral density (PSD) function for the input and the output.

Alternatively, frequency response function for the apparent mass can be
calculated using power spectral density (PSD) method:

$$|H(f)|^2 = \frac{S_{yy}(f)}{S_{xx}(f)}$$

(2.4)

The coherence function describes the extent of the linear relationship between the
input and the output signals and it is defined as:

$$\gamma_{xy}^2(f) = \frac{|S_{xy}(f)|^2}{S_{xx}(f)S_{yy}(f)}$$

(2.5)

Where $\gamma_{xy}^2(f)$ is the coherency of the system with a value between 0 and 1. The
coherency has a maximum value of 1.0 in a linear single-input system with no
noise - the output motion is entirely caused by, and linearly correlated with, the
input motion.
When the coherence function falls substantially from unity, the use of a linear model to describe the system is in question. In practice, four main reasons exist as to why a computed coherence function may not equal to unity at all frequencies (Bendat, 1998). They are:

Extraneous noise in the input and output systems;
Bias and random errors in the spectral density estimates;
The output $y(t)$ is due in part to an input other than the measured $x(t)$;
Nonlinear system operation between $x(t)$ and $y(t)$.

Good data acquisition practice, signal processing and physical understanding of the system under test can eliminate the coherence drops due to the first three reasons.

**Biomechanical response of the human body**

Biomechanical responses are measured and represented by the frequency response functions (FRFs). These FRFs fall into two categories, transmissibility and driving-point response. Those involving the measurement and ratios of motions at different locations on the human body are grouped as transmissibility. The other type which measures and calculates the complex ratio of forces and motion at the same point (e.g. acceleration or velocity) is grouped as the driving-point response (e.g. apparent mass or mechanical impedance).

Biomechanical responses, either forces or accelerations, have been classified as inline or cross-axis responses based on the direction of measurement. The responses which are measured in the direction same as the direction of excitation are referred as inline responses whereas the responses which are measured in directions other than the direction of excitation are referred as cross-axis response. For example, a semi supine subject with horizontal excitation, the fore-and-aft cross-axis apparent mass is calculated by taking the output force in the direction perpendicular to the horizontal input acceleration.

Apparent mass or driving point apparent mass, $M(f)$ at a frequency $f$ is defined as the complex ratio of force, $F(f)$, and acceleration, $a(f)$, at the same location on the interface between human body and the seat:

$$M(f) = \frac{F(f)}{a(f)} \quad (2.6)$$
The mechanical impedance or driving-point mechanical impedance, $Z(f)$ at a frequency $f$ is defined as the complex ratio of the output (or driving) force, $F(f)$, to the input velocity, $v(f)$, measured at the seat-subject interface:

$$Z(f) = \frac{F(f)}{v(f)} \quad (2.7)$$

In the literature, there are more studies using apparent mass than the mechanical impedance. The main reason for this is that the apparent mass can be directly obtained from the measured acceleration and force. And also, the primary resonance frequency in the mechanical impedance is either the same or higher than the primary resonance frequency in the apparent mass (Mansfield, 2005). At low frequencies, the human body has the behaviour of a rigid mass, i.e. the apparent mass represents the static weight of the body (Griffin, 1990).

Transmissibility of the body represents the amount of motion transmitted between two locations. The acceleration is widely used for convenience of measurement. The transmissibility is defined as the complex ratios of the motion measured at the output location to the motion measured at the input reference location. The input reference motion is usually measured at the seat-subject interface. For example:

$$T(f) = \frac{a_{\ell 3}(f)}{a_{b}(f)} \quad (2.8)$$

where $T(f)$ is the transmissibility between the vertical acceleration at the seat base, $a_{b}(f)$, and the vertical acceleration at the third vertebra of lumbar spine, $a_{\ell 3}(f)$.

The apparent mass or the mechanical impedance is a representation of the overall dynamic response of the human body above the force sensing platform which takes into account the local body system and the whole-body movement. The transmissibility is often used to investigate the vibration transmission path and modes contributing to the resonances of the human body.

Let’s simplify the human body to be represented by the single degree-of-freedom linear lumped parameter model as in Figure 2.2.
Figure 2.2 Schematic showing the linear single degree-of-freedom lumped parameter model.

The equation of motion becomes:

\[ m \ddot{z}_1 + c \dot{z}_{01} + k z_{01} = 0 \]  

(2.9)

where \( m \) is the mass of the human body supported on the seat, \( k \) and \( c \) are linear stiffness and damping respectively. Apparent mass \( M \) and transmissibility \( T \) are calculated as shown in Equations (2.10) and (2.11).

\[ M(s) = m \frac{cs + k}{ms^2 + cs + k} \]  

(2.10)

\[ T(s) = \frac{cs + k}{ms^2 + cs + k} \]  

(2.11)

Assuming the values \( m=50 \text{ kg} \), \( k=40000 \text{ N m}^{-1} \) and \( c=1000 \text{ Ns m}^{-1} \), the modulus of the calculated vertical apparent mass and the transmissibility to human body are shown in the Figure 2.3.

It is shown that at low frequencies, the modulus of apparent mass is close to the mass seated on the seat while the transmissibility is close to unity. The resonance frequency of apparent mass is the same as the resonance frequency of transmissibility (Figure 2.3).
2.2.2 Apparent mass of a human body

The apparent mass of the human body has been measured in different postures with single and multi-directions of excitation (e.g. Fairley and Griffin, 1989; Zheng, 2012). It has been long observed that the resonance frequency of the apparent mass decrease with increasing vibration magnitude. Experimental studies have shown that many factors can affect the apparent mass of the human body (e.g., vibration magnitude, vibration spectrum, seating conditions, intra-subject variability and inter-subject variability). The following sections mainly review the effect of vibration magnitude and vibration spectrum on the inline and cross-axis apparent masses.

2.2.2.1 Apparent mass with vertical excitation

Previous studies investigating apparent mass have reported a first resonance at around 4 to 6 Hz and a second resonance in the region of 8 to 12 Hz for the vertical apparent mass (e.g., Fairley and Griffin, 1989; Kitazaki and Griffin, 1998; Mansfield and Griffin, 2000; Mansfield and Griffin, 2002; Matsumoto and Griffin, 2002a; Matsumoto and Griffin, 2002b; Nawayseh and Griffin 2003, Mansfield and Maeda 2006; Qiu and Griffin, 2010). Some studies have reported considerable forces in the fore-and-aft direction on the seat with vertical excitation with the principal resonance frequency in the vicinity of 5 Hz (e.g. Nawayseh and Griffin 2003; Mansfield and Maeda, 2005a; Qiu and Griffin, 2010).

Figure 2.3  Modulus of calculated apparent mass (top) and transmissibility (bottom) of a seated human body.
Effect of vibration magnitude

The previous studies have constantly reported that resonance frequency of the apparent mass of the human body decreases with increasing vibration magnitude, this has been referred as biomechanical nonlinearity (e.g. Fairley and Griffin, 1989; Mansfield, 1998; Matsumoto and Griffin, 1998b; Mansfield and Griffin, 2000; Matsumoto and Griffin, 2002a and 2002b; Nawayseh and Griffin 2005a; Subashi et al., 2006 and Subashi et al., 2009).

Fairley and Griffin (1989) has reported that the resonance frequency of apparent mass of seated subjects exposed to vertical random vibration (1 to 20 Hz) decreased from about 6 to 4 Hz as the vibration magnitude increased from 0.25 to 2.0 ms\(^{-2}\) r.m.s. (Figure 2.4).

The apparent masses and transmissibilities of twelve subjects were measured with six excitation magnitudes, 0.25, 0.5, 0.75, 1.0, 1.5 and 2.5 ms\(^{-2}\) r.m.s., of random vibration in the frequency range of 0.2 to 20 Hz (Mansfield and Griffin, 2000). The apparent mass resonance frequency reduced from 5.4 to 4.2 Hz as the magnitude of the vibration increased from 0.25 to 2.5 ms\(^{-2}\) r.m.s. With the eight male subjects exposed to vertical random vibration (0.5 to 20 Hz) at five magnitudes (0.125, 0.25, 0.5, 1.0 and 2.0 ms\(^{-2}\) r.m.s.), Matumoto and Griffin, 2002a has observed that the resonance frequency of the normalised apparent mass decreased from 6.4 Hz to 4.75 as the vibration magnitude increased from 0.125 to 2.0 ms\(^{-2}\) r.m.s.

The effect of muscle tension on nonlinearity in the apparent mass of seated subjects exposed to vertical whole-body vibration was investigated by Matsumoto and Griffin (2002b). The study found that with increases in the magnitude of random vibration from 0.35 to 1.4 ms\(^{-2}\) r.m.s., the apparent mass resonance frequency decreased from 5.25 to 4.25 Hz with normal muscle tension, from 5.0 to 4.38 Hz with the buttocks muscles tensed, and from 5.13 to 4.5 Hz with abdominal muscle tension.
An experimental study has been designed to investigate the effect of variations in posture and vibration magnitude on apparent mass with vertical random vibration over the frequency range of 1.0 to 20 Hz (Mansfield and Griffin, 2002). Each of 12 subjects was exposed to 27 combinations of three vibration magnitudes (0.2, 1.0 and 2.0 ms$^{-2}$ r.m.s.) and nine sitting postures 'upright', 'anterior lean', 'posterior lean', 'kyphotic',
'back-on', ‘pelvis support’, ‘inverted SIT-BAR’ (increased pressure beneath ischial tuberosities), ‘bead cushion’ (decreased pressure beneath ischial tuberosities) and ‘belt’ (wearing an elasticated belt) (Figure 2.5). In all postures, the resonance frequencies in the apparent mass decreased with increased vibration magnitude, indicating a nonlinear softening system. There were only small changes in apparent mass with changes in posture. The changes in apparent mass caused by changes in vibration magnitude were greater than changes caused by variation in posture.

Figure 2.5  Schematic representation of the nine postures used in the experiment (adapted from Mansfield and Griffin, 2002).

Many experimental studies show considerable cross-axis forces on a seat induced by vertical whole-body vibration (e.g. Nawayseh and Griffin 2003; Nawayseh and Griffin 2005a and Nawayseh and Griffin 2009). The forces show that the seated
human body moves in at least two dimensions when exposed to vertical vibration, consistent with rotational modes of the pelvis, the spine, and the upper body. Such motion will result in both axial and shear cross-axis forces in the spine.

Nawayseh and Griffin (2003) investigated the nonlinearity in the fore-and-aft cross-axis apparent mass of seated human body. Twelve male subjects were exposed to random vibration in the frequency range of 0.25-25 Hz at four vibration magnitudes (0.125, 0.25, 0.625 and 1.25 ms\(^2\) r.m.s.). The subjects sat in four sitting postures having varying foot heights so as to produce differing thigh contact with the seat (feet hanging, feet supported with maximum thigh contact, feet supported with average thigh contact, and feet supported with minimum thigh contact). Forces were measured in the vertical, fore-and-aft, and lateral directions on the seat.

Figure 2.6 Median cross-axis apparent mass of 12 subjects in the fore-and-aft direction: effect of vibration magnitude. ——, 0.125 ms\(^2\) r.m.s.; · · · ·, 0.25 ms\(^2\) r.m.s.; — — —, 0.625 ms\(^2\) r.m.s.; — — —, 1.25 ms\(^2\) r.m.s. (adapted from Nawayseh and Griffin, 2003).
There were considerable forces on the seat in the fore-and-aft direction when the subjects exposed vertical vibrations. In all postures, the resonance frequencies of fore-and-aft apparent mass were around 5 Hz which were similar to that for the vertical apparent mass. There were high correlations between the resonance frequencies in the vertical response and the resonance frequencies of the fore-and-aft response. The resonance frequency in the fore-and-aft cross-axis apparent mass decreased as the magnitude of vibration increased (Figure 2.6). This suggested that the high forces measured in the fore-and-aft direction might be attributed to some combination of bending or rotational modes of the upper thoracic and cervical spine at the principle resonance frequency or a bending mode of the lumbar and lower thoracic spine.

**Effect of vibration spectrum**

Majority of the previous studies have measured the apparent mass of the human body with random vibration (e.g. Fairley and Griffin, 1989; Mansfield and Griffin 2000) while a few have studied the response to sinusoidal vibration (e.g., Mansfield and Maeda, 2005b). In random vibration, excitation signal contains all the possible frequencies with energy distributed equally. In sinusoidal vibration, the excitation signal is composed of a single frequency and repeated measurement is required if the dynamic response at more than one frequency is of interest.

The difference in vibration spectra has been observed to induce the nonlinearity in biomechanical responses (e.g., Toward, 2002; Mansfield *et al.*, 2006). Toward (2002) investigated the apparent mass of 12 seated subjects exposed to random broadband vibration (0.125 to 25 Hz, at 0.25 ms$^{-2}$ r.m.s.) on which nine 1/2-octave narrow-band inputs were superposed at four magnitudes (0.25, 0.4, 0.63 and 1.0 ms$^{-2}$ r.m.s.). Consistent with other studies, the frequency of the first and second resonances in the apparent mass decreased with increasing input magnitude. The apparent masses of the subjects also depended on the frequency of the narrow-band inputs. The magnitude of vibration at frequency below 4 Hz had the greatest effect on the apparent mass at resonance, while vibration at frequencies below 8 Hz had the greatest effect on the resonance frequency.

*Mansfield et al.* (2006) studied the effect of vibration spectra and waveform on the primary resonance frequency in the vertical apparent masses of 12 seated male
subjects exposed to vibration, where the vibration spectrum was dominated by either low-frequency motion (2 to 7 Hz), high-frequency motion (7 to 20 Hz) or a 1.0 ms$^{-2}$ r.m.s. sinusoidal vibration at the frequency of the second peak in the apparent mass (10 to 14 Hz) added to 0.5 ms$^{-2}$ r.m.s. random vibration. The results showed that both the resonance frequency and peaks of apparent mass were lower for low frequency dominated vibration than high frequency dominated vibration or sinusoidal vibration (Figure 2.7).

![Figure 2.7](image.png)  
Figure 2.7 Median normalised apparent mass of 12 subjects exposed to random vibration: effect of vibration spectrum in a relaxed upright posture —□—: high frequency; —○—: low frequency; —x—: sine (adapted from Mansfield et al., 2006)

### 2.2.2.2 Apparent mass with horizontal excitation

High magnitude vibration has been experienced in the horizontal directions of off-road vehicles (e.g. Lundström and Lindberg, 1983). The biomechanical responses of the seated human body with horizontal vibration show a resonance at lower frequency, compared to the biodynamic response with vertical excitation (e.g., Fairley and Griffin, 1990; Mansfield and Lundström, 1999a; Hinz et al., 2006b; Qiu and Griffin, 2010).
The apparent mass in the fore-and-aft and lateral directions with eight subjects and broadband vibration (0.25 to 20 Hz) in both horizontal directions with and without a backrest have been investigated (Fairley and Griffin, 1990). When there was no backrest, the first resonance at 0.7 Hz and a second less evident resonance were found for the response in both the fore-and-aft and the lateral directions. It was suggested that peak moduli in the fore-and-aft apparent mass arise from rotation of the whole upper-body. When the pitching motion of the upper-body was reduced, the peak modulus of the apparent mass was reduced. Only one mode was observed for backrest contact condition, with the resonance frequency in the region of 3 Hz for fore-and-aft apparent mass and 1.5 Hz for lateral apparent mass.

The horizontal apparent masses of 13 subjects exposed to various combinations of vibrations were determined (Hinz et al., 2006b). There was single-axis excitation in each of the three translational directions, combined fore-aft and lateral excitation, and combined vertical, fore-aft and lateral excitation. The magnitude of vibration in each axis was 0.25 ms\(^{-2}\) r.m.s., 1.0 ms\(^{-2}\) r.m.s. and 2.0 ms\(^{-2}\) r.m.s. A primary peak in the fore-and-aft apparent mass was found in the region of 2.18 and 2.94 Hz, but was not observed with all subjects or at all vibration magnitudes. Peaks in the apparent mass increased with increasing body mass and decreased with increasing chest circumference in the subjects. Two modes of lateral apparent mass were registered: one below 1 Hz and another between 1.61 and 2.19 Hz. With vibration in the lateral direction (i.e. \(y\)-axis), the upper-body was observed to sway from side to side and the buttocks moved out of phase with the upper-body and shoulders.

Considerable vertical forces were found on the seat and the footrest when there was fore-aft excitation (Nawayseh and Griffin, 2005b). The effect of thigh contact on the vertical cross-axis apparent mass on the seat was not consistent across the frequency range. A significant difference was found in the vertical cross-axis apparent masses measured with and without a backrest at 0.78, 2.15, 6.05, 8.0, and 12 Hz. Greater vertical forces on the feet below 4 Hz was observed with subjects without a backrest.

### 2.2.2.3 Apparent mass with dual-axis excitation

Previous study which used vibration in both vertical and fore-and-aft directions found that, resonance of vertical apparent mass and fore-and-aft cross-axis apparent mass
shifted to lower frequency compared to those with single-axis vertical excitation (Qiu and Griffin, 2010). Apart from the decrease in the peak frequency, the coherency between the vertical acceleration and the vertical force was lowered by the addition of fore-and-aft excitation but raised by increasing the magnitude of the vertical excitation. The coherency between fore-and-aft forces and vertical acceleration was reduced compared to that with single-axis vertical vibration. With increasing vibration magnitudes in the fore-and-aft direction, the coherency was found to be further decreased. A similar trend was observed for vertical cross-axis apparent mass. Peaks in the vertical cross-axis apparent mass with dual-axis vibration were less clear than that with single-axis vibration.

An experimental study was designed to examine how the apparent mass and transmissibility of the human body depend on the magnitude of vertical excitation and the addition of fore-and-aft excitation, and the relation between the apparent mass and the transmissibility of the body (Zheng et al., 2012). The study used 12 male subjects sitting with their hands on their laps during random vertical vibration excitation (over the range of 0.25 to 20Hz) at three vibration magnitudes (0.25, 0.5 and 1.0 ms\(^2\) r.m.s.). At the highest magnitude of vertical excitation (1.0 ms\(^2\) r.m.s.) the effect of adding fore-and-aft vibration (at 0.25, 0.5, and 1.0 ms\(^2\) r.m.s.) was investigated. The forces in the vertical and fore-and-aft directions on the seat surface were measured so as to calculate apparent masses.

With single-axis and dual-axis excitation, both the vertical apparent mass and the fore-and-aft cross-axis apparent mass showed a primary resonance at about 5 Hz, consistent with previous studies. A second resonance in the vertical apparent mass was observed around 8 Hz in subject 5 and in the region of 10 to 12 Hz with subjects 6 and 12. With single-axis vertical excitation, the resonance frequency evident in the median vertical apparent mass decreased as the vibration magnitude. The reduction was 0.54 Hz, 0.42 Hz and 0.96 Hz respectively as the vibration magnitude increased from 0.25 to 0.5 ms\(^2\) r.m.s. With dual-axis vertical and fore-and-aft excitation, the resonance frequency in the vertical apparent mass also reduced as the magnitude of the additional fore-and-aft excitation increased (Figure 2.8a). The reduction was 0.35 Hz when the magnitude of the fore-and-aft excitation increased from 0.25 ms\(^2\) r.m.s. to 1.0 ms\(^2\) r.m.s. The fore-and-aft cross-axis apparent mass exhibited a similar nonlinear characteristic: the resonance
frequency decreased with increasing magnitude of vertical excitation and with the addition of fore-and-aft excitation (Figure 2.8b). With single-axis vertical excitation, a softening of the tissues with increasing vibration magnitude may explain the nonlinearity. With dual-axis excitation, the nonlinear behaviour may be explained by coupling between the mode associated with vertical excitation and the cross-axis influence of the fore-and-aft excitation.

![Figure 2.8](image.jpg)

**Figure 2.8** Effect of magnitude of the additional fore-and-aft excitation on: (a) vertical apparent mass; (b) fore-and-aft cross-axis apparent mass. ——, a<sub>x</sub>=0.25 m/s<sup>2</sup> r.m.s., a<sub>z</sub>=1.0 m/s<sup>2</sup> r.m.s.; ——, a<sub>x</sub>=0.5 m/s<sup>2</sup> r.m.s., a<sub>z</sub>=1.0 m/s<sup>2</sup> r.m.s.; ---, a<sub>x</sub>=1.0 m/s<sup>2</sup> r.m.s., a<sub>z</sub>=1.0 m/s<sup>2</sup> r.m.s. (medians of 12 subjects; adapted from Zheng et al., 2012).

### 2.2.3 Human body transmissibility

Previous studies have measured the transmissibility at different locations of the human body which reflects how much of the vibration on the seat is transmitted to the pelvis, spine, abdomen wall, or the head (e.g., Paddan and Griffin 1988a; Paddan and Griffin 1988b; Messenger and Griffin, 1989; Matsumoto and Griffin, 1998a; Kitazaki and Griffin, 1998; Paddan and Griffin 1998; Mansfield and Griffin, 2000). Transmissibility has been measured to understand the transmission of
vibration through and to the body, to study the disturbances at the head (Griffin and Lewis, 1978) during manual control (Lewis and Griffin, 1978), or to help explain incidences of low back pain (Seidel and Heide, 1986). Measurement of vibration transmitted to different parts of the body also helps identify dynamic modes contributing to resonances seen in apparent mass and mechanical impedance. This section mainly reviews some of the previous studies to identify the modes that have primarily contributed to the resonance in apparent mass and the effect of vibration magnitude on these modes.

A previous study has measured a translational vibration (fore-and-aft, lateral and vertical) and the rotational motion (roll, pitch, and yaw) of the head with a gripped bite-bar (Paddan and Griffin, 1988a). The study has reported that with vertical seat motion, the head motion up to 25 Hz occurred principally in the fore-and-aft, vertical and pitch axes of the head. With fore-and-aft seat motion, the head motion up to 16 Hz also occurred principally in the fore-and-aft, vertical and pitch axes of the head. The resonance frequency was located around 2 Hz in the fore-and-aft head motion. With lateral seat motion, the principal head motion occurred in the lateral direction with peak transmissibility also around 2 Hz. This study also showed that the magnitude of head vibration was increased when the back rest was applied during vertical seat vibration and fore-and-aft seat vibration but had little effect on the head motion with lateral seat vibration.

The motions of the head, spine, pelvis and viscera in the mid-sagittal plane of eight subjects exposed to vertical random vibration of 0.5 to 30 Hz at a magnitude of 1.7 ms$^{-2}$ r.m.s. were measured in three different postures (Kitazaki and Griffin, 1998). Eight modes of body deformation were extracted from the median transfer functions of the subjects in a normal sitting posture below 10 Hz (Figure 2.9). The authors attributed the fourth mode at 4.9 Hz to the principal resonance observed in apparent mass at around 5 Hz. This mode consisted of ‘an entire bod mode in which the head, spinal column and the pelvis moved vertically due to axial and shear deformations of the buttocks tissue’. The fifth mode at 5.6 Hz contained a bending mode of the lumbar and the lower thoracic spine. The sixth mode at 8.1 and the seventh at 8.7 Hz were caused by pitching modes of the pelvis with varying pivoting points. The eighth mode at 9.3 Hz was due to a visceral movement. The
sixth, seventh and eighth modes were considered to be related to the secondary resonance in apparent mass at around 8 Hz.

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**Figure 2.9** Vibration mode shapes below 10 Hz in the normal posture extracted from the mean transfer functions of the eight subjects exposed to vertical random vibration from 0.5 to 30 Hz at a magnitude of $1.7 \text{ ms}^{-2}$ r.m.s. — —, the initial position of body parts; ——, vibration modes (adapted from Kitazaki and Griffin, 1998).

Matsumoto and Griffin (1998b) measured the vertical, fore-and-aft, and pitch transmissibilities to the head, at six locations along the spine (T1, T5, T10, L1, L3, L5) and the pelvis. The vertical transmissibilities to the spine and pelvis showed a peak in the vicinity of the apparent mass resonance frequency for each subject, i.e. 4.75 to 5.75 Hz (Figure 2.10). The authors also noticed that the vertical spinal and pelvic transmissibilities at peak tended to be higher with lower locations, but except for T1 – its movement might be amplified by the head. The fore-and-aft transmissibilities were much smaller than those in the vertical direction and occurred
at a higher frequency range than the resonance frequency in apparent mass (Figure 2.10).

Figure 2.10  Transmissibilities from vertical seat to T1 (1, 3, 5) and L3 (2, 4, 6) in the vertical (1, 2), fore-and-aft (3, 4), and pitching (5, 6) axes of eight upright seated subjects exposed to vertical broadband (0.5 to 20 Hz) random vibration at 1.0 ms\(^{-2}\) r.m.s. (adapted from Matsumoto and Griffin, 1998b).

Over the frequency range of the apparent mass resonance, the pitch transmissibilities to the head and T1 were found to be greater than those to the other locations. These results showed that relative motions of rocking and bending over the spine might have contributed to the resonance in the apparent mass at around 5 Hz (Figure 2.11). The authors commented that the primary resonance in the apparent mass was caused by a combination of translational and rotational
modes in the mid-sagittal plane: a bending mode of the spine, a rocking mode of the thoracic spine, a pitch mode of the pelvis, and axial and shear deformations of the tissue beneath the pelvis. The authors also pointed out that high damping property of the body made it difficult to determine the degree of contribution of each mode to the resonance.

Figure 2.11  Movement of the upper body at the principal resonance frequency of the apparent mass of a single subject at 5.25 Hz when the seat moved upward. The units for both axes are metres, and the scale of the movement is exaggerated for clarity (adapted from Matsumoto and Griffin, 1998a).

With six magnitudes of vertical random vibration from 0.25 to 2.5 ms\(^{-2}\) r.m.s., the transmissibilities from the seat to the lumbar spine (L3), pelvis (posterior-superior iliac spine and iliac crest) and abdominal wall of twelve subjects were found to be nonlinear (Mansfield and Griffin, 2000). The authors found that the primary peak frequencies of the transmissibilities to the pelvis and the lumbar spine were in the
same range as the resonance frequency in apparent mass (around 4 Hz). A degree of nonlinearity in the spine vertical to abdomen vertical transmissibility was found with individual subjects. But this nonlinearity was less apparent comparing with the nonlinearity found in the seat-to-spine and seat-to-abdomen transmissibilities. Since the spine-to-abdomen transmissibility reflected the dynamic response of the viscera, the authors concluded that the viscera alone could not account for the primary resonance in apparent mass, and that the nonlinearity might have been caused via a transmission path common to the spine and the abdomen. The authors speculated the cause of the nonlinearity to be a combination of factors such as that the spinal muscular activity did not increase proportionally with increasing vibration magnitude, and that the passive property of buttocks tissues had a softening effect with increasing vibration magnitude.

The nonlinearity has also been reported in the vertical, fore-and-aft and pitch transmissibilities to the head, spine (T1, T5, T10, L1, L3, L5) and pelvis (posterior superior iliac spine) during vertical random vibration at five magnitudes from 0.125 to 2.0 ms$^{-2}$ r.m.s. (Figure 2.12 and Figure 2.13; Matsumoto and Griffin, 2002a). The peak frequency of the vertical transmissibility to the L3 decreased from 6.27 to 4.75 Hz while the vibration magnitude was increased from 0.125 to 2.0 ms$^{-2}$ r.m.s. The authors pointed out that the transmissibilities showing the relative motions at locations above L5 exhibited less degree of nonlinearity comparing with that in the transmissibilities between the seat and various body locations. This is consistent with the less nonlinear relative motion measured with spine-to-abdomen transmissibility by Mansfield and Griffin (2000). Matsumoto and Griffin (2002a) attributed the nonlinear responses above L5 to the coupling between the spinal column and its surrounding tissues and structures, such as postural muscles and intra-abdominal pressure.
Figure 2.12  Median transmissibilities from vertical seat vibration to vertical vibration at each measurement location of eight upright seated subjects exposed to vertical broadband (0.5 to 20 Hz) random vibration at five magnitudes: 0.125 (· · · · ·), 0.25, 0.5, 1.0, and 2.0 (———) ms$^2$ r.m.s. (adapted from Matsumoto and Griffin, 2002a).

Figure 2.13  Median transmissibilities from vertical seat vibration to fore-and-aft vibration at each measurement location of eight upright seated subjects exposed to vertical broadband (0.5 to 20 Hz) random vibration at five magnitudes: 0.125 (· · · · ·), 0.25, 0.5, 1.0, and 2.0 (———) ms$^2$ r.m.s. (adapted from Matsumoto and Griffin, 2002a).
An experimental study was designed to examine how the transmissibility of the human body depends on the magnitude of vertical excitation and the addition of fore-and-aft excitation (Zheng et al., 2012). The movement of the body (over the first, fifth and twelfth thoracic vertebrae, the third lumbar vertebra, and the pelvis) in the fore-and-aft and vertical directions (and in pitch at the pelvis) was measured in 12 male subjects sitting with their hands on their laps during random vertical vibration excitation (over the range 0.25 to 20Hz) at three vibration magnitudes (0.25, 0.5 and 1.0 ms$^{-2}$ r.m.s.). At the highest magnitude of vertical excitation (1.0 ms$^{-2}$ r.m.s.) the effect of adding fore-and-aft vibration (at 0.25, 0.5, and 1.0 ms$^{-2}$ r.m.s.) was investigated. Authors noted that with the single axis excitation, the primary resonance frequencies in the vertical transmissibility to each measurement location decrease with increasing vibration magnitude (Figure 2.14). The vertical transmissibility to the pelvis had a higher resonance frequency at 8 Hz. The primary resonance frequency in the transmissibility to the thoracic spine was located at 5 Hz.

Figure 2.14 Median transmissibility of 12 subjects with single-axis vertical excitation: ---, $a_z=0.25$ ms$^{-2}$ r.m.s.; ----, $a_z=0.5$ ms$^{-2}$ r.m.s.; ---, $a_z=1.0$ ms$^{-2}$ r.m.s. (adapted from Zheng et al., 2012).
The fore-and-aft transmissibility to all locations along the spine showed a primary peak at 5 Hz. The fore-and-aft transmissibility to the pelvis had peaks at about 3, 8 and 12 Hz. With dual-axis excitation, the resonance frequency in the vertical transmissibility decreased as the magnitude of fore-and-aft excitation increased with constant magnitude of vertical excitation (Figure 2.15). The study also showed that increasing the magnitude of vertical excitation reduced the resonance frequency in the fore-and-aft cross-axis transmissibility to T5 and T1, but not to T12 or L3. However, the resonance frequency in the fore-and-aft motion of the pelvis reduced with the increasing magnitude of the vertical excitation and the addition of fore-and-aft excitation. The resonance frequency in the pitch motion of the pelvis reduced with increasing magnitude of the single-axis excitation but not with increasing magnitude of the additional fore-and-aft excitation.

Figure 2.15 Median vertical transmissibility of 12 subjects exposed to dual-axis excitation: ———, $a_x=0.25\text{ ms}^{-2}\text{ r.m.s.}$, $a_z=1.0\text{ms}^{-2}\text{ r.m.s.}$; ———, $a_x=0.5\text{ ms}^{-2}\text{ r.m.s.}$, $a_z=1.0\text{ms}^{-2}\text{ r.m.s.}$; ———, $a_x=1.0\text{ms}^{-2}\text{ r.m.s.}$, $a_z=1.0\text{ ms}^{-2}\text{ r.m.s.}$ (adapted from Zheng et al., 2012).
The authors concluded that transmissibility measures in this study are consistent with complex modes contributing to motion of the body at the principal resonance: pitch motion of the upper thoracic and lumbar spine, and vertical and fore-and-aft motion of the pelvis and spine. The mode varies with the magnitude of the vertical and fore-and-aft excitations.

2.2.4 Reported causes of biomechanical nonlinearity

Nonlinear biomechanical responses, characterised by the resonance frequency of the apparent mass or transmissibility decrease with increasing vibration magnitude, have been extensively reported (e.g., Fairley and Griffin, 1989; Smith, 1994; Lundström and Holmlund, 1998; Mansfield and Griffin, 2000; Matsumoto and Griffin 2002a; Matsumoto and Griffin, 2002b; Mansfield and Maeda, 2005a; Mansfield et al., 2005b; Qiu and Griffin, 2010; Huang and Griffin, 2006, 2008a; 2008b and 2009). The previous studies has speculated that the primary variables which are contributing to the biomechanical nonlinearity could be related to factors such as some active muscle activity or some passive property of soft tissues (i.e. thixotropy) at the interface (e.g. the buttocks of a seated and the back of a recumbent subject).

It has been suggested that a softening effect with increasing vibration magnitude is caused by a reduced stiffness of the musculo-skeletal structure due to a greater movement of a body with high magnitudes of vibration (Fairley and Griffin 1989). The resonance frequency changed less at higher magnitude of vibration. The study has suggested that subjects may involuntarily increase muscle tension to reduce the motion, or there may be limited ability to vary body stiffness. Mansfield and Griffin (2000) observed the nonlinearity along a transmission path common to the spine and the abdomen. The following factors have been suggested to be a cause for the nonlinearity observed: i) softening response of the buttocks tissue; ii) bending or buckling response of the spine (i.e. a geometric nonlinearity – physically equivalent to inverted pendulum); iii) different muscular forces at different magnitude of vibration – doubling of vibration magnitude did not result in doubling of the muscle activity.

Nine different sitting conditions ‘upright’, ‘anterior lean’, ‘posterior lean’, ‘kyphotic’, ‘back-on’, ‘pelvis support’, ‘inverted SIT-BAR’, ‘bead cushion’ and ‘belt’, have been designed as shown in (Figure 2.5) to investigate the cause of the nonlinearity in the apparent mass (Mansfield and Griffin 2002). No significant difference was found in
the apparent mass with different postures at higher vibration magnitude (i.e. 1.0 and 2.0 ms$^{-2}$ r.m.s.). However the resonance frequency of the normal upright sitting posture (median 5.27 Hz), the resonance frequencies of the kyphotic (median 6.25 Hz) and the anterior lean (median 6.06 Hz) postures were found to be higher but only at 0.2 ms$^{-2}$ r.m.s. The stud also noticed that the ‘anterior lean’ was one of the posture showed the most variability at all three magnitude investigated (i.e. 0.2, 1.0, and 2.0 ms$^{-2}$ r.m.s.). To understand the influence of the dynamics of the tissue beneath the ischial tuberosities, the test was designed with the ‘inverted SIT-BAR’ and ‘cushion’ conditions. Comparison with the upright posture, increasing the loading area (i.e., the ‘cushion’ condition showed a significant decrease in the apparent mass resonance frequencies at 1.0 and 2.0 ms$^{-2}$ r.m.s. The small and inconsistent differences in the nonlinearity were reported in some of the postures, and therefore it is difficult to interpret. However, most studies with different sitting postures agree that a more erect or tensed posture results in a higher resonance frequency - higher effective stiffness of the human body.

It has been found that increased pressure in the buttocks tissues slightly reduced the nonlinearity during vertical random vibration (Nawayseh and Griffin 2003). The study also reported that minimum thigh contact posture produced less degree of nonlinearity compared with, the maximum thigh contact and the feet hanging postures at the two highest magnitudes (0.625 and 1.25 ms$^{-2}$ r.m.s.).

The study of Huang and Griffin (2006) found that voluntary periodic movement significantly reduced the difference in resonance frequency at two vibration magnitudes (0.25 and 2.0 ms$^{-2}$ r.m.s.) compared with the difference in static sitting condition. This indicates that muscles (i.e. back) or tissues in the upper body, influence biomechanical responses of the human body to vibration, and that voluntary muscular activity or involuntary movement of these parts can alter their equivalent stiffness.

Subashi et al. (2006) found the resonance frequency decreased from 6.39 to 5.63 Hz with increasing vibration magnitude from 0.125 to 0.5 ms$^{-2}$ r.m.s. by using an upright standing posture. The study found that nonlinear change in resonance frequency to be significant between the three vibration magnitudes (0.125, 0.25 and 0.5 ms$^{-2}$ r.m.s.) with the upright posture, but insignificant between 0.25 and 0.5 ms$^{-2}$ r.m.s. when a lordotic, a knee bent, or a knee more bent posture was adopted by subjects.
The authors speculated that the change in the nonlinearity with different standing postures was caused by modified voluntary and involuntary muscle activity. The median fore-and-aft cross-axis apparent mass resonance frequency of the standing subjects tended to decrease with increasing vibration magnitude from 0.125 to 0.5 ms\(^{-2}\) r.m.s. However, this nonlinear response was not significantly difference in any of the five standing postures. The authors reckoned that the mechanism causing the nonlinearity in the direction of excitation might be different from that in the fore-and-aft cross-axis direction.

**Thixotropic behaviour of soft tissue**

Thixotropy refers to the recovery behaviour of colloidal materials after the breakdown of structural linkages (Tanner, 1985). It was also suggested to be a passive property of human tissues (i.e., the stiffness of tissues increases with prior stillness or low magnitude stimuli but decreases with prior perturbation). The passive tissue performance has been observed with the human wrist (Lakie *et al.*, 1979), finger flexor (Hagbarth *et al.*, 1985; Lakie, 1986), finger extensor (Lakie, 1986) and the rib cage respiratory muscles (Homma and Hagbarth, 2000).

Thixotropy of soft tissues, in which the stiffness of tissues reduces during, or immediately after excitation, could have caused the nonlinear responses at various body locations and with different sitting and standing postures. It has been suggested that the buttocks tissues are associated with the vertical and fore-and-aft cross-axis mode of the body at the primary resonance. A softening thixotropic behaviour in the buttocks could have contributed to the nonlinearity found in both vertical and fore-and-aft cross-axis responses of seated persons. Fairley and Griffin (1989) has speculated that thixotropic behaviour of the musculo-skeletal structure could be a cause of the nonlinear change in the dynamic stiffness of the body during whole body vibration. However, no experimental evidence has been provided.

Previous studies investigating the nonlinearity used sitting or standing conditions which required considerable muscular postural control of posture. Some relaxed conditions, such as a supine postures, allow the dynamic responses of the body to be measured when there is minimal, or at least reduced, active muscle control. This allows further investigations of the passive thixotropic hypothesis during whole-body vibration. Through a series of experimental investigation, the primary cause of the
nonlinear magnitude dependence was found to be a softening effect in the superficial soft tissue at the excitation-subject interface, i.e. the buttocks of a seated or the soles of a standing or the back of a recumbent person (Huang and Griffin, 2006; Huang and Griffin, 2008a; Huang and Griffin, 2008b; Huang and Griffin, 2009).

The study was conducted with 12 semi-supine subjects exposed to two types of vertical vibration (in the x-axis of the semi-supine body): (i) continuous random vibration (0.25-20 Hz) at five magnitudes (0.125, 0.25, 0.5, 0.75 and 1.0 ms\(^2\) r.m.s.); (ii) Intermittent random vibration (0.25-20 Hz) alternately at 0.25 and 1.0 ms\(^2\) r.m.s. (Huang and Griffin, 2008; Figure 2.16).

![Figure 2.16](image)

Figure 2.16  Schematic and photographic representation of the experimental set-up for semi-supine subjects: showing the semi-supine position and the axes of the force (z-axis and x-axis) and the acceleration (x-axis) transducers (adapted from Huang and Griffin, 2008a).

The resonance frequency at the low magnitude (0.25 ms\(^2\) r.m.s.) was lower with intermittent vibration than with the continuous vibration, whereas the resonance frequency at a high magnitude (1.0 ms\(^2\) r.m.s.) was higher with intermittent vibration than with continuous vibration (Huang and Griffin, 2008). The authors attributed that lower resonance frequency at lower vibration magnitude with intermittent vibration to be the effect of prior high magnitude “perturbation”. Similarly, the higher resonance frequency at high vibration magnitude with intermittent vibration was attributed to be the effect of prior low magnitude “perturbation”. No significant effect of intermittent vibration on the horizontal cross-axis apparent mass was observed. This is possibly due to the low magnitude of the response in the cross-axis direction. It was also observed that the absolute difference between the resonance frequencies of vertical
apparent mass (x-axis) at 0.25 and 1.0 ms\(^{-2}\) r.m.s. was significantly less with intermittent random vibration than with the continuous random vibration. However, absolute differences between the peak frequencies of horizontal (z-axis) cross-axis apparent mass at 0.25 and 1.0 ms\(^{-2}\) r.m.s. were not significantly different between the intermittent random vibration and the continuous random vibration.

The resonance frequencies of horizontal (z-axis) apparent masses of 12 subjects with intermittent longitudinal random vibration at 0.25 ms\(^{-2}\) r.m.s. were found to be significantly lower than those with continuous random vibration at the same magnitude (Huang and Griffin, 2008a, b). The absolute difference between the resonance frequencies at 0.25 and 1.0 ms\(^{-2}\) r.m.s. was less with intermittent random vibration than with the continuous random vibration for all 12 subjects. The peak frequency of vertical cross-axis apparent mass with intermittent random vibration was also significantly lower than the peak frequency with continuous random vibration at 0.25 ms\(^{-2}\) r.m.s. (Huang and Griffin, 2008a, b). The authors concluded that the passive thixotropic properties of the body could be the principal cause of the nonlinearity seen in measures of the apparent mass and transmissibility of the human body.

An experimental study has designed to explore the effects of body location on the nonlinearity of the body in the supine posture (Huang and Griffin, 2009). With a group of 12 subjects, the apparent mass and transmissibility to the sternum, upper abdomen, and lower abdomen were measured in three postures (relaxed semi-spine, flat supine and constrained semi-spine) with vertical random vibration (0.25-20 Hz) at seven vibration magnitudes (0.0313, 0.0625, 0.125, 0.25, 0.5, 0.75 and 1.0 ms\(^{-2}\) r.m.s.). In all three postures, the apparent mass resonance frequencies and the primary peak frequencies of the transmissibilities to the upper and lower abdomen decreased with increasing vibration magnitude. Authors attributed that nonlinearity that was generally apparent in transmissibility to the abdomen was less evident in transmissibility to the sternum and less evident in transmissibilities to the abdomen at vibration magnitudes less than 0.125 ms\(^{-2}\) r.m.s.. It was also observed that the nonlinearity was more apparent in the flat supine posture than in the semi-supine postures. The findings suggest that the nonlinearity is caused by the response of soft tissues’ thixotropic behaviour than the active muscle activity.
2.2.5 Biomechanical modelling

The measured biodynamic responses, apparent mass and transmissibility, from the experimental study have been widely used to identify mechanical-equivalent properties of the human body. This helps in developing and validating mathematical models for analytical study in order to obtain a better insight of the human body behaviour under vibration. These mathematical models can be further used to help in developing anthropodynamic manikins for vibration assessment and to design anti-vibration seats and devices.

In previous studies, various biodynamic models have been developed to depict human motion from single-DOF to multi-DOF models. These models can be divided as distributed (finite element) models, lumped parameter models and multi-body models. The distributed model treats the spine as a layered structure of rigid elements, representing the vertebral bodies, and deformable elements representing the intervertebral discs by the finite element method. Multi-body dynamic models comprise several rigid bodies interconnected by revolute joints (i.e. rotational springs and dampers). The lumped parameter models consider the human body as several rigid bodies and spring-dampers.

2.2.5.1 Lumped parameter models

Lumped-parameter models are widely used as they are simple to analyse and easy to validate with experiments. In this type of model the human body is represented by several masses which are not necessarily anatomically representative. Masses are interconnected by springs and dampers and generally limited to move in just one direction. Some models include rotational degrees-of-freedom to investigate the pitch motion. Many lumped-parameter models have been developed based on certain experimental data and specific testing conditions. Some of these models are reviewed here for convenience.

Linear models

A single-degree-of-freedom mass-spring-damper model is the simplest and easiest form to represent the biomechanical responses of a seated human body in vertical direction. A single-degree-of freedom linear lumped-parameter model was used to reproduce the vertical apparent masses of 60 seated subjects (Fairley and Griffin, 1989; Figure 2.17). The body mass that moved relative to the seat, represented by
sprung mass $m_1$, and the body mass and leg that did not move relative to the seat, represented by un-sprung mass $m_2$, were connected to a linear spring and damper. The interaction between legs and stationary footrest was simulated by the mass $m_3$ which provides additional degree of freedom. The model was not capable of representing the effect of increased muscle tension, contact with backrest, or vibration magnitude.

Figure 2.17  Lumped-parameter model of the seated human body (adapted from Fairley and Griffin, 1989).

Wei and Griffin (1998a) derived single- and parallel two-degree-of-freedom models to reproduce the apparent mass to predict the seat transmissibility (Figure 2.18). Model (a) is a single-degree-of-freedom linear model where the mass of the person is divided into two parts: a support structure $m_1$, and a sprung mass $m_2$. The soft tissue which supports the mass of the body is represented by the spring, $k_1$, and damping, $c_1$. Model (b) is a two-degree-of-freedom human body model. The mass $m_2$ represent the mass of the head, the mass $m_1$ represents the main part of the body and the mass $m$ comprises the skeleton. The vertical apparent mass of 60 seated subjects

2-31
was used to calibrate these two models. The subjects were exposed to vertical broadband random vibration (0.2 - 20 Hz) at 1.0 ms\(^{-2}\) r.m.s.. Models were fitted to the individual data of 60 subjects and the average data of the whole group. The models were not fully anatomy-based but proposed to use as mathematical tools to represent the modulus and phase of apparent mass. The two-degree-of-freedom model provided a better fit to the phase of the apparent mass at frequencies greater than about 8 Hz. The model also provided a better fit to the modulus of the apparent mass at frequencies around 5 Hz than the single-degree-of-freedom model. It was found that frame mass, \(m\) – two-degree-of-freedom model, and \(m_1\) – single-degree-of-freedom model, helped to improve the fitting results.

![Figure 2.18](image)

Figure 2.18 Lumped-parameters models developed to represent the individual and mean modulus and phase of apparent mass: (a) single-degree-of-freedom model; (b) two-degree-of-freedom model (adapted from Wei and Griffin, 1998a).

A lumped parameter model with a vertical, a horizontal and a rotational degree-of-freedom has been developed to fit the moduli and phases of the vertical apparent masses and fore-and-aft cross-axis apparent masses of persons sitting on a rigid seat with no backrest during vertical excitation (Nawayseh and Griffin, 2009; in Figure 2.19). The model is a three degree-of-freedom model with vertical, fore-and-aft, and rotational motion. The rotational degree of freedom is used to predict the fore-and-aft force on the seat induced by vertical excitation. The translational and rotational springs and dampers used in the model have linear excitation-response relationships. The model is not intended to represent the full complexity of motion.
occurring in the seated human body exposed to vertical vibration, it is developed to provide an approximation to the vertical apparent mass and cross-axis fore-and-aft apparent mass of the body.

![Diagram of lumped parameter model](image)

**Figure 2.19** Lumped parameter model for the seated human body exposed to vertical vibration (adapted from Nawayseh and Griffin, 2009).

The equations of motion of the model were derived using Lagrange’s equations. The parameters of the model were optimised by comparing the model with the vertical apparent mass and fore-and-aft cross axis apparent mass data obtained by Nawayseh and Griffin (2003). The results showed that the same model can provide close fits to the responses of each of 12 individual subjects.

**Nonlinear models**

Nonlinear refers to the behaviour of a system that does not obey the superposition principle. For example, a linear spring will maintain the same stiffness at different ranges of displacement. But the stiffness of a nonlinear spring can be dependent on the magnitude of displacement, velocity or acceleration, or alternatively, dependent on some function that is not proportional to the displacement (e.g. a cubic spring). The biomechanical responses of a human body to vibration showed a nonlinear behaviour (the resonance frequency of the FRFs decreases with increasing vibration magnitude). Models with embedded ‘nonlinear’ components or ‘nonlinear’ geometric
arrangements have been used in the previous studies to represent particular ‘nonlinear’ behaviours of the human body (e.g. Hopkins, 1971; Muksian and Nash, 1974; Muksian and Nash, 1976; Mansfield, 1998). The review of these nonlinear models embedded with nonlinear components, or nonlinear geometric arrangements, is to identify any possible representative mechanisms that could represent the characteristic nonlinearity.

Nonlinear lumped parameter models were developed using nonlinear geometric arrangements to incorporate nonlinear characteristics of the body. Hopkins (1971) modelled the nonlinear motion due to the geometry of the visceral mass by a non-rigidly attached visceral mass (Figure 2.20 a), and the nonlinear mechanisms of the lungs by a piston in a cylinder with an orifice (Figure 2.20 b). The models were used as mathematical tools to simulate the mechanical impedance and phase angle of the body measured with sinusoidal vibration of 1/2 to 1/4 g at frequencies 0 to 15 Hz. The author has included some nonlinear characteristics of the strain at the abdomen and the colon pressure when simulating of the mechanical impedance. But the effect of these nonlinearities was not quantified in the mechanical impedance when the target fitting criteria was defined.

Figure 2.20 The nonlinear geometry models of the seated human body to incorporate: (a) nonlinear visceral mass motion reflected by the strain of the upper and lower abdomen; (b) nonlinear mechanism of the lungs reflected by the colon pressure (adapted from Hopkins, 1971).
A modelling study used a multi-degree-of-freedom model to simulate the anatomical path from the pelvis to the head to describe the ‘nonlinearities’ of the body (adapted from Muksian and Nash 1974; Figure 2.21a). The model was embedded with the nonlinear cubic spring and damper between the back ($m_2$) and torso ($m_3$). Coulomb friction forces were used to represent the sliding surfaces between the back and torso. The diaphragm ($m_5$) muscle forces were simulated from half of the heartbeat rate. The model was calibrated to represent the transmissibilities to the head, back, torso, thorax, diaphragm, and abdomen using vertical sinusoidal vibration. The goodness of fittings from 1 to 7 Hz was better than those from 7 to 30 Hz. The stabilised relationship between the ‘nonlinear’ behaviours and the transmissibilities is missing. Although this information is necessary if the model is to represent the frequency response function of the human body.

Muksian and Nash (1976) used a modified three degree-of-freedom model of the human body in the sitting position that contained a parallel connection between the pelvis and the head (Figure 2.21b).

![Figure 2.21](image-url)

**Figure 2.21** The multi-degree-of-freedom model: (a) a nonlinear model of the human body in the sitting position (adapted from Muksian and Nash, 1974). (b) a dual pelvis to head path model of the human body in the sitting position (adapted from Muksian and Nash, 1976).
The modified model was used to simulate the seat-to-shoulder (body) transmissibility and the seat-to-head transmissibility. The model incorporated a frequency dependent nonlinear 'parabolic' damper between the pelvis ($m_3$) and the body ($m_2$) at frequency higher than 10 Hz. The linear stiffness and damping was employed at the frequencies less than 10 Hz. It was concluded that frequency-dependent active components (e.g. muscles) of the body should be included in the biomechanical models. However, this argument was based on the fact that the proposed passive linear model could not represent the responses of the human body at the full range of frequencies from 1 to 30 Hz which might not be the case in other studies.

Mansfield (1998) used a single-degree-of-freedom models with linear quasi-static variable parameter procedure and a nonlinear quasi-static variable parameter procedure to predict the median apparent mass modulus at six magnitudes (0.25, 0.5, 1.0, 1.5, 2.0 and 2.5 ms$^{-2}$ r.m.s.) of broadband random vibration (0.5 to 20 Hz). In the linear procedure, a set of mass, stiffness, and damping parameters were obtained by minimizing the error between the median apparent mass and the predicted apparent mass at all six magnitudes. Then the optimized parameters were fixed and one parameter at a time was allowed to vary to minimize the error at each magnitude. It was found that optimizing the stiffness and mass had greater effect of reducing the error than changing the damping. When optimizing all parameters, the error was further reduced. The stiffness and the damping decreased with increasing vibration magnitude. The nonlinear procedure started with the optimized parameters determined by the linear procedure error than changing the damping. When optimizing all parameters, the error was further reduced. The stiffness and the damping decreased with increasing vibration magnitude.

The nonlinear procedure started with the optimized parameters determined by the linear procedure. Then one of the nonlinear parameters (a softening cubic spring, a nonlinear friction damper, a nonlinear sprung mass) at a time was allowed to change to minimize the error at each magnitude (Figure 2.22). The error was reduced by varying the stiffness or the sprung mass, but not the damping. The results suggested that the change in the apparent mass resonance frequency due to vibration magnitude could be caused by variations in the effective stiffness or in the effective sprung mass of the body, or both.
Figure 2.22 The predicted apparent masses at six magnitudes of vibration ((0.25, 0.5, 1.0, 1.5, 2.0 and 2.5 ms\(^{-2}\) r.m.s.) using nonlinear single-degree-of-freedom models by: (A) varying the nonlinear stiffness only; (B) varying the nonlinear damping only; (C) varying the sprung mass; and (D) measured median apparent mass of twelve upright seated subjects (adapted from Mansfield, 1998).

2.2.5.2 Multi-body dynamic models

Multi-body human models are made of several rigid bodies interconnected by pin (two-dimensional) or ball and socket (three-dimensional) joints, and can be further separated into kinetic and kinematic models. These bodies are expected to be anatomically representative and each of them has three degrees of freedom in the sagittal plane, namely vertical, fore-and-aft, and rotational. Therefore motion of each body segment is predictable with a multi-body model.

A modelling study used a five-degree-of-freedom model to evaluate ride comfort in terms of transmissibility to head, back and hip with the vertical vibration (Cho and Yoon, 2001; Figure 2.23).
Figure 2.23 The model proposed to represent mean transmissibility to the head, back and hip of 5 subjects exposed to vertical random vibration (1-25 Hz) at 1.0 ms$^{-2}$ r.m.s. (adapted from Cho and Yoon, 2001).

The whole-body was simplified into three rigid bodies in 2-D sagittal plane (i.e. lower body incorporating sacrum, thighs and legs, upper body with arms, heads and so on). Backrest support was taken into account in light of its contribution to maintain posture and decrease muscle tension. However, foot support was ignored. Each body of the model was interconnected by linear translational springs and dampers together with rotational springs and dampers. Three vertical and horizontal spring damper units representing the mechanical properties of seat and backrest cushions are serially connected to lower bodies and upper bodies. The mean mass properties of each segment were from literature with standard deviations while the centre of each body was assumed to be at the middle of two joints. The joints and contact positions were measured. Seat cushion parameters were extracted from measured seat transmissibility and the other parameters of model were identified by matching
predicted transmissibility to experiment value. The five-degree-of-freedom of model can describe not only vertical motion of hip and head but also fore-and-aft motion of back which a lumped parameter model cannot generally do.

Zheng et al. (2011) developed a seven degree-of-freedom multi-body model to represent the dynamic response of the human body when seated with (Figure 2.24b) or without (Figure 2.24a) a backrest and exposed to vertical vibration excitation. When sitting without a backrest, the model represents both the vertical apparent mass and the fore-and-aft cross-axis apparent mass on the seat. When sitting with a backrest, the model also represents the vertical apparent mass and the fore-and-aft cross-axis apparent mass at the back. Study concluded with sensitivity analysis that the vertical apparent mass and the fore-and-aft cross-axis apparent mass on the seat and the backrest were all highly sensitive to the axial stiffness of the tissue beneath pelvis. Pitch motion of the upper-body contributed to the vertical apparent mass and the fore-and-aft cross-axis apparent mass on the seat. The apparent mass at the back was more sensitive to the stiffness and damping of the lower back than the properties of the upper back.

![Figure 2.24](image-url) The human body model without backrest (a) and with backrest (b) (adapted from Zheng et al., 2011).
2.2.5.3 Finite element models

Finite element model of the human body and seat consist of numerous elements interconnected by nodes (e.g., Verver, 2004; Pankoke and Siefert, 2008) or lumped spring damper (e.g., Kitazaki and Griffin, 1997; Pankoke et al., 1998). Such a model is usually accurately and anatomically depicted with detailed skeleton structure and muscle in regions of interest such as spine, back, pelvic and thigh. This type of model can be developed to predict not only body motion but also internal forces contributing to injury. Various commercial finite element human body models were developed for different applications as summarised by Verver (2004). However, properties of elements are difficult to obtain and validate. Furthermore, finite element models of human body are generally complex and computationally inefficient.

Some FE models of the human body embedded with body structures such as buttocks and thighs, where deformable elements representing the buttocks tissue and rigid parts representing the pelvis and femurs, were used (e.g. Choi et al., 2007; Verver et al., 2004; Siefert et al., 2008; Zheng et al., 2012). These FE models vary in the complexity of the structures representing the upper-body. Whole skeletons (skull, spine and ribs) with viscera are modelled in PAM-Comfort (Choi et al., 2007), while the lumbar spine with muscles in the lumbar area are modelled in CASIMIR (Siefert et al., 2008a). The spinal structures in PAM-Comfort and the muscles in CASIMIR were included in the models partly because they are thought to be required to predict spinal loads. However, the role of spinal structures and muscles in influencing the vertical apparent mass on the seat is not clear. Unnecessary detailed representation of the anatomy in a FE model will unnecessarily increase the computation time. Mathematical models with the upper-body represented simply with several rigid bodies are capable of representing the apparent mass at a seat (e.g. Matsumoto and Griffin 2001; Zheng et al., 2011).

Some models were developed to investigate relationships between mode shapes and human response. For instance, a modal analysis using finite element methods was performed and seven modal shapes below 10 Hz were extracted (Kitazaki and Griffin, 1997; Figure 2.25). The results showed that the fourth calculated mode shape which consisted of entire body mode with vertical and fore-and-aft pelvic motion due to deformation of tissue beneath pelvis and in phase with vertical viscera motion corresponded to the primary resonance. The second resonance was found to be
related to second viscera mode and pelvic rotation which was dominant in the sixth and seventh predicted mode shape respectively.

Effect of postures on the resonance frequency shift was also investigated with the finite element model. Changing from erect to normal posture with pelvis rotation backward, which led to an increase of the axial stiffness of buttocks tissue and a higher resonance frequency. Changing from normal to slouched posture, head and spine tended to incline forward, increasing contact between thigh and seat. Tissue also became softer and resonance frequency was decreased.

Figure 2.25 Planar finite element model of human body with normal posture developed to investigate modes relating to vibration response up to 10 Hz (adapted from Kitazaki and Griffin, 1997).

FE model with homogeneous soft tissue, surrounding rigid bones (representing the pelvis and femurs), and rigid bodies interconnected with rotational joints (representing the torso of the body) was developed to represent the vertical in-line apparent mass and fore-and-aft cross-axis apparent mass of the seated human body during vertical
vibration excitation (Liu et al., 2015a; Figure 2.26). Deformable elements were used to represent the soft tissue of the buttocks and the thighs.

The three vibration modes, at frequencies less than 15 Hz, of the current model were consistent with the first, second and fourth modes of the model of Matsumoto and Griffin (2001). This is partly because the two models had similar structures for the torso and the current model was optimised with these three vibration modes. The principal resonance frequency in both the vertical in-line apparent mass and the fore-and-aft cross-axis apparent mass increased from 5.37 to 6.12 Hz when Young’s modulus of the soft tissue was increased from 0.15 to 0.195 MPa. However, the resonance frequency did not change when Young’s modulus decreased from 0.15 to 0.105 MPa. The apparent mass at lower frequencies (i.e. less than the principal resonance frequency) decreased with increasing Young’s modulus of the soft tissue, while the apparent mass at higher frequencies (i.e. greater than the principal resonance frequency) increased with increasing Young’s modulus. It is worth to note that overall stiffness of the soft tissue was modelled as linear elastic material.

Figure 2.26 FE model of the seated human body: (left) the complete model; (right) the pelvis-thigh segment (adapted from Liu et al., 2015a).
More complex models do not always provide more accurate predictions of the dynamic responses of human body, because the complex nature of the human body is not well understood (e.g., insufficient anatomical information, complex geometry and complex structure of the body, unknown material properties of tissues, and nonlinearity). The study agrees with the fact that scarceness of the human soft tissue mechanical properties is one of the main problems in these finite element modelling studies.

2.2.6 Summary

The biomechanical responses (i.e. apparent mass and transmissibility) of the human body exposed to vibration have been reported to be nonlinear. Previous attempts have been made using different postures, positions, and single axes of excitation, to study the change in resonance frequency at varying vibration magnitudes, but little effect was reported on the nonlinearity (e.g. Nawayseh and Griffin, 2003; Matsumoto and Griffin, 2003; Subashi et al., 2006). With simultaneous vibration in two orthogonal axes and gradually increasing magnitude in one axis, researchers observed similar nonlinear responses in apparent mass in seated subject (Zheng et al., 2012). With experimental evidence, the primary cause of the nonlinearity in apparent mass of the supine bod was found similar to a passive ‘thiotropic’ property of the soft tissue at the subject-excitation interface (Huang and Griffin, 2008 a, b, 2009). The term thixotropy was used to describe a decrease in dynamic stiffness when subjected to perturbation.

The lumped parameter models with single (Fairley and Griffin, 1989) or multiple degrees of freedom (Zheng, 2012) were studied, to describe the magnitude-dependent whole body vibration, with seated (e.g. Fairley and Griffin, 1989; Wei and Griffin, 1998b), standing (e.g. Matsumoto and Griffin, 2003; Subashi et al., 2009) and supine postures (Vogt et al., 1978). Several studies have included rotational elements in the biodynamic models to allow two-dimensional motion (e.g. Matsumoto and Griffin, 2001; Qiu, 2007; Nawayseh and Griffin 2009). But, these models have not described the mechanism responsible for the magnitude-dependent whole-body vibration and hence are not able to predict the responses at varying magnitudes. Recent modelling studies trying to describe cross-axis coupling of the seated human body using finite element models have highlighted the lack of understanding in the interface soft tissue dynamics, i.e. at the buttocks (Zheng, 2012 and Liu et al., 2015a).
2.3 Biomechanics of soft tissues

Many biological soft tissues such as, muscle, tendons and ligaments are nonlinear, anisotropic, inhomogeneous and nearly incompressible. They can undergo large deformations *in vivo*, both under physiological conditions and during injury. The muscle tissue can be categorised into three groups (Fung, 1993): (1) smooth muscles which are found in blood vessels and intestines whose stimulation is not under our voluntary control; (2) skeletal muscles which are attached to bones (via tendons), causing bones to move or apply force; (3) heart muscle, generates self-stimulating electrical signals that controls the beating of heart. Different muscles generate different passive properties. Some are stiffer than others, and generate more force during a given amount of elongation or compression. The force generated by a muscle fiber is directly proportional to the number of myosin cross-bridges that are strongly bound to the actin filament (Figure 2.28). In this section, the mechanical behaviour of skeletal muscle is mainly reviewed.

2.3.1 The human skeleton muscle structure

Skeletal muscles are surrounded by a thick connective tissue layer, the fascia. The inner layer of the fascia is called the epimysium and contains large blood vessels that branch from the main artery and strain feeding and draining the muscle. This outer layer of extracellular matrix continues as the perimysium, separating the muscle into fascicles (Figure 2.27), which represent bundles of muscle fibres. A muscle fibre is a multinucleated elongated cell with a diameter of 10 to 100μm (Susan, 2015), surrounded by a third thin layer of extracellular matrix, the endomysium. The endomysium contains blood and lymph capillaries.

Each muscle fibre is composed of fibrils, which represent repeated units of sarcomeres and are responsible for the contractile properties of muscles (Figure 2.28 A). They consist of actin and myosin molecules, which can actively slide along each other, thereby producing muscle contraction. The energy needed for this process comes from either aerobically or anaerobically produced, depending on muscle fibre type. The fibre composition of a muscle corresponds to its main function. Type I or slow fibres are mainly dependent on aerobic production for postural, high endurance muscle activity. Type II or fast contract fibres are
responsible for fast, short contractions and need the rapid availability of energy, which they predominantly obtain from anaerobic metabolism (Susan, 2015).

**Figure 2.27** Skeletal muscle structure: the entire muscle is surrounded by the epimysium, which continues as perimysium covering the fascicles, and finally as endomysium which surrounds the individual muscle fibers (adapted from Susan, 2015).

**Mechanical behaviour of the skeletal muscle tissue**

Dynamic force developed during stretch is thought to be associated with increased strain of attached cross-bridges between actin and myosin myofilaments (see Figure 2.28 C). In this situation, cross-bridges act like springs and develop force that increases with their strain. The time history of muscle force during stretch, it is found to be dependent on several factors: rate of stretch, stretch magnitude, initial muscle length, stimulation intensity, muscle fatigue, and temperature. The force stemming from a contractile response within a single muscle fiber is transmitted from the muscle fiber to bone via a complex network of connective tissue: the intramuscular connective tissue (endomysium, perimysium, and epimysium), tendons, inter-muscular connective tissue of adjacent muscles, and structures other than the muscles (blood vessels, tissue that support nerves). While the network of
connective tissue certainly plays a crucial role for the structural organization of a muscle, the bulk of a skeletal muscle is made up of muscle fibers. Previous studies showed that external compressive loading could affect skeletal muscle tissue without compromising skin tissue. And also they (i.e. Shabshin et al., 2010) have found that skeletal muscle tissue deformation was more than 50% larger than its adjacent fat tissue deformation.

Figure 2.28  The structure of a skeletal muscle fiber: A - showing the arrangement of sarcomere within myofibril; B - showing zones within a sarcomere - narrow, plate shaped regions of dense material called Z discs separate one sarcomere from the next; a narrow H zone in the centre of each A band contains thick but no thin filament; supporting proteins that hold the thick filaments together at the centre of the H zone form the M line ; C - showing the arrangement of filaments within the sarcomere (adapted from Susan, 2007).

A human skeletal muscle tissue has been found to show some thixotropic behaviour (Fung, 1993). The thixotropy has been used to describe as passive dynamic property of human tissues. The nature of thixotropy is such that the stiffness of the relaxed
body tissues reduces during and immediately after prior high magnitudes of excitation, while the stiffness increases during and immediate after prior low magnitudes of excitation. In other words, the dynamic stiffness of tissues depends on the ‘shear history’ i.e. velocity of the citation.

The cause of the thixotropy in muscle tissues is thought to be the breakdown of the bonds between actin and myosin (Hill, 1968). Myosin and actin are contractile proteins in muscles. The thick myosin filament in the isotropic (I) band is overlapped by the thin actin filament in the anisotropic (A) band (Figure 2.28 C). A large muscle protein called titin was found to be the primary contributor to the stiffness of relaxed muscles (Wolfgang et al., 1996). The stiffness of the titin protein is dependent on its length. For example, some sections of the titin, such as the ‘PEVK’ and ‘poly-Ig’, are suggested to contribute to tissue stiffness.

The nonlinear softening effect observed with the relaxed or partially contracted muscles during whole-body vibration could be due to a combination of the dynamic properties of the passive titin filament and the active contractile myosin and actin filaments as discussed in the previous Section 2.24. The titin filament could have a high stiffness in response to low magnitudes of excitation but a reduced stiffness during high magnitudes of excitation. Likewise, the breakdown of the myosin-actin bonds develops during high magnitudes of vibration due to high levels of inertial forces, and these bonds recover with increased tissue stiffness during low magnitudes of vibration or stillness. A totally relaxed ‘switched-off’ muscle fibre would be more liable to break down than an activated ‘switched-on’ fibre.

2.3.2 The dynamic characterisation of soft tissues

Mechanical properties of soft tissue is usually studied using specimen size in the order of 2 to 20mm in height with 5 to 40 mm footprint at least two-ends of the specimen constrained. They can be divided into static and dynamic behaviours in general. The static property is often quantified by Young’s modulus ratio of stress to strain on the loading plane along the loading direction), bulk modulus (ratio of applied pressure over volumetric strain), shear modulus (ratio of shearing stress over shearing strain), and Poisson’s ratio (the ratio of lateral strain and a axial strain). These were usually obtained by quasi-static loading conditions where the specimen was tensed, compressed or sheared slowly to provide incremental changes in
displacement and force. For dynamic property, the storage (or the real part, representing elasticity) and loss (or the imaginary part, representing viscosity) moduli of the complex Young’s, bulk or shear modulus describing the energy storage and dissipation capacity of a material were measured using cyclic excitations at discrete frequencies.

The studies provided micro-scale force-displacement (or strain-stress) relationship so called ‘constitutive’. Models derived from these studies were constrained to limited magnitude of the strain rate to allow linear formulation of the force-displacement relations, i.e. the isotropy (or geometry) and homogeneity (e.g. Sims et al., 2010). ‘Nonlinear’ models were able to consider disproportionate force arising from large displacement excitation, direction of loading and non-homogeneity (e.g. Then et al., 2012). The viscoelastic combined variation of elastic ‘storage’ and viscous ‘loss’ modulus and hyperelastic nonlinear force-displacement variation without yielding) behaviour of soft tissue have been studied using constitutive models.

The experimental and analytical studies on constitutive description of soft tissue using quasi-static (e.g. Song et al., 2007; Loocke et al., 2006), transient (e.g. Ozcan et al., 2011; Then et al., 2007, 2012) and cyclic (e.g. Aimedieu et al., 2003; Loocke et al., 2009) excitations provide readily assistance on developing more relevant approach for understanding the magnitude dependency observed with whole-body vibration. With careful conversion, these studies could only provide useful comparison of the type and size of the specimens, loading rate or frequency component, magnitude, and boundary conditions, i.e. how the specimen is constrained.

Viscoelastic properties of biological soft tissues have been measured under quasi-static and cyclic compression and tension usually with one end of the specimen fixed and another subjected to the load. Literature about cyclic compression of muscle tissue using free load, base excitation vibration is sparse. One particular study looked into the dynamic behaviour of porcine skeletal muscle subjected to base excitation loaded with a sprung mass as an analogue SDOF (Aimedieu et al., 2003).

With measured transmissibility between base and the resultant motion of the sprung mass, authors extracted dynamic stiffness and damping parameters. Most other
studies were conducted with two ends of the specimen clamped, and the load is applied to one end.

Compressive shear and bulk modulus of human heart in the clamped setup have been found to be in the range from 0.25 to 0.38 GPa (bulk) and 60 to 148 kPa (shear) using compressive strain rate from 300 to 5000 s\(^{-1}\) and shear strain rate from 200 to 2800 s\(^{-1}\) (Saraf et al., 2007). With porcine muscle fibre orientated 0°, 45° and 60°, the compressive stress-strain behaviour with different strain amplitude (0.02 and 0.1) was reported using compression at frequencies from 5 to 80 Hz (Loocke et al., 2009). Dynamic and quasi-static stress-strain behaviour of porcine muscle was reported with strain rate ranging from 0.7x10\(^5\) to 3.7x10\(^5\) s\(^{-1}\) (Song et al., 2007). For transient excitation, the strain rate quantifies the rate at which load is applied, similar to the frequency for cyclic or steady-state random stimuli. The magnitude of excitation defined by excitation acceleration, strain amplitude or impact velocity in these studies usually governed by the type of loading and boundary conditions.

Hyperelastic formulation provides a base for numerical modelling of nonlinear force displacement behaviour of soft tissue by allowing large deformation without yielding the material. The commonly used hyperelastic material models to predict the soft tissue responses are Neo Hookean model (Loocke et al., 2009), Mooney-Rivlin model (Song et al., 2007) and the Ogden model (Shergold et al., 2006). A time domain quasi-linear viscoelasticity formulation is of particular interest in constitutive modelling of biological soft tissue (Fung, 1993), which uses a ranges of frequencies to calculate parameters of the formulation. Stress-strain relationship of porcine skin was modelled using ‘3\(^{rd}\) order Ogden’ function with the strain rate of 4x10\(^{-3}\) to 4x10\(^3\) s\(^{-1}\) (Shergold et al., 2006).

The Ogden model for an incompressible, hyperelastic solid describes a wide range of stress-strain characteristics for the soft tissue. Viscoelastic stress-strain behaviour of porcine subdermal fat was examined using Neo Hookean model (Sims et al., 2010) with the indentation loading range from 19.6 to 98.1 N. A combined experimental and numerical approach was used to calculate the time dependent shear moduli of buttock adipose and skeletal muscle tissue (Then et al., 2007, 2012), the study performed displacement-controlled ramp-and-hold indentation relaxation test using human subject and they used time domain quasi-linear viscoelastic formulation with prony series and hyperelastic model of ‘Ogden’ to examine the viscoelastic stress-
strain behaviour. These formulations were not directly ready to offer analytical solutions and predictions of global responses of WBV.

The following section dedicated to summarise some soft tissue studies which are most relevant to WBV. The findings from each study have been summarised in Tables 2.1A, 2.1B and 2.1C.

**Aimedieu et al., 2003**

The study examined the mechanical properties of the porcine muscles, using base-excited shaker test (5-30 Hz), which is in the range of whole body vibration. Tests were conducted in vitro on porcine muscles, using a lever arm device, which applied a static load onto cylindrical samples; load was allowed to move freely while the sample was subjected to cyclic excitation at the base. A two parameter viscoelastic model (Kelvin-Voigt) was used to calculate the frequency dependent stiffness and damping of the samples using measured transmissibility. The study reported that average stiffness curve showed a monotonous increase (5 Hz: 8.5x10³ N/m and then increase till 30 Hz: 347x10³ N/m). For the damping, between 5 and 20 Hz, values were typically inferior to 300 Ns/m, which then increased till 30 Hz (556 Ns/m).

**Loocke et al., 2006**

The study provided an experimental basis for theoretical modelling of quasi-static compressive stress-strain behaviours of porcine muscle in three dimensions. Experiment was carried out using aged and fresh porcine muscle specimens with the loading at fibre and cross fibre directions to quantify the Young’s modulus and Poisson’s ratio at each direction. The values obtained for Poisson’s ratios showed that muscle is stiffer in fibre direction than cross fibre direction in expansion. Calculated Young’s modulus in both fibre and cross fibre directions showed that during compression porcine muscle is stiffer in cross fibre direction than in fibre direction. These three dimensional quasi static Young’s modulus and Poisson’s ratio are based on compressive behaviour.
Table 2.1A  Soft tissues: comparison of experimental conditions and methods

<table>
<thead>
<tr>
<th>Authors</th>
<th>Samples</th>
<th>Excitation, samples, measures and results</th>
</tr>
</thead>
<tbody>
<tr>
<td>AP et al., 2003</td>
<td>Porcine muscle, Silicon</td>
<td>Sample size: Cylindrical sample, diameter 38.5-mm and height 20-mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Boundary condition: specimen was placed inside a bath filled with saline water (0.9%, 37 °C), bottom end was fixed and top end was attached with a free mass, specimen was excited at the bottom.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Cyclic compression</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Frequency: 5-30 Hz, in 5 Hz increments</td>
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<tr>
<td></td>
<td></td>
<td>Magnitude: 1.5 ms(^{-2})</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Static loading 5.3 kg load - 400 mmHg static pressure</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Measurements: two accelerations, at top and bottom of the specimen, measured at sampling rate 0.001s</td>
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<tr>
<td></td>
<td></td>
<td>Result: Dynamic stiffness 8.5 to 347x10(^3) Nm(^{-1}) (5-30 Hz)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Damping 300 to 556 Ns(^{-1}) (5-30 Hz)</td>
</tr>
<tr>
<td>LV et al., 2006</td>
<td>Porcine muscle (fresh and aged)</td>
<td>Sample size: Cubic sample of 5x10 mm (samples were tested in the fibre and cross-fibre directions and at 45° and 60°)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Boundary condition: Two ends fixed</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Quasi-static Compression, magnitude of strain up to 0.3 at strain rate 5x10(^{-4}) s(^{-1}).</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Result: Young’s modulus of fresh porcine muscle in fibre and cross fibre direction (E_L = 15.6x10^3 \text{ Nm}^{-2}) and (E_T = 11.1x10^3 \text{ Nm}^{-2})</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Poisson's ratios (v_{LT}: 0.5, v_{TL}: 0.36) and (v_{TT1}: 0.64) ((v_{TL}) donates the Poisson's ratio that corresponds to an extension in direction L when a compression is applied in direction T).</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(L)- fibre direction (longitudinal) and (T) and (T1)- cross fibre directions (transverse).</td>
</tr>
</tbody>
</table>

Saraf et al., 2007

The study provided an experimental basis for finite element modelling of dynamic compressive and shearing behaviours of soft human tissues, regarded as hyperelastic and viscoelastic. Hyperelastic (sometimes, nonlinear elastic) refers to variations in the force-displacement relation of a material without yielding, viscoelastic refers to variations in viscosity with changes in elasticity.

The dynamic compressive bulk modulus was approximated by linear regression between the applied pressure and resultant volumetric strain with no further demonstration of the effect of variation in strain rate. The dynamic shearing modulus was found to be of typical exponential form with low stress at low magnitudes of shear strain but exponentially higher stress as the strain increased. The heart and stomach tissues, both primarily consisting of muscles, exhibited considerable variations in response to different shearing strain rates but the variations were not quantified.

The authors pointed out in a supplementary document that explicit models were used to take into account the wave propagation in hyperelastic-viscoelastic shearing behaviour due to the time requirement for developing uniform stress state. The equivalent frequency and magnitude range (converted from the strain rate) used in the study were much higher than the range relevant to whole-body vibration studies. However, for mechanical shocks and prediction of injury, this range of data can provide a basis to quantify the physical responses of local tissues.

Wu et al., 2007

The study evaluated the frequency dependence of the dynamic strains in a fingertip model subjected to normal and tangential vibration. The authors found shear vibration introduced considerable shear strain but little normal strain, but normal vibration introduced both normal and shear strain but the shear strain introduced by normal vibration was less than 0.3 mm.

At the single excitation magnitude investigated the motion transmitted into the tissues tended to decrease with increasing excitation frequency. The author commented that motion transmitted deep into the tissue, e.g. 1-3 mm, at low frequencies may cause damage to certain neural receptors, while at higher frequencies, e.g. >1000 Hz, there was potential for damage to local tissue.
Table 2.1B Soft tissues: comparison of experimental conditions and methods

<table>
<thead>
<tr>
<th>Authors</th>
<th>Samples</th>
<th>Excitation, samples, measures and results</th>
</tr>
</thead>
<tbody>
<tr>
<td>SH et al 2007</td>
<td>Iced-fresh tissues of: Heart, Stomach, Liver, Lung</td>
<td>Tested at 20°C room temperature immediate form thawing. Compressive bulk modulus – Kolsky bar (impact) – test: Tissue samples 12.7 mm in diameter, 1-2 mm thick. Confined compression at nominal strain rate 300 to 5000 s⁻¹ (Strain rate $E' = v / \dot{\varepsilon}$, where $v$ = impact velocity; $\dot{\varepsilon}$ = original length of specimen, a term similar to the onset rate of shocks). Duration of compressive pulse = 100x10⁻⁶ or 140x10⁻⁶ s. Bulk modulus of heart = 0.25 - 0.38 GPa (Bulk modulus is the ratio of applied pressure over volumetric strain – obtained by linear fitting), stomach 0.48 GPa. Shear (impact) test: Tissue samples 9x20 mm, 1-2 mm thick. Effect of shearing strain rate on shear modulus (i.e. shearing stress / shearing strain) – obtained by linear fitting, $E'$ ranges from about 200⁻¹ to about 2800⁻¹. Shear modulus of heart = 60 - 148 kPa, stomach 8 - 45 kPa.</td>
</tr>
<tr>
<td>WJZ et al 2007</td>
<td>Modelling of index fingertip</td>
<td>Finite element model (16 mm in width, 12 mm in height): Nonlinear elastic and viscoelastic: skin epidermis and dermis, subcutaneous tissue. Linear elastic: bone, nail, contact fingertip PVC support. Pre-deformation in the normal to contact surface: 0.5, 1.0, 1.5, 2.0 mm. Normal and tangential continuous harmonic vibration: 0.5 mm peak to peak, 16 to 2000 Hz octave bands. Fingertip major resonance found at 100 - 125 Hz, secondary resonance at around 250 Hz in both normal vertical and tangential shearing directions. At low frequencies (around the major resonance), dynamic strain tended to penetrate into the tissue more than about 3 mm. at higher frequencies (around 1000 Hz), the depth was less than 1 mm.</td>
</tr>
</tbody>
</table>

Ozcan et al., 2011

The study characterised the frequency dependent material properties (i.e. storage and loss modulus) of human liver and the silicon rubber. Impact excitation was applied on the samples and response displacement was recorded to estimate the receptance FRF. By applying the linear viscoelastic formulation using the receptance in the frequency domain, study has calculated dynamic stiffness and damping first and then storage and loss modulus. The study has reported the storage modulus of human liver varied from 10 to 20 kPa and 20 to 50 kPa. The silicone rubber showed the similar order of storage modulus. With the different thicknesses of silicone rubber specimen, author has reported that the resonance frequency decreased with increasing thickness. Similar observation was found to the stiffness.

Takaza et al., 2013a

The study examined the viscoelastic properties of porcine skeletal muscle under drop impact in loading rates relevant to the car crashworthiness industry. The constitutive equations were formulated to provide fibre and cross-fibre direction stress-stretch data. These data were usually fed to numerical models of the human soft tissue so as to assess damage at higher frequencies and magnitudes of loading than those seen in human vibration. These formulations were not directly ready to offer analytical solutions WBV.

Sparks et al., 2015

The study has investigated the use of silicone materials to simulate tissue biomechanics as related to deep tissue injury. Unconfined compression test were conducted to quantify the mechanical properties of three different variant of silicone rubber (Ecoflex 0030, Ecoflex 0010 and DragonSkin). The study also conducted the indentation test on both silicone rubber and the porcine muscle to investigate the silicone’s ability to mimic the non-uniform stress distribution muscle demonstrates under concentrated loadings. All the 3 silicone formulations demonstrated shear moduli within the range of published value for biological tissue.
Table 2.1C Soft tissues: comparison of experimental conditions and methods

<table>
<thead>
<tr>
<th>Authors</th>
<th>Samples</th>
<th>Excitation, samples, measures and results</th>
</tr>
</thead>
<tbody>
<tr>
<td>OUM et al. 2011</td>
<td>Porcine muscle, Silicon</td>
<td>Sample size: Whole human liver and cylindrical silicon samples. Boundary condition: specimen was placed on a rigid foundation and impact load was applied on top of a small (400 g) sprung mass. Impact load Frequency: 0 to 100 Hz Measurements: Receptance was measured to estimate storage modulus and loss modulus. Result: storage modulus 10 - 20 and 20 - 50 for the frequency range 0 - 100 Hz.)</td>
</tr>
<tr>
<td>TM et al. 2013a</td>
<td>Porcine muscle (fresh)</td>
<td>Sample size: Cubic sample with width and depth nominally at 20 mm and the height of approximately 10 mm. (samples were tested in the fibre and cross-fibre directions) Boundary condition: Fixed base with drop impact on top. Impact velocity at 1.16, 2.2, 3.78, 2.2 and 3.78 m/s. Result: Fibre and cross-fibre direction stress-stretch data on the impact response of fresh porcine muscle tissue at rates between 11,600%/s and 37,800%/s - rates relevant to automotive impact.</td>
</tr>
<tr>
<td>SJL et al. 2015</td>
<td>Porcine Muscle, Silicon</td>
<td>Sample size: Silicon - 35.8 mm diameter and 24.5 mm height, 59.3 mm diameter and 26 mm height. Porcine specimen - 60.2 mm diameter and average height of 28.2 mm. Boundary condition: uni-axial cyclic compression and shear. Results: shear modulus: 75.44 kPa - Dragonskin, 12.6 kPa - Ecoflex0010 and 22.08 kPa - Ecoflex 0030. Shear modulus porcine muscle: longitudinal - 51 - 105 kPa Transverse - 11 - 54 kpa.</td>
</tr>
</tbody>
</table>

2.3.3 Summary

Existing biomechanics studies of soft tissues examined viscoelastic properties under quasi-static and cyclic compression and tension usually with one end of the specimen fixed and another subjected to the load. It is reported that the measured viscoelastic properties may vary depends on the way soft tissue specimen is clamped and the excitation levels used. The microstructure of the soft tissue also contributes to the changes in the viscoelastic properties (i.e. Takaza et al., 2013a). Two particular studies are of interest (Aimedieu et al., 2003; Ozcan et al., 2011). Both studies used free mass on top of the specimen with the excitation level relevant to WBV. However the studies using base-excitation with varying magnitudes are still missing.

Considering the similarities in physical and biological composition, the number of experiments conducted using porcine muscle tissue to understand passive mechanical property of the human equivalent, and the scarceness of human tissue, it is tenable to use porcine muscles to represent mechanical property of human’s (e.g. Fung, 1993; Loocke et al., 2009). An alternative approach was to substitute the biological tissue with artificial materials with similar viscoelastic characteristics. The silicone rubber was widely used as a biological soft tissue substitute in many studies (e.g. Ozcan et al., 2011; Sparks et al., 2015). The silicone rubber has an ability to be reconstructed to mimic wide range of stiffness values without complicating the time dependent behaviour 'creep' (Sparks et al., 2015).

2.4 Conclusions

Biomechanical responses of human body

The resonance frequency in the apparent mass and transmissibility associated with vertical excitation is decreased with increasing vibration magnitude. The main mechanism causing this nonlinearity has been recently found to be caused by the soft tissue at the excitation-subject interface (Huang and Griffin, 2009). Despite the experimental evidence, formulation of dynamic forces at such interface and the prediction of body movement with varying vibration magnitudes remain absent. This is partly because, the soft tissue biomechanics at the interface is not well understood. Recent modelling studies have highlighted that the lack of
understanding in the interface soft tissue dynamics, i.e. at the buttocks (Zheng, 2012 and Liu et al., 2015a).

Various lumped parameter models with single and multiple degrees of freedom were developed to represent the biomechanical responses usually in frequency domain. But, these models have not described the mechanism responsible for the magnitude-dependent whole-body vibration responses and hence are not able to predict the responses at varying magnitudes. Sensible predictions of the pressure distribution on an occupied soft seat were provided with a finite element model of the buttocks which included linear skin, rigid bony structure and nonlinear hyper elastic isotropic soft tissue (Verver, 2004). However, it is unclear that whether the nonlinear force-deflection properties of tissue based on the quasi-static compression (i.e., due to slow loading, the inertial effects could be ignored) is efficient in representing the variation in the resonance in the dynamic response (e.g., vertical apparent mass) with dynamic excitation at different magnitudes. The recent study (e.g. Liu et al., 2015b) has reported that the frequency of the principal resonance in the vertical apparent mass was higher at the front thighs (8 to 10 Hz) than at the ischial tuberosities (around 5 Hz). The author has speculated that the factors causing this difference could be caused by different thickness, contact area and the pressure of the tissue under things and at ischial tuberosities. However it is difficult to examine how these factors (i.e. tissue thickness and contact area) affect the global biomechanical response of the human body using conventional WBV experiments. This begs the need for a controlled experiment with a scaled model of the excitation-subject interface. Unfortunately, this has not been done yet.

**Biomechanical responses of soft tissue**

Studies examining the mechanical properties of soft tissues may provide alternative measures for quantifying the nonlinear behaviour by using the strain rate, or loading rate, and defining the onset rate of motions transmitted to local tissue. The section (Section 2.3) dedicated to review the biomechanics of soft tissue also established that sensation. Majority of the biomechanical studies have measured mechanical properties of soft tissue under quasi-static, cyclic compression and tension usually with one end of the specimen fixed and another subjected to the load. The excitation techniques and the boundary conditions used in these studies
were not the realistic experimental condition of soft tissue *in vivo* at the excitation-subject interface of the WBV.

There has been a lack of knowledge on the physical responses of soft tissue to oscillatory motions using free load boundary condition (e.g. excitation is applied to one end of the specimen and other end is attached to the sprung mass which is allowed to move freely) at varying magnitude of base excitation. This shortage may partly stem from a lack of established relationships between the measurable micro-structural properties (e.g. compressive bulk modulus and shear modulus) and macro behaviours (e.g. resonances in apparent mass and transmissibilities). The links between the two levels of behaviour have the potential to contribute to improvements in human body models. But this not has been done yet.

*The research scope of the thesis*

In the light of the state of knowledge summarised in this chapter, the research undertaken for this thesis was conducted mainly to answer the following questions:

1) Whether typical commercially available silicone rubber formulae (i.e. Ecoflex 0010) when excited at the base with a free mass on top could reproduce the magnitude dependent biomechanical behaviour of the human body during WBV?

2) How close the silicone rubber and the fresh porcine muscle could mimic the soft tissue biomechanics at the excitation-subject interface of the WBV?

3) Can single degree of freedom linear and nonlinear models (i.e. Zener or Bouc-Wen) be used to formulate the dynamic responses at the excitation-subject interface of the whole-body vibration in time domain? (This is a preliminary study).
CHAPTER 3
Methodology

This chapter describes the apparatus, specimen preparation and data analysis methods used in the studies. The majority of experiments were conducted in the laboratory of the Mechanical Behaviour of Materials (MBM), School of Engineering, University of Portsmouth, Portsmouth, UK. The impact hammer test on fresh porcine muscle tissue was conducted at the Trinity Centre for Bioengineering, Trinity College Dublin, Dublin, Ireland. Ethical review was conducted to ensure that research project complies with the requirements of the school’s research ethics policy. And the ethical approval was granted by the ethical committee before research project commences. The experiments were also reported to and checked by the health and safety personnel at the University of Portsmouth.

3.1 Cyclic compression test

The cyclic compression test was designed to obtain the dynamic stiffness and compressive elastic modulus (or Young’s modulus). During the test, one end of the specimen is fixed and the other end is subjected to a cyclic compressive force continuously at different frequencies. The specimens used were in the shape of a circular disk with two diameters of 50 mm and with thickness of 10 mm. The maximum thickness of the specimen was chosen to be as thin as possible to eliminate the buckling of the lateral surface of the specimen. Elimination of buckling helps maintain the specimen in uni-axial state of stress so as to approximate true uni-axial compression. The specimens were prepared in the MBM lab using silicone rubber. The material is assumed to be isotropic and homogeneous.

3.1.1 Specimen preparation

In the present study, a commercial silicone grade rubber product Ecoflex 0010 was selected as a dummy material for passive human soft tissues. The mechanical behaviour of this silicone rubber is reported to be close to those of the human skin (Shergold et al., 2006), liver (Ozcan et al., 2011) and heart (Sarah et al., 2007). The different degrees of stiffness of the specimen can be obtained to mimic the mechanical properties of the biological tissue by adjusting the mixing ratio of the two part (part A and B) liquid form of the silicone rubber. The silicone rubber has an
ability to retain its shape and resistant to degradation (Shergold et al., 2006). However it has a somewhat higher elastic modulus, a lower rate of strain hardening but with a comparable toughness (Shergold and Fleck, 2005). Silicone rubber does not strain crystallize, and its crystallization temperature is well below room temperature. This is not the case for natural rubber and natural butyl rubber. The crystallization of a rubber leads to a rapid increase in the shear modulus.

The silicone rubber material is frequently used in movie industry for replicating the human parts like arm, hand, and even face. The material has been used by medical research industries for its human-tissue like characteristics. Moreover, it is easily obtainable from local distributors in many countries. For the present study, the commercial-grade silicone rubber Eco-flex 0010 soft (Smooth-On Inc.) was supplied by Bentley Advanced Materials, Frederick Road, Hoo Farm Industrial Estate, Kidderminster, Worcestershire, UK. The material contains two parts, part A is a base material that forms the main of the final specimen, and part B is a catalyst that hardens the silicone solution. The pilot study using different variants of silicone rubber (i.e. Dragon skin10, 30, 60 and Eco-Flex 0010 and 0030.) showed that the chosen Eco-flex 0010 having its overall best and realistic properties and composition compared with other variations of silicone rubber materials tested.

The tissue-like cylindrical samples were prepared by mixing the part A and B in the volume ratio of 1:2. The selected volume ratio provides the possible lowest stiffness value of the specimen - providing the lower resonance frequency while loaded with the sprung mass setup. Frequent stirring was necessary for obtaining a homogenous mixture with the approximate mixing time of 5 min. The poor mixing forms air bubbles within the mixture – result in difference in material properties locally. Average curing time for the current mixing ratio is 48 hours at a room temperature around 24 °C. Once the specimen is cured, it was taken out from the mould and left at 24 °C for another 48 hours before continuing with the actual test.

Homemade Plexiglas hollow-cylindrical blocks with different sizes were prepared as moulds for the silicone solution (see Figure 3.1). The Plexiglas was preferred since it has some advantages over other moulding materials. The Plexiglas does not form any chemical reaction with silicone solution. The cured silicone samples do not stick with Plexiglas mould’s surfaces and hence it is easy to remove the cured specimen from the mould. The samples with different foot-print and thicknesses were prepared.
for all the designated experiments (see Figure 3.2). A lot of experimentation and trials were required as the samples did not necessarily react in the same way as instructed. Some of the contributing factors for this could be ambient temperature, mixing time etc.

Figure 3.1 Photo of the equipment used in the Silicone rubber casting process: a - Plexiglas mould and b - Silicone solution (mixture of the two parts Silicone rubber).

Figure 3.2 Photo of the families of silicone specimens used in the designated experiments: dimensions showing the nominal diameter of 100, 75 and 50 mm and three different nominal thicknesses 10, 15 and 20 mm.
3.1.2 Apparatus

The dynamic compressive force-displacement tests were performed on a Bose Electro-Force 3200 testing machine under load control. The machine is capable of handling a loading frequency range from 0.00001 to 200 Hz, a maximum peak displacement of ± 6.5 mm, and a peak-to-peak force of ± 225 N. The excitation force of 100 sinusoidal cycles and ensuing displacement were recorded with different amplitudes and frequencies. The excitation force and the response displacement time histories were acquired for each condition at a sampling rate of 1000 points per cycle.

The preconditioning has been recommended for stabilising internal structure of soft tissues by Fung (1993). The number of cycles was chosen to be large enough to capture the steady-state response after a transient response and the preconditioning period. The specimen was first compressed with 10N static preload and then the dynamic peak-to-peak loads were applied at 20, 50, 75 and 100 N each with sinusoidal excitation frequencies of 2, 5, 10, 15, 20 and 40 Hz. The force was measured with a 225 N capacity load cell calibrated in the full load range according to ISO 7500-1:2004 class 0.5. See Figure 3.3 for a schematic of equipment used to acquire force and displacement.

![Diagram](image)

Figure 3.3 Schematic of the Bose Electro-Force 3200 for compression experiment: a - specimen, b - top moving platen, c - fixed bottom platen, d - excitation force applied from the top moving platen and e - load cell.
Two aluminium platens (b and c in Figure 3.3) were used to sandwich the specimen in between the upper and the lower clamps of the machine. Surfaces of the platens were well polished to avoid any frictional effects during the test. No glue was applied at the interface surfaces in between the platen and the specimen as the designated tests were pure compression. The machine was calibrated for misalignment beforehand according to the manufacturer’s recommendation. Acquired force and displacement signals were not filtered in order to avoid any phase disturbance between them. The time domain analogue filters would introduce phase shifts, which is not desirable when measuring the phase between the force and displacement.

3.1.3 Analysis

Based on a conventional mass-spring-damper single degree of freedom (SDOF) system, one could utilise the measured force and the displacement to calculate the complex elastic modulus (Young’s modulus). Consider the forced vibration of the SDOF with the rigid foundation, and the equation of motion becomes:

$$m \ddot{z} + c \dot{z} + k z = F(t)$$  \hspace{1cm} (3.1)

where $m$ is the mass, $k$ is the dynamic stiffness, $c$ is the damping, $F(t)$ is cyclic excitation force, and $\dot{z}$, $\ddot{z}$ and $z$ are response acceleration, velocity and displacement respectively.

The equation 3.1 can be rewritten by introducing the complex stiffness as discussed in (Snowdon et al., 1968):

$$m \ddot{z} + k^* z = F(t)$$  \hspace{1cm} (3.2)

where $k^*$ is the complex dynamic stiffness as a function of excitation frequency, $\omega$, and defined as $k^*(\omega) = k(\omega)(1 + i\eta(\omega))$, where $k(\omega)$ is the frequency dependant dynamic stiffness and $\eta(\omega)$ is the frequency dependant loss factor.

The Young’s modulus measured by the slope of a stress-strain curve is equivalent analogously to the stiffness defined by the slope of a force-displacement curve. The complex Young’s modulus ‘$E^*(\omega)$’ can thus be estimated using the same analogy used in complex stiffness.

$$E^*(\omega) = E(\omega)(1 + i\eta(\omega))$$  \hspace{1cm} (3.3)
where \( E(\omega) \) is the frequency dependant compressive elastic modulus (Young’s modulus). The first objective of the experimental study was to estimate the Young’s modulus, i.e. the real part of the complex modulus of the specimen subjected to dynamic compressive loads. \( E(\omega) \) can be calculated using the dynamic stiffness ‘\( k(\omega) \) (N/m)’, the cross-sectional area ‘\( A \) (m\(^2\))’ and the thickness of the specimen ‘\( L \) (m)’.

\[
E(\omega) = \frac{k(\omega) L}{A}
\] (3.4)

Hooke’s law emphasises that the stiffness is the slope of the line between the points of maximum force-maximum displacement and minimum force-minimum displacement. With the hysteretic behaviour, where the loading and unloading paths differ, Hooke’s law is no longer valid since the maximum displacement does not occur at the maximum force giving rise to an elliptical displacement-force curve (Montalvao et al. 2013). In the present study, the entire force-displacement curve was fitted with the first order polynomial function to estimate the dynamic stiffness and the frequency dependant elastic modulus.

### 3.2 Base-excited shaker test

The experiment is designed to investigate whether typical commercially available silicone rubber formulae (i.e. Ecoflex 0010 and Ecoflex 0030, Dragon Skin) when excited at the base could reproduce the dynamic behaviour of the human body during whole-body vibration. Such rubber has been widely used in the commercial world as ‘artificial soft tissue’ but its dynamic behaviour, particularly when one end is excited and the other end is free to move with an attached sprung mass, has not been examined. The experimental setup provide a more controlled environment to study the base-excited biodynamic system compared to a conventional test involving a live human subject. In the latter case there is considerably more variation in inertial and spatial parameters, such as the sprung mass, shape of the sprung mass, tissue thickness and size.
3.2.1 Apparatus

The testing apparatus was mainly composed of a mechanical shaker and a rigid mass-silicone rubber system shown in Figure 3.4. The base motion on the shaker table was controlled by a specialist electronic module via a PC. The base-excited rigid mass-silicone rubber system was considered a close representation of the *in vivo* loading scenario of the soft tissue at the excitation-subject interface. The interface could be the back of a recumbent, the sole of a standing or the buttocks of a seated human subject during whole-body vibration.

![Figure 3.4](image)

**Figure 3.4** Schematic representation of the experimental setup: setup showing base-excited sprung mass-soft tissue system, the three accelerometers and the two types of the sprung mass contact surfaces (each with three different weights 1.5, 2.5 and 5 kg).

3.2.1.1 Electro-dynamic shaker

The shaker was used to produce vibration in the vertical direction (i.e., z-axis of the specimen). The vertical electro-dynamic, shaker, Derritron VP85, was capable of accelerations up to ±550 ms⁻², a peak-to-peak displacement of 25.4 mm, and a
dynamic load of 38.5 kg in the vertical direction. The shaker table was made of a round rigid 20 mm thick and 200 mm diameter aluminium disc, which was rigidly bolted on top of the shaker armature – the moving end of the shaker. The other experimental equipment, such as the base platen of the specimen, accelerometers, accelerometer's mountings, were mounted rigidly onto the round shaker table (Figure 3.5). The excitation signal as input acceleration to the system is generated and sent to the shaker by a Data Physics shaker amplifier (DSA5-1K) and controller (SignalStar Vector II).

![Figure 3.5 Photographic representation of the experimental setup: the equipment and setup showing the base-excited rigid mass-silicone rubber system, the four accelerometers and shaker table.](image)

### 3.2.1.2 Sprung mass

Sprung mass assembly composed of two aluminium plates, an aluminium shaft and several lead weight discs (Figure 3.6). The lead weight discs with a hole in the middle were stacked together in-between two aluminium plates and rigidly screwed together (Figure 3.7). The bottom aluminium plate has three screw holes to attach an indenter plate. Two different contours of the sprung mass indenter surface were used in the test, one with a flat surface and another with protruded ‘hip-bone’ surface mimicking the bony structure of the pelvis. The flat indenter was made of Plexiglas and the hip-
bone indenter was made of aluminium alloy. Indenter surfaces were polished to the surface roughness $Ra$ of 1.817 $\mu$m. Figure 3.6 shows the sprung mass assembly setup with bottom aluminium plate rigidly screwed with a flat indenter plate made from Plexiglas.

**Figure 3.6** Photo of the lead weight disc used in the sprung mass assembly: disc having a diameter of approximately 100 mm and a thickness of 1 to 2 mm, mass of each about 160 (±10) g.

**Figure 3.7** Photo of the sprung mass assembly: setup showing a – aluminium shaft, b - top flat aluminium plate, c - lead weight discs, d - bottom flat aluminium plate and e - flat indenter plate (Plexiglas). Sprung mass has a circular cross-section and a diameter of approximately 110 mm and a height of 60 mm for 5 kg of mass.
The lead was chosen as a material to make the sprung mass as it has higher material density compared with other reasonably low cost alloys. It has an advantage of easiness of handling during the preparation process.

Four hip-bone knobs (indenter) were rigidly mounted to the base plate by screws (Figure 3.8 (d)). The knobs were equally spaced in the perimeter of the circle of approximately 40 mm diameter centred at the base plate.

![Figure 3.8](image)

Figure 3.8 Schematic and photographic representation of the two types of sprung mass indentation setup: (a) and (b) flat indenter; (c) and (d) hip-borne indenter (each indenter knob has a base diameter of 15mm and tapered thickness of 10mm).

3.2.1.3 Accelerometers

In all experiments, the input motion on the shaker table was measured and monitored using an IEPE type accelerometer Dytran 3097A3 with a range ±10 g, and a sensitivity of 500 mV/g (Figure 3.9 (a)). The ‘response’ accelerations on top of the sprung mass were measured in three orthogonal directions – vertical z-axis, horizontal x- and y-axis, by using three identical B&K 4507 B 006 ±14g accelerometers (Figure 3.9 (b), (c) and (d)). The three accelerometers (b, c and d)
had a sensitivity of approximately 490 mV/g with an operating range of ±14 g. And each had the same size of 10 by 10 mm and weight of 4.6 gram.

Figure 3.9  Photo of the accelerometers and their mounting studs used to measure accelerations at: (a) base (Dytran 3097A3 ± 0 g) in vertical ‘z-axis’; (b) on top of sprung mass in vertical ‘z-axis’ (B&K4507 B 006 ±14g); (c) on top of sprung mass in horizontal ‘x-axis’ (B&K4507 B 006 ± 4g); (d) on top of sprung mass in horizontal ‘y-axis’ (B&K4507 B 006 ± 4g).

The Dytran 3097A3 ±10 g accelerometer was attached to the base plate of the shaker via a piece of 20 x 20 x 15 mm aluminium mounting stud (6240A) (Figure 3.9). The accelerometer was rigidly screwed on top of the mounting stud and the mounting stud was rigidly screwed on top of the shaker table (Figure 3.5). The three B&K4507 B 006 ±14g accelerometers were attached separately to the top aluminium plate of the sprung mass via three identical pieces of 12 x 12 x 2 mm rigid plastic mounting stud B&K UA1407 (see Figure 3.9). The three identical accelerometer mounting studs were glued to the top aluminium plate of the sprung mass (Figure 3.10) using a commercial grade super glue. The interface between the B&K accelerometer and the mounting stud was slightly lubricated using commercial grade grease as recommended by the manufacturers. The resonance frequency of the lubricated mounting stud was higher than 2000 Hz which is beyond the frequency range of interest in the study (2 to 80 Hz).
Figure 3.10 Photo showing the orientation of the three B&K accelerometers mounted on top of the sprung mass: vertical z-axis, horizontal x- and y-axis.

Calibration of accelerometers using PCB hand held vibrator

The sensitivity of the accelerometers were checked and calibrated using portable hand held vibrator (PCB 394C06) before they were checked using electro-dynamic shaker. The PCB portable vibrator is a small, self-contained, battery powered, vibrator specially designed to conveniently verify sensitivity of the accelerometer. The accelerometer is mounted on top of the moving armature of PCB vibrator table using the accelerometer’s mounting stud. The accelerometer was connected to the DataPhysics ABACUS system in conjunction with a Dynamic Signal Analyzer software (SignalCalc 730). The user interface of the transducer calibration of the Signal Analyzer software provides the option for visualising and recording the calibrated sensitivity of the accelerometer. The time history and PSD of the calibration excitation and error tolerances can also be recorded. The vibrator is driven with a sinusoidal vibration at frequency and magnitude of 159.2 Hz and 9.8 ms\(^2\) r.m.s. respectively. And the vibrator can take the maximum load of 210 grams on top of its armature assembly. The sensitivity of the accelerometer is measured and recorded at that particular frequency of 159.2 Hz.
Calibration of accelerometers using electro-dynamic shaker

All the accelerometers were calibrated before each experiment and checked after the experiment. The accelerometers were mounted on top of the shaker table using the same mounting studs described above (see Figure 3.9). They were connected to the DataPhysics ABACUS system in conjunction with Vibration controller. The shaker was driven with a vertical z-axis broadband (2 to 80 Hz) random vibration at 0.5, 1.0, 1.5 and 2.0 ms\(^2\) r.m.s. and the response acceleration in vertical z-axis on top of the shaker table was acquired at the sampling rate of 204.8 Hz for 80 seconds using the four accelerometers.

The transfer function (transmissibility) between two accelerometers was computed using the measured accelerations. Ideally transfer function between two calibrated accelerometers rigidly placed at two adjacent points of the shaker table when set into motion should give a modulus of 1.0 and a phase delay of zero. Figure 3.11 shows the modulus and the phase of the computed transmissibility between two calibrated accelerometers.

![Transmissibility modulus and phase between two calibrated accelerometers](image)

**Figure 3.11** Transmissibility modulus and phase between two calibrated accelerometers rigidly mounted on top of the shaker table using broadband (2 to 80 Hz) random vibration at 2.0 (a and b) and 0.5 (c and d) ms\(^2\) r.m.s.

The modulus and phase are flat at 1.0 and 0 respectively throughout the frequency range of interest of this calibration (2 to 80 Hz). The standard error of the flatness is
within 1% for both modulus and phase. The coherence between two measured accelerations was flat at 1.0 throughout the frequency range of interest.

**Shaker rigidity check**

After the accelerometers were calibrated, the non-rigidities of the shaker in the cross-axis directions were checked by measuring the transmissibility between the input acceleration in z-axis and the cross-axis accelerations in y- and x-axis using the same accelerometers described above. The shaker was driven with a vertical z-axis broadband (2 to 80 Hz) random vibration at 0.5, 1.0, 1.5 and 2.0 ms\(^{-2}\) r.m.s. and the response acceleration in three orthogonal directions z-, y- and x-axis on top of the shaker table was acquired at the sampling rate of 204.8 Hz for 80 seconds. The cross-axis transmissibility between the input acceleration (vertical z-axis) and the acceleration perpendicular to the direction of input acceleration, i.e. the x- and y-axis were computed. Given entirely rigid mounting of accelerometers, the modulus of the cross-axis transmissibility should be zero for an ideal shaker whose motion is entirely in the vertical axis. **Figure 3.12** shows the modulus and the phase of the computed cross-axis transmissibility in x- and y-axis – they were flat at zero throughout the frequency range of interest except two spikes found at the frequencies 20 Hz and 30 Hz.

![Figure 3.12](image)

**Figure 3.12** The modulus of cross-axis transmissibility: longitudinal (z-x) during vertical excitation at 2.0 (a) and 0.5 (b) ms\(^{-2}\) r.m.s. and lateral (z-y) during vertical excitation at 2.0 (c) and 0.5 (d) ms\(^{-2}\) r.m.s.
These two spikes were consistently found with all the vibration levels tested i.e. 0.5, 1.0, 1.5 and 2.0 ms\(^{-2}\) r.m.s. These two spikes were caused by the spatial instability characteristics of the shaker at those frequencies - the motion of a body (shaker table) considered to be spatially unstable when it can execute the motion orthogonal to the direction of the driving force. The spatial instability occurs in shakers in connection with, either shifting of the centre of mass of the shaker table assembly relative to the symmetry axis of the shaker - along which the driving force acts, or nonlinearity of the elastic couplings of the moving part of the shaker with the frame. The coherence between input vertical acceleration and the cross-axis response accelerations in x- and y-axis shows the relatively high energy at those frequencies (see Figure 3.13).

![Figure 3.13 Coherences of cross-axis transmissibility: longitudinal (z-x) during vertical excitation at 2.0 (a) and 0.5 (b) ms\(^{-2}\) r.m.s. and lateral (z-y) during vertical excitation at 2.0 (c) and 0.5 (d) ms\(^{-2}\) r.m.s.](image)

3.2.2 Data acquisition

Signals - one excitation acceleration and three response acceleration channels - were acquired using a built-in 4-channel data acquisition system as part of the shaker controller (ABACUS, DataPhysics, software version V4.9.333). The controller uses a real-time control algorithm based on continuous convolution method to match the desired frequency band, waveform, excitation magnitude, sampling rate, and
sampling duration with the real time excitation acceleration signal monitored using an oscilloscope (Figure 3.14).

![Figure 3.14](image_url)  
**Figure 3.14** Schematic representation of the shaker controller used to control the shaker motion and to acquire the acceleration signals for the base excited experiment: blue lines showing the input/control signals and red lines showing the output/feedback signals.

A 1000-W power amplifier (DataPhysics DSA5-1K) was used to supply a voltage and a current to the parts of the shaker system. The ABACUS shaker controller uses a combination of a single analogue anti-aliasing low-pass filter at around 49 kHz and many digital anti-aliasing low-pass filters for each of its 83 frequency ranges executed in the digital signal processor (DSP) to provide a continuous stream of low-pass filtering. The digital anti-aliasing filter cut-off frequency is selected automatically by the controller software according to the frequency range of interest. The input and the output time histories were acquired at the sampling rate of 204.8 Hz for 80 seconds.

### 3.2.3 Analysis

After the data in the time domain had been acquired by the ABACUS system, the data were transferred to another computer where the frequency response functions and other analysis, such as the calculation of dynamic mechanical properties i.e.
stiffness, damping, elastic modulus and curve-fitting, was performed using MATLAB (version 10.0.1, R14).

### 3.2.3.1 Frequency response functions

For the transmissibility measurements, the vertical (z-axis) and horizontal (x- and y-axis) accelerations measured at the sprung mass were analysed relative to the vertical (z-axis) acceleration measured at the base (input motion). Three frequency response functions: one inline transmissibility (where the input acceleration was in-line with the acceleration measured at the sprung mass in the vertical direction, i.e. z-axis) and two horizontal cross-axes transmissibility (where the vertical acceleration measured at the base was perpendicular to the horizontal accelerations measured at the sprung mass, i.e. the x- and y-axis) were calculated using the cross-spectral density method:

The transfer function, $H(f)$, was determined as the ratio of the cross-spectral density of the input acceleration at base and the output acceleration at sprung mass, $G_{io}(f)$, to the power spectral density of the input acceleration, $G_{ii}(f)$:

$$H(f) = \frac{G_{io}(f)}{G_{ii}(f)}$$

(3.5)

where, $f$ is the frequency, in Hz; $H(f)$ is the transmissibilities, unit less parameter (inline or cross-axis transmissibilities).

The mathematical nonlinearity, the amount of the output motion that is not linearly correlated or not entirely caused by, the input motion in the calculated frequency response functions was investigated using the coherency:

$$\gamma^2_{io}(f) = \frac{|G_{io}(f)|^2}{G_{ii}(f)G_{oo}(f)}$$

(3.6)

Where $G_{oo}(f)$ is the PSD of the sprung mass acceleration and $\gamma^2_{io}(f)$ is the coherency of the system with a value between 0 and 1. The coherency has a maximum value of 1.0 in a linear single-input system with no noise - the output motion is entirely caused by, and linearly correlated with, the input motion.

The cross-spectral densities and the power spectral densities were both estimated via Welch’s method at frequencies between and 80 Hz. The frequency response functions for each of the 80s continuous random signals used a fast Fourier
transform (FFT) windowing length of 512 samples, a hamming window with 50% overlap, a sampling rate of 204.8 samples per second and an ensuing frequency resolution of 0.4 Hz with 128 degrees of freedom.

3.2.3.1 Dynamic stiffness and damping calculation

The lumped parameter models calibrated in frequency domain to represent the dynamic characteristics of the human body offer limited physical interpretation of the dynamic property of the interface soft tissue. The optimised parameter values (i.e. stiffness and damping) represented by these models were limited to a single frequency (i.e. the resonance frequency) and magnitude. The analytical technique described in this section aimed to evaluate the frequency dependant behaviour of these parameters using the base-excited rigid mass-silicone rubber system for whole frequency range of interest (2 to 80 Hz).

The rigid mass-silicone rubber system was modelled as a SDOF mass-spring-damper system with base excitation. Based on the SDOF system and an analytical formulation developed using the linear viscoelastic formulation in Appendix G, one could utilise the measured transmissibility to compute the frequency dependent dynamic stiffness $k$ (N/m) and damping $c$ (Ns/m).

3.3 Impact hammer test

The experiment is designed to investigate the frequency dependant mechanical properties (i.e. stiffness, damping constant and complex elastic modulus) of silicone rubber and the porcine muscle. The impact hammer test involves the use of a hand-held hammer to apply a light impact force on a rigid pre-load mass (sprung mass) placed on top of the test specimen, which is free to move. The experimental setup is expected to be less time consuming comparing with a base-excited shaker test - providing more chance for experimenters to execute the test using practically as many samples as possible within a relatively short period of time.

3.3.1 Specimen preparation

The impact hammer tests were carried out using silicone rubber specimen and the fresh porcine muscle specimen. The silicone rubber specimens were prepared as described in the Section 3.1.1. The fresh porcine specimen preparation and test were conducted at the Centre for Bioengineering, Trinity College Dublin, Dublin, Ireland. Porcine skeletal muscle tissue (*Gluteus Maximus*) dissected from a three
A month old female pig was chosen as a material for the specimen. After sacrifice of the animal (Time of death 12:45), specimens were prepared using the muscle of back leg. The specimens nominally 50 by 50 mm square with 10, 15 and 20 mm thicknesses were cut from the muscle using a scalpel so that the muscle fibres were roughly parallel to the plane of the flat test bench surface (Figure 3.15).

![Figure 3.15](image) Photo of the fresh porcine specimen with a nominal dimension of 50 ±2 mm and three different nominal thicknesses 10, 15 and 20 mm.

The entire specimens were ready for the test at the time of 13:30 and the test were started immediately once the specimens were ready and finished within two hours of animal death to minimise rigor mortis effects.

### 3.3.2 Apparatus

The sprung mass-soft tissue SDOF system was placed on a horizontally flat test bench that is rigidly attached to the ground (see Figure 3.16). The vertical impact force is aligned with the centre of the sprung mass shaft, and the horizontal impact force is in line with the centre edge of the sprung mass as shown in Figure 3.17. The impact hammer is equipped with a built-in force sensor (Dytran, model 5800B4) which has a sensitivity of 2.2 mV/N. Three B&K accelerometers - described in the Section 3.2.1.3 - were rigidly mounted on top of the sprung mass to measure the output accelerations in three orthogonal directions.
Figure 3.16 Schematic representation of the experimental setup showing impact hammer with vertical (-z) and horizontal inputs (-x, -y), the three accelerometers and the two types of sprung mass contact surfaces each with masses of 1.5, 2.5 and 5 kg.

Figure 3.17 Photos of the impact hammer test setup: (a) the equipment and setup showing the (1) - impact hammer (vertical input), (2) - specimen, (3) - sprung mass, (4) - accelerometers; (b) the same equipment and setup as in (a) but impact hammer showing the horizontal input (1).
3.3.2.1 Dynamic calibration

Calibration involves testing the sensitivity of an impact hammer with instrumented force sensor and an accelerometer to measure the response motion. A hammer can be calibrated by hitting a suspended rigid mass with an attached accelerometer. The impact force exerted on the rigid mass is produced by the product of the acceleration of the hammer head and the mass of the hammer head. According to Newton’s Second Law of motion, at any instant in time, the force experienced by the rigid mass is the product of the mass and the measured acceleration.

\[
\text{Force} = \text{Mass} \times \text{Acceleration} \quad (3.7)
\]

The frequency response function (FRF) implements the Newton’s Second Law of motion and is expressed as transfer functions describing the excitation-response relationships. The FRFs of the different formulations, accelerance - the response acceleration per unit force, mobility - the response velocity per unit force and receptance - the response displacement per unit force, are the commonly used ones in the experimental modal analysis. The accelerance was used in the present dynamic calibration. Acceleration divided by force (inverse of accelerance) is the reciprocal of the (dynamic) apparent mass of the structure between the measured points.

If a hung-rigid mass (block of metal) is impacted at the same point where the response is measured (driving point measurement), any resonances of the structure will typically be higher in frequency than those of interest (0 to 80 Hz) for this calibration. Below the first structural mode of the test specimen the value of the FRF will be ‘flat’ over a wide frequency range and will be equivalent to the reciprocal of the actual mass of the specimen.
The dynamic calibration was carried out on known hung-rigid masses of 1.5 kg and 2.5 kg. The impact hammer has a nominal sensitivity of 2.2 mV/N and an accelerometer has a nominal sensitivity of 500 mV/g. The response accelerometer was placed on the same end of the mass as the applied impact force - driving point measurement (Figure 3.18).

The accelerance were computed using the measured input impact force and the response acceleration. Five repeat measurements of hits were taken, to capture each hit, the time histories of the impact force (z-axis) and the response inline acceleration (z-axis) were acquired at 107520 samples per second for 1.219 second. By applying a Fast Fourier Transform (FFT) at this full length of data and a rectangular window, the accelerance was computed at a frequency resolution of 0.8203 Hz. In theory reciprocal of this accelerance should give the static weight of the known hung-mass in kg, so the inverse of the accelerance were taken and the data format was converted to magnitude-phase (see Figure 3.19).
Figure 3.19  The modulus of inverse of accelerance (1/accelerance; (kg)) for known masses of 2.5 kg (a) and 1.5 kg (b).

The results from the dynamic calibration (Figure 3.19) shows that the magnitude of the inverse of accelerance is flat over the frequency range of interest in this calibration (10-80 Hz) and approximately equivalent to 2.5 kg (Figure 3.19a) and 1.5 kg (Figure 3.19b) – confirming the known hung-masses of 2.5 kg and 1.5 kg respectively. The standard error of the flatness is within 2%. The small fluctuations in the magnitude of the inverse of accelerance below 5 Hz were suspected to be caused by the suspension arrangement of the hung-masses.

3.3.3 Data acquisition

The accelerometers and the force sensor of the impact hammer were connected to the same shaker controller but in conjunction with a Dynamic Signal Analyzer software of DataPhysics (SignalCalc 730). Signals – one input impact force and three response acceleration channels – were acquired using an inbuilt 4-channel data acquisition system as part of the shaker controller (Software version V4.9.329). Three to five repeat measurements of hits were taken from each test specimen. To capture each hit, the time histories of the impact force and three accelerations were acquired at 107520 samples per second for 1.219 second, i.e. a total of 131072 data points. The data acquisition system is schematically explained in Figure 3.20.
Figure 3.20  Schematic of the experimental set-up used to trigger the impact force and to acquire the output acceleration signals: blue lines showing the input impact force signal and red lines showing the output acceleration signals.

3.3.4 Analysis

Based on the SDOF mass-spring-damper model with the rigid foundation exposed to force vibration and an analytical formulation in Appendix H called ‘omega arithmetic’, one could utilise the measured complex FRF accelerance $A(\omega) = \ddot{Z}(\omega) / F(\omega)$, the ratio between resultant acceleration and input force in the angular frequency domain $\omega$ (rad/s), to calculate the complex receptance $R(\omega) = Z(\omega) / F(\omega)$, the ratio between resultant displacement and input force. By applying linear viscoelastic formulations, it is possible to compute the frequency dependent dynamic stiffness $k$ (N/m) and damping $c$ (Ns/m).

The time domain measured data was transformed into the frequency domain in the form of FRFs by applying a fast Fourier transform (FFT) at the full length of data and a rectangular window, the PSDs and FRFs were computed at a frequency resolution of 0.8203 Hz.
3.4 **Time domain analytical modelling (Chapter 4)**

Lumped parameter models developed in the frequency domain have been extensively used to represent the dynamic characteristics of the human body. But these models are restricted to single magnitudes of vibration (e.g. Wei and Griffin, 1998a; Matsumoto and Griffin, 2001; Nawayseh, 2003). Without knowing the response of the body at different magnitudes, a model giving a close fit to the measured response at one magnitude is not able to ‘predict’ the responses at the other magnitudes.

The classical linear and nonlinear viscoelastic models were chosen as numerical tools to evaluate their performance in predicting the biomechanical responses at different magnitudes of excitation. These time domain mathematical formulations (i.e. Bouc-Wen and Zener models) have been widely used in structural dynamics but never been tried with the whole-body vibration data.

**3.4.1 Linear and nonlinear lumped parameter model formulation**

The single-degree-of-freedom linear and nonlinear viscoelastic models were calibrated in time domain using the human body vibration experimental data of previous study (data from Huang and Griffin, 2009; Chapter 4).

![Schematic showing the linear single degree of freedom lumped parameter model with base excitation.](image-url)
Figure 3.22  Schematic showing the non-linear single degree of freedom lumped parameter models: (2a) Cubic stiffness model, (2b) Zener model and (2c) Bouc-wen model.

The selected models have their own purposes and the detail discussion is given in the individual chapters (Appendix C).

Variables in Figure 3.21 and 3.22:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ddot{z}_0$</td>
<td>Vertical excitation acceleration measured at base</td>
</tr>
<tr>
<td>$\ddot{z}_I$</td>
<td>Vertical acceleration at the intermediate point ‘I’</td>
</tr>
<tr>
<td>$\ddot{z}_1$</td>
<td>Vertical response acceleration at the sprung mass1</td>
</tr>
<tr>
<td>$F_{TF}$</td>
<td>Measured dynamic force at excitation-subject interface</td>
</tr>
<tr>
<td>$F_{RF}$</td>
<td>Dynamic restoring force</td>
</tr>
<tr>
<td>$m_0$</td>
<td>Unsprung mass</td>
</tr>
<tr>
<td>$m_1$</td>
<td>Sprung mass1</td>
</tr>
<tr>
<td>$c_L$</td>
<td>Linear damping constants</td>
</tr>
<tr>
<td>$k_L$, $k_{L1}$, $k_{L2}$</td>
<td>Linear stiffness constants</td>
</tr>
<tr>
<td>$k_{NL}$</td>
<td>Non-linear cubic stiffness constant</td>
</tr>
<tr>
<td>$F_{BW}$</td>
<td>Bouc-Wen force</td>
</tr>
</tbody>
</table>

3.4.1.1  Bouc-Wen model

Hysteretic nonlinear behaviour is often observed in wide ranges of processes in which the input–output dynamic relations between variables involve “memory dependent” effects. Examples can be found in magnetism, electricity, material and elastoplasticity of solids and among other areas (Worden and Tomlinson, 2001). In mechanical vibrations, hysteresis appears as a natural mechanism of materials to supply restoring forces against movements and dissipate energy. In these systems,
hysteresis refers to the memory dependent of inelastic behaviour where the restoring force depends not only on the instantaneous deformation but also on the history of the deformation.

The detailed modelling of these systems is often a difficult task. A model, which describes accurately many random non-linear vibrations of the above type, is the Bouc-Wen model. This model does not come, in general, from the detailed analysis of the physical behaviour of the systems with hysteresis. But, it combines some physical understanding of the hysteretic system along with the mathematical formulations. In the context of the single-degree-of-freedom (Figure 3.22c), equation of motion become:

\[ m_1 \ddot{z}_1 + c_L \dot{z}_{01} + k_L z_{01} + F_{BW} = 0 \]  

(3.8)

where \( F_{BW} \) is the hysteretic restoring force or Bouc-Wen force.

The hysteretic restoring force \( F_{BW} \) is given by the Bouc-Wen model of (Bouc, 1967):

\[
\dot{F}_{BW} = \begin{cases} 
A \dot{z}_{01} - \alpha \left| \dot{z}_{01} \right| F_{BW}^n - \beta \left| F_{BW} \right| \dot{z}_{01}, & \text{n odd,} \\
A \dot{z}_{01} - \alpha \left| \dot{z}_{01} \right| F_{BW}^{n-1} \left| F_{BW} \right| - \beta \dot{z}_{01} F_{BW}^n, & \text{n even} 
\end{cases}
\]

(3.9)

The model was "firstly introduced by Bouc in its basic form with \( n \) equal to unity (Bouc, 1967). This was able to describe the hysteresis loop based on the relationship between the restoring force, \( F_{BW} \), and the relative displacement, \( Z_{01} \). The parameters \( \alpha \) and \( \beta \) govern the shape of the loop. Wen generalized the Bouc equation to its final form by introducing the parameter \( n \) as given in Equation 3.9 (Wen, 1976). This parameter controls the smoothness of the hysteresis loop. The parameter \( A \) contributes to the linear stiffness of the oscillator. Due to the versatility of this model to represent various shapes of hysteresis loops, it has attracted much attention in the area of hysteretic vibrations.

### 3.4.1.2 Parameter effects in the Bouc-Wen model

The Bouc-Wen model is capable of producing different hysteretic behaviours. Three hysteretic parameters \( \alpha \), \( \beta \) and \( n \) and their interactions determine the basic hysteresis shape. Absolute values of \( \alpha \) and \( \beta \) inversely influence hysteretic stiffness.
and strength, as well as the smoothness of the hysteresis loops. For \( n=1 \), the relationships between \( \alpha \) and \( \beta \) and their effects on hysteresis are described below and shown in Figure 3.23 (Sengupta and Li, 2011).

![Figure 3.23 Possible Hysteresis Shapes for \( n=1 \) (Sengupta and Li, 2011).](image)

Figure 2.23 illustrates mainly five possible hysteretic shapes for different combination of \( \alpha \) and \( \beta \) for \( n=1 \). They can be summarized as follows (Sengupta and Li, 2011):

- **Case 1**, Weak Softening: \( \alpha + \beta > 0 \)
  \[ \beta - \alpha < 0 \]

- **Case 2**, Weak Softening on loading, mostly linear unloading
  \[ \alpha + \beta > 0 \]
  \[ \beta - \alpha = 0 \]

- **Case 3**: Strong Softening on loading and unloading, narrow loop
  \[ \alpha + \beta > \beta - \alpha \]
  \[ \beta - \alpha > 0 \]

- **Case 4**, Weak Hardening: \( \alpha + \beta = 0 \)
  \[ \beta - \alpha < 0 \]

- **Case 5**, Strong Hardening: \( \alpha + \beta > \beta - \alpha \)
  \[ \beta + \alpha < 0 \]

Human responses to vibration showed a memory dependent or the hysteretic behaviour, force-displacement characteristics of the WBV produced a hysteretic loop
Thus, it is plausible that Bouc-Wen model may well describe such hysteretic behaviour of the human body to vibration. In chapter 4, Bouc-Wen model, with $n$ is equal to 1, is used in time domain to predict the human responses at two different magnitudes (0.125 and 1.0 ms$^2$ r.m.s.).

**Curve-fitting and optimisation in time domain**

The equations of motion of the above described four models were formulated in Simulink and integrated forward in time, in each optimisation iteration, using fixed-step forth-order Runge-Kutta algorithm. The equations of motion and Simulink models are described in Appendix C.

The unknown parameters (i.e. $m_0$, $m_1$, $k_L$, $c_L$) in the above described time domain models were optimised by comparing, measured and calculated response of the whole-body vibration data (In Chapter 4) using a MATLAB version 10.0.1, R14. The adaptive differential evolution algorithm ‘JADE’ (Zhang and Sanderson, 2008) was used as an optimisation tool to minimise the error between measured and the calculated response. The procedure involved in JADE is described in Appendix D. The custom made MATLAB code was implemented for the JADE algorithm and the m-file of the script was given in Appendix F.

**3.5 Summary**

This chapter describes the methodology adopted for this research study. The research study includes four technical chapters, Chapter 4, to 7. The chapter 4 is purely an analytical study using an experimental data from the previous study (Huang and Griffin, 2009). The formulation of the mathematical models, description of the optimisation procedure used for the particular chapter (Chapter 4) were described in this chapter. The rest of the three studies are experimental studies. The short description of the experimental procedure, apparatus used in the studies, description of the specimen preparation, description of the data capturing techniques and the description of the analysis used to interpret the data were explained and discussed in this methodology chapter.
CHAPTER 4
A preliminary study on analytical modelling of biomechanical responses of whole-body vibration in time domain

4.1 Introduction

The frequencies of the major resonance of the human body during vertical whole-body vibration were found around 5 to 6 Hz for a seated or 8 to 10 Hz for a supine subject seen in ordinary FRFs of transmissibility and apparent mass. This resonance frequency decreases with increasing excitation magnitude. The biodynamic nonlinearity has been consistently observed with all different stationary sitting, standing, and recumbent postures, and different directions of excitation and different axes of responses (e.g. Fairley and Griffin, 1989; Nawayseh and Griffin, 2005a, b; Huang and Griffin, 2008a, b).

The ordinary coherence functions associated with the response acceleration and dynamic forces exhibited values above 0.8 to 0.9 in the frequency range of the major resonances. This suggested that the nonlinear magnitude-dependent responses were ‘coherent’ to their excitations – not seen in applications of nonlinear structural dynamic analysis such as conditioned reverse path (CRP) or restoring force surface (RFS) (e.g. Kerschen, 2002).

The nature of the softening effect in the human soft tissue may be considered as a result of the changes in viscoelastic properties at different magnitudes of excitation. Typically the effective stiffness and damping at the excitation-subject interface may depend on the relative displacement and relative velocity between the un-sprung and sprung mass if considering a SDOF model of the major resonance of the human body. The lumped parameter models with single (Fairley and Griffin, 1989) or multiple degrees of freedom (Zheng, 2012) were studied, to describe the magnitude-dependent whole body vibration, with seated (e.g. Fairley and Griffin, 1989; Wei and Griffin, 1998a), standing (e.g. Matsumoto and Griffin, 2003; Subashi et al., 2009) and supine postures (Vogt et al., 1978). Several studies have included rotational elements in the biodynamic models to allow two-dimensional motion (e.g. Matsumoto and Griffin, 2001; Qiu, 2007; Nawayseh and Griffin 2009). But, these models have not described the mechanism responsible for the magnitude-dependent whole-body
vibration and hence are not able to predict the responses at varying magnitudes. Hysteretic or memory dependent phenomena are observed in many areas of structural dynamics. One of many available models of hysteresis, one which is often used in the context of dynamics is the Bouc-Wen model. The recent book (Ikhouane and Rodellar, 2007) provides detailed summary of the research on Bou-Wen model and can be consulted as a comprehensive guide to the literature.

To predict the restoring force, Bouc-Wen formulation of the nonlinear component of the effective stiffness offers a possible solution to describe the magnitude dependence in the time domain. Recent development in differential evolution (DE) algorithm and its self-adaptive variation (SADE) and (JADE) made it possible to obtain optimisation results with a varying exponential parameter in the Bouc-Wen model (Worden and Manson, 2012).

The objective of this preliminary study was to investigate Bouc-Wen model along with other single degree of freedom lumped parameter models in time domain to predict the dynamic forces at the excitation subject interface of the Whole-Body vibration. This study was designed as the preliminary study and the first attempt of the preliminary observation will only be reported.
4.2 Experimental data

Preliminary models of the restoring force used the vertical (x-axis) excitation acceleration and vertical response acceleration at the sternum of a supine subject (S12) during two magnitudes of vibration (Huang and Griffin, 2009). The supine human body was exposed to vertical (x-axis) random vibration, nominally flat from 0.25 to 20 Hz, at two vibration magnitudes, nominally 0.125 and 1.0 ms\(^{-2}\) r.m.s. (Figure 4.1). The acquired data included: the vertical excitation acceleration (\(\ddot{x}_0(t)\) in Figure 4.1), the vertical response acceleration at the sternum of the subject (\(\ddot{x}_1(t)\) in Figure 4.1), and the vertical force at the excitation-subject interface of the back support (\(F_{TF}(t)\) in Figure 4.1). All data were acquired for 90 seconds at 200 samples per second with an anti-aliasing filter of 67 Hz.

In the following sections, the measured excitation and response accelerations (\(\ddot{x}_0(t)\) and \(\ddot{x}_1(t)\)) were used to ‘calculate’ the total restoring force, and then the ‘calculated’ compared with the measured total restoring force (\(F_{RF}(t)\)).

![Figure 4.1](image)

Figure 4.1 Schematic showing experimental setup for the vertical whole-body vibration (adapted from Huang and Griffin, 2009).
The vertical point apparent masses and transmissibility between base and sternum of one semi-supine subject (S12) measured at two magnitudes of continuous broadband random (nominally flat 0.25 to 20 Hz) vibration, i.e. 0.125 and 1.0 ms$^{-2}$ r.m.s., are used in this study (data from Huang and Griffin, 2009). Figures 4.2, 4.3, 4.4, 4.5 are based on data of this experimental study.

**Figure 4.2** Apparent mass (a), phase (b) and coherence (c) of the semi-supine subject (S12) exposed to vertical random vibration at 1.0 ms$^{-2}$ r.m.s. (data from Huang and Griffin 2009).
Figure 4.3  Power spectral densities for the vertical excitation acceleration ($G_{00}$) and vertical dynamic force ($G_{11}$) measured at the excitation-subject interface (or driving point) of the semi-supine subject (S12) exposed to vertical random vibration at 1.0 ms$^{-2}$ r.m.s. (data from Huang and Griffin, 2009).

4.3  Model formulation and data processing
4.3.1  Integration of time histories

Worden and Tomlinson (2001) showed that numerical integration procedures can be used to produce estimated velocity and displacement data from acceleration measurements by a variety of methods. Time domain trapezium integration method was used in the present study. Due to accumulated error at low frequencies, the method is prone to a linear ‘drift’ in the mean level of the integrated signal as the time elapses (Worden and Tomlinson, 2001). Nevertheless, such drawback is resolved by introducing a high-pass filter (with zero phase shifts) before and after an integration to remove the low frequency error. Careful selection of the high-pass cut-off frequency is needed. If the frequency is too low the data remains ‘drifted’ and if the cut-off frequency is too high, the filter removes useful data which should remain
in the signal. In the present study, all signals were high-pass filtered at 1.5 Hz using 4-pole Butterworth zero-phase filter before and after the first integration, and low-pass filtered at 30 Hz using 4-pole Butterworth zero-phase filter once before the first integration. The signals were normalised before and after each integration to remove their means. Figure 4.4 shows the velocity and displacement time histories obtained by integrating the measured accelerations, and, the measured dynamic force time histories for the two magnitudes of excitation. Figure 4.5 shows the power spectral density for the measured accelerations, calculated velocities and displacements for the two magnitudes of excitation. Refer to Appendix E for the integration procedure.

Figure 4.4  Time histories of the vertical excitation acceleration at the base (black solid line) and at the sternum (red solid line), velocity after first integration v_L and v_H for the low magnitude 0.125 ms^{-2} r.m.s.) and v_H and ‘H’ for the high magnitude 1.0 ms^{-2} r.m.s.), displacement after the second integration d_L and d_H and measured vertical restoring force f_L and f_H with each having a duration of 90 seconds. Data from Huang and Griffin (2009).
4.3.2 Analytical models formulation

Four different analytical models are formulated, details of the models are described in Chapter 3 and Appendix C.

4.3.3 Optimisation

The parameters in the described time domain models (Chapter 3 and Appendix C) were optimised by comparing, measured and calculated restoring force $F_{RF}$ at the excitation subject interface. JADE algorithm (refer to Appendix D for detailed description) was implemented in MATLAB to execute the optimization process, using a MATLAB version 10.0.1, R14. The normalised mean square error (MSE) was used to evaluate the goodness of fit:

$$MSE = \frac{100}{N \sigma} \sum \left[ F_{RF} - \hat{F}_{RF}(a) \right]^2$$  \hspace{1cm} (4.1)

where, $a$ is model parameters, $F_{RF}$ is the measured restoring force and $\hat{F}_{RF}(a)$ is the calculated restoring force and $N$ is the number of samples and $\sigma^2$ the variance (or standard deviation) of the measured restoring forces. The procedure was used to calculate MSE when the parameters were optimised using measured and calculated response displacement data.
4.4 Preliminary result and discussion

As a preliminary result, first optimised curve-fit for the Bouc-Wen and the SDOF linear models were compared (Figure 4.6).

![Figure 4.6](image)

*Figure 4.6*  The measured and predicted time histories of the restoring force at low magnitude 0.125 ms$^{-2}$ r.m.s. (left) and at high magnitude 1.0 ms$^{-2}$ r.m.s. (right). Zoomed time history slice was shown out of 90 s data.

The time histories of the restoring force generated from identified parameters using the basic Bouc-Wen and the linear SDOF were shown not to agree well with those measured. Similar observation was found with cubic stiffness and Zener and the cubic damping model. The SDOF mass-spring-damper model gives better fit than the basic Bouc-Wen model. The basic Bouc-Wen model, with exponential term ‘$n$’ equals to 1, was not able to capture small variation of high frequency components and especially so at the lower magnitude of 0.125 ms$^{-2}$ r.m.s. These high frequency components were thought be caused by noise and distortion at the lower magnitude. But an inspection of the ordinary coherence functions between the measured dynamic force and the input excitation (i.e. coherence of the apparent mass) reveals high coherence between 1 and 20 Hz, and any higher-frequency components of the response force were rolled off in the original data (see $G_{00}$ in Figure 4.3).
4.5 Conclusions

The dynamic force of the semi-supine human body at the excitation-subject interface is characteristic of a SDOF system with a major resonance at a relatively low frequency around 8 to 10 Hz comparing to nonlinear structural dynamic systems. The numerous viscoelastic models for biological soft tissues were primarily based on quasi-static and cyclic loading at limited magnitudes of excitations. There has been no reported analytical study on the base-excited nonlinear magnitude dependence of driving point response dynamic force of viscoelastic materials. The present study tried to model the restoring forces at the excitation-subject interface using four different time domain models. The model failed to predict the responses at multiple vibration excitations. This was suspected to be caused by the integration technique used in this study ‘Runge-Kutta’.

Further study has been done using the simulated data of SDOF linear model to see the ability of JADE along with Runge-Kutta to identify the true parameters of the simulated data. For the time being in the thesis writing, the results were not shown here. Two different configuration of the SDOF linear model, base-fixed and base-excited were used to generate the simulated data. Then JADE was used along with Runge-Kutta to identify the parameters of the simulated data. Interestingly, the parameters of the base-fixed configuration were identified by the JADE accurately (less than 0.1 %). However the estimated parameters using base-excited configuration, showed relatively large error between true value and identified value. This may be partly because in the base-excited configuration relative motion is involved and integration error could be large compared with the Base-fixed configuration. This begs more research on this area.
CHAPTER 5

Cyclic compression test on silicone rubber specimen

5.1 Introduction

Biomechanical behaviour of soft tissue at the excitation-subject interface play vital role in transmitting motion to and through the human body during the whole-body vibration. The soft tissue at such interface has been recently found to show thixotropic or memory dependent behaviour.

Assessment of elastic and damping properties - complex modulus - of the biological and artificial soft tissue materials are in growing needs with the intensive current use of biomechanical properties of these materials for assessment of current human body models.

One of the most frequently used experimental methods for characterizing the viscoelastic material properties of biological and artificial soft tissue materials is the dynamic oscillation experiment. In this test, small periodic displacements at varying frequencies are applied to the specimen and the force response is measured. The cylindrical viscoelastic sample is placed between the two plates which are attached to each end of the machine clamp - excitation is applied at the one end and the other end is fixed.

The present study intends to characterise dynamic property of a silicone using uni-axial cyclic compression test. The test was designed to examine the biomechanical properties, i.e. stiffness and elastic modulus, of silicone rubber: first to compare these properties with the published data for the soft tissue and then to examine whether they can produce any thixotropic or memory effect observed in human vibration response when one end of the specimen is fixed and another subjected to cyclic loadings. It was suspected that silicone rubber would not be able to represent the thixotropic behaviour observed in WBV.
5.2 Method

The experimental study is conducted using a cyclic compression test using silicone rubber specimen. Specimens were cyclically excited with different magnitude of the force and the frequency in force-controlled mode. Input force and the resultant displacement were recorded for the analysis. Refer to Section 3.1.2 in chapter 3 for the methodology.

5.2.1 Apparatus

The dynamic compressive force-displacement tests were performed on a Bose Electro-Force 3200 testing machine under load control. The machine is capable of handling a loading frequency range from 0.00001 to 200 Hz, a maximum peak displacement of ± 6.5 mm, and a peak-to-peak force of ± 225 N. The excitation force of 100 sinusoidal cycles and ensuing displacement were recorded with different amplitudes and frequencies. The excitation force and the response displacement time histories were acquired for each condition at a sampling rate of 1000 points per cycle. The specifications of the machine and the equipment were described in Sections 3.2.1 to 3.2.2 in Chapter 3.

Figure 5.1 Photo of the Bose Electro-Force 3200 for compression experiment: a – specimen and b – force cell.
5.2.2 Specimen

Silicone rubber specimens were prepared with 50-mm diameter and 10-mm thickness using commercial grade silicone rubber Ecoflex 0010 *(Smooth-On Inc., 2014)*. The procedure involved in specimen preparations was described in Section 3.1.1 in Chapter 3. Table 5.1 described some of the mechanical properties of particular silicone rubber provided by the supplier. The results in table 5.2 and 5.3 were obtained using the specimens which were prepared using the volume ratio of 1:2 of A and B.

Table 5.1 Mechanical properties of the silicone rubber Ecoflex0010 measured by supplier using 1:1 volume ratio of part A and part B *(Smooth-On Inc., 2014)*.

<table>
<thead>
<tr>
<th>Shore hardness*</th>
<th>Viscosity (mPa.s)</th>
<th>Density (kg/m$^3$)</th>
<th>Tensile strength (kPa)</th>
<th>Tearing Strength (kN/m)</th>
<th>Elongation at break (%)</th>
<th>Shrinkage (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>00-10</td>
<td>14000</td>
<td>1040.6</td>
<td>827.4</td>
<td>3.853</td>
<td>800</td>
<td>&lt; 0.001</td>
</tr>
</tbody>
</table>

* Shore hardness: A measure of resistance to indentation – scale 00 measures very soft rubbers and gels. A value between 0 and 100 indicates softness within that scale with lower values indicating a softer material *(Smooth-On Inc., 2014)*.

5.2.3 Analysis

First, force - displacement characteristic diagram was generated using the measured input force and the response displacement. The dynamic stiffness was estimated using the gradient of the force - displacement graph. First order polynomial function was used to curve-fit the force - displacement graph to obtain the gradient. Using the Dynamic stiffness value, the elastic modulus was estimated using the procedure described in Section 3.13 in Chapter 3.
5.3 Results

A total of 5 silicone rubber specimens were tested. First, 4 specimens of 50-mm diameter and 10-mm thickness were tested with two different excitation force (20 and 50 N) and two different excitation frequencies (2 and 5 Hz) for intra- and inter-specimen variability. Then 1 specimen of 50-mm diameter and 10-mm thickness was examined for four different excitation force and six different excitation frequencies. The dependent variables were dynamic stiffness and compressive elastic modulus.

In the first set of experiment, each of the four specimens was subjected to five repeated tests to show their intra-specimen variability, mean and the standard deviation of the mean were used. Inter-specimen variability was then tested using the 4 nominally identical specimens. Parametric statistical tests were employed using analysis of variance (ANOVA) method. For inter-specimen variability, one-factor analysis of variance was used with main effect as four identical specimens.

In the second set of experiment, the specimen with 50-mm diameter with 10-mm thickness was investigated with four different excitation force and six different excitation frequencies but only one specimen with each test specification. This allows examination of effects of magnitude of excitation force and the frequency. Two-factor ANOVA was used to determine whether there was a difference between different level of measures or dependent variables (i.e. stiffness and elastic modulus) with different experimental conditions or independent variables (i.e. magnitude of excitation force and frequency).

One fact that should emphasised that the correlation coefficients were close to unity ($r^2=0.999$) in the linear-fits for all the curve-fit in this study.

5.3.1 Inter-specimen and intra-specimen variability

With the uni-axial cyclic compression, force - displacement curves were shown for two different magnitudes of excitation force and frequencies (Figure 5.2) for the specimen S1. With each specimen subjected to five repeated tests, identical force - displacement curve was observed. This was consistent for all four excitation magnitudes and frequencies. The standard deviation of the mean of the different repeated measurements of the same specimen tends to be similar for the entire specimen range and it is less than 3% for all cases (refer to Table 5.2). Amongst
the four nominally identical specimens, no significant difference was found in stiffness and elastic modulus ($p>0.05$, one-factor ANOVA).

Figure 5.2  Hysteretic force-displacement diagram of the specimen S1 with different frequencies (2 and 5 Hz) at compressive force of 20 and 50 N peak to peak. The negative values in the force - displacement graph indicating the pure cyclic compression.
Table 5.2  Mean dynamic stiffness $k$ and Elastic modulus $E$ (± standard deviation of the mean) over five repeats extracted using two different excitation frequencies (2 and 5 Hz) and magnitude of force (20 and 50 N). Silicone rubber specimen with 50 mm diameter and 10 mm thickness was used. Results from four nominally identical specimens S1 to S4 were shown.

<table>
<thead>
<tr>
<th>Specimen number</th>
<th>Excitation load (N)</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k$ (kN/m)</td>
</tr>
<tr>
<td>S1</td>
<td>20</td>
<td>40.6 ± 0.9</td>
</tr>
<tr>
<td>S1</td>
<td>50</td>
<td>49.3 ± 0.9</td>
</tr>
<tr>
<td>S2</td>
<td>20</td>
<td>40.9 ± 0.8</td>
</tr>
<tr>
<td>S2</td>
<td>50</td>
<td>49.9 ± 0.9</td>
</tr>
<tr>
<td>S3</td>
<td>20</td>
<td>41.1 ± 0.8</td>
</tr>
<tr>
<td>S3</td>
<td>50</td>
<td>49.5 ± 1.1</td>
</tr>
<tr>
<td>S4</td>
<td>20</td>
<td>40.1 ± 0.6</td>
</tr>
<tr>
<td>S4</td>
<td>50</td>
<td>50.1 ± 0.9</td>
</tr>
</tbody>
</table>
5.3.2 Effect of excitation force

The dynamic stiffness and elastic modulus increased as the peak to peak excitation force increased from 20 to 50, 75 and 100 N (Figure 5.3). This was consistent with all the excitation frequencies tested (refer to Table 5.3 for values). The effect of peak to peak excitation force on the dynamic stiffness ($p=0.000$) and the elastic modulus ($p=0.000$) were significant ($p<0.05$, two-factor ANOVA for both).

Figure 5.3 The variation in the dynamic stiffness (left) and elastic modulus (right) with the four different peak to peak excitation forces (20, 50, 75 and 100 N): with 2 Hz (hollow square), 5 Hz (hollow diamond), 10 Hz (upward-pointing triangle), 15 Hz (circle), 20 Hz (cross) and 40 Hz (asterisk) of excitation frequencies. Specimen with 50-mm diameter and 10-mm thickness was used. Refer to Table 5.3 for values.
Table 5.3  Mean dynamic stiffness $k$ and Elastic modulus $E$ (± standard deviation of the mean) over five repeats extracted using six different excitation frequencies (2, 5, 10, 15, 20 and 40 Hz) and four different magnitudes of force (20, 50, 75 and 100 N). Silicone rubber specimen with 50 mm diameter and 10 mm thickness was used.

<table>
<thead>
<tr>
<th>Force(N)</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$k$ (kN/m)</td>
<td>$E$ (kPa)</td>
<td>$k$ (kN/m)</td>
<td>$E$ (kPa)</td>
<td>$k$ (kN/m)</td>
<td>$E$ (kPa)</td>
</tr>
<tr>
<td>20</td>
<td>40.6 ± 0.9</td>
<td>55.2 ± 0.9</td>
<td>65 ± 0.1</td>
<td>73 ± 0.9</td>
<td>80.3 ± 0.7</td>
<td>89.7 ± 1.3</td>
</tr>
<tr>
<td></td>
<td>211 ± 4.2</td>
<td>280 ± 4.2</td>
<td>330 ± 5.4</td>
<td>370 ± 4.2</td>
<td>410 ± 4.2</td>
<td>460 ± 5.9</td>
</tr>
<tr>
<td>50</td>
<td>49.3 ± 0.9</td>
<td>61.6 ± 0.9</td>
<td>78.5 ± 0.8</td>
<td>89.9 ± 0.9</td>
<td>96.8 ± 1.2</td>
<td>132 ± 0.9</td>
</tr>
<tr>
<td></td>
<td>251 ± 4.6</td>
<td>311 ± 4.7</td>
<td>400 ± 4.5</td>
<td>460 ± 4.6</td>
<td>490 ± 5.7</td>
<td>670 ± 4.6</td>
</tr>
<tr>
<td>75</td>
<td>60.9 ± 0.8</td>
<td>84 ± 0.8</td>
<td>93.4 ± 0.7</td>
<td>102 ± 0.8</td>
<td>117 ± 0.9</td>
<td>143 ± 1.1</td>
</tr>
<tr>
<td></td>
<td>310 ± 4.6</td>
<td>430 ± 4.4</td>
<td>480 ± 4.5</td>
<td>520 ± 4.6</td>
<td>600 ± 4.6</td>
<td>730 ± 5.6</td>
</tr>
<tr>
<td>100</td>
<td>74 ± 0.9</td>
<td>88.9 ± 0.9</td>
<td>117 ± 1.1</td>
<td>126 ± 0.9</td>
<td>126 ± 1.2</td>
<td>164 ± 1.2</td>
</tr>
<tr>
<td></td>
<td>380 ± 4.6</td>
<td>450 ± 4.6</td>
<td>600 ± 5.7</td>
<td>640 ± 4.6</td>
<td>640 ± 5.8</td>
<td>830 ± 5.7</td>
</tr>
</tbody>
</table>
5.3.3 Effect of excitation frequency

The dynamic stiffness and elastic modulus increased as the excitation frequency increased from 2 to 5, 10, 15, 20 and 40 Hz (Figure 5.4). This was consistent with all the excitation peak to peak forces tested (refer to Table 5.3 for values). The effect of excitation frequencies on the dynamic stiffness ($p=0.000$) and the elastic modulus ($p=0.000$) were significant ($p<0.05$, two-factor ANOVA for both).

![Figure 5.4](image)

Figure 5.4  The variation in the dynamic stiffness (left) and elastic modulus (right) with the six different excitation frequencies (2, 5, 10, 15, 20 and 40 Hz): with 20 N (hollow square), 50 N (hollow diamond), 75 N (upward-pointing triangle) and 100 N (circle) of peak to peak excitation forces. Specimen with 50-mm diameter and 10-mm thickness was used. Refer to Table 5.3 for values.

5.4 Discussion and conclusions

Before the experiment described in this chapter started, a pilot study was conducted to examine different variation of silicone rubber compounds (e.g. Dragon Skin, Ecoflex 0010 and Ecoflex 0030) to see which compound gives the lowest stiffness and static Youngs' modulus which are close to the biological soft tissue. Based on the pilot study, Ecoflex 0010 was chosen as a candidate for this experiment. Static Youngs' modulus of the Ecoflex 0010 was $25.6 \pm 0.8$ kPa which is in the range of published value for the porcine skeletal muscle tissue (Loocke et al., 2006; Sparks et al., 2015).

The uni-axial cyclic compression test was designed to examine the biomechanical properties, i.e. stiffness and elastic modulus, of silicone rubber: first to compare
these properties with the published data for the soft tissue and then to examine whether they can produce any thixotropic or memory effect observed in human vibration response when one end of the specimen is fixed and another subjected to cyclic loadings. The stiffness values obtained for the silicone rubber were in the same range of that published for the porcine muscle (Aimedieu et al., 2003; see Table 2.1A for values). The compressive elastic modulus shows relatively higher values than most of the published data for the porcine muscle and human tissue (Loocke et al., 2009; Ozcan et al., 2011; Sparks et al., 2015; see Table 2.1 A, B for values). However one particular study showed similar order for the elastic modulus reported in this study (Saraf et al., 2007; see Table 2.1B for values).

The test showed that the stiffness of the silicone rubber increased (stiffening) with increasing excitation force and the frequency. Similar stiffening effect was reported in the compression tests of skin and adipose tissue (Wu et al., 2007). However this observation deviated from the thixotropic effect or softening effect of excitation-subject interface of WBV where the stiffness of the soft tissue at such interface decreased with increasing excitation magnitude and frequency. Biomechanical properties largely depend on excitation levels (Fung, 1993). In the cyclic compression test, the specimen is placed between two clamps, the excitation is applied on one end and other end is fixed. This is not the realistic boundary condition of the soft tissue at the excitation-subject interface of the WBV. In the next chapter, Chapter 6, the silicone rubber specimen will be tested using the base-excited shaker test using broadband random vibration with different magnitudes of excitation.
CHAPTER 6
Base-excited shaker test on rigid mass-silicone rubber system

6.1 Introduction

Biomechanical responses, i.e. apparent mass and transmissibility, play vital role in assessing health and comfort risk caused by whole-body vibration (WBV). Key to understanding of any biomechanical responses during WBV is the dynamic behaviour of soft tissues at the excitation-subject interface. Such interface could be buttock of seated, back of a recumbent or soles of a standing subject (Huang and Griffin, 2009). Skeletal muscle tissue deforms almost 50% more than its adjacent fat tissue (Shabshin et al., 2010). The skeletal muscle tissue at the interface is likely to play a crucial role in transmitting force and motion to the human body.

Usually a base-excited linear single-degree-of-freedom (SDOF) mass-spring-damper system was employed to describe the principal resonance of a seated, standing or recumbent person (e.g. Huang and Griffin, 2008b). Such system requires a base excitation with the sprung mass free to move. The parameters in this linear model, i.e. stiffness, damping and mass, were obtained by fitting the model with experimental results in the form of either time histories or FRFs. Such approach offers limited interpretation of the dynamic properties of the soft tissue at interface. Damping and stiffness characteristics of real viscoelastic materials are usually frequency dependent or strain rate dependent (Loocke et al., 2009; Ozcan et al., 2011). Constant stiffness and damping values obtained by fitting linear models for the full frequency range of interest are questionable.

The dynamic property of porcine skeletal muscle subjected to base excitation loaded with a sprung mass as an analogue based excited SDOF was investigated by Aimedieu et al. (2003). The authors extracted frequency dependent dynamic stiffness and damping parameters from transmissibility so as to facilitate a finite element analysis of a buttocks model for evaluating discomfort of automotive seats. The study was limited to base excitation acceleration of 1.5 ms$^{-2}$ r.m.s. due to limited displacement of the shaker. The stiffness and damping increased with increasing excitation frequency from 20 to 30 Hz. The damping constant was around 300 Ns/m from 5 to 20 Hz and around 556 Ns/m at 30 Hz. The average stiffness ranged from
8.5 kN/m at 5 Hz to 347 kN/m at 30 Hz. The specimens were tested after 24 hours of animal death, storing at 4°C and testing at 37°C in a bath. Most other biomechanical studies were performed within 2 hours of animal death to minimise rigor mortis effect (e.g. Takaza et al., 2013a). This study did not offer any information about how changes in the magnitude of base excitation affect the resonance frequency, stiffness and damping.

An alternative approach was to substitute the biological tissue with artificial materials with similar viscoelastic characteristics and reconstruct the excitation-subject interface. But most studies characterising biological and artificial soft tissues employed quasi-static (e.g. Loocke et al., 2006; Song et al., 2007), cyclic compression and tension (e.g. Loocke et al., 2009; Takaza et al., 2013b) usually with one end of the specimen fixed and another subjected to the load. These studies offer limited understanding of dynamic response of soft tissue to oscillatory motions at varying magnitudes of excitation. Experimental study of muscle tissue using base excitation with sprung mass free to move has been sparse. This shortage may partly stem from a lack of established relationships between the measurable micro-structural properties i.e. compressive elastic modulus and shear modulus and ‘macro’ behaviours i.e. resonances in apparent mass and transmissibilities. However, the links between the two levels of behaviours may have the potential to improve current understanding of interaction between the excitation and the human body.

Mechanical behaviour of silicone rubber was found close to those of human skin (Shergold et al., 2006), heart (Saraf et al., 2007) and liver (Ozcan et al., 2011). A recent study has examined mechanical behaviours of different silicone rubber materials (e.g. Ecoflex 0030, Ecoflex 0010 and Dragon Skin) to mimic deep tissue injury (Sparks et al., 2015). The silicone formulations demonstrated shear moduli within the range of published values for biological tissue.

The present study intends to characterise dynamic property of a set of scaled silicone rubber specimens using FRFs generated from base-excited shaker tests. This scaled rigid mass-silicone rubber system is considered a close representation of the in vivo loading condition of the soft tissue at the excitation-subject interface during WBV. This information is the prerequisite for mechanistic models of body movement. It is hypothesised that primary resonance frequencies of FRFs, stiffness
and damping constants of silicone rubber change with specimen thickness, diameter, sprung mass and contact contour of the sprung mass. Resonance frequency refers to the frequency at which the frequency response function shows a peak. It was suspected that silicone rubber would not be able to reproduce the magnitude dependent biomechanical characteristics of soft tissues at the interface during WBV.

6.2 Method

The experimental study is conducted using a base-excited scaled rigid mass-silicone rubber analogue SDOF system to examine the dynamic biomechanical properties of silicone rubber. Band limited random excitation with four different excitation magnitudes was applied on the shaker table in the vertical (z-axis) direction of the rigid mass-silicone rubber system and response tri-axial accelerations on top of the sprung mass was measured. Specimen with three different diameters and thicknesses, three sprung masses and two sprung mass contact contours were employed to examine the effect of these parameters on the measured biomechanical properties (i.e. stiffness and damping). The measured time histories, i.e. the base excitation acceleration and the resultant tri-axial accelerations measured on top of the sprung mass were processed and transformed into frequency response functions (FRFs), transmissibility.

6.2.1 Apparatus

The sprung mass-silicone rubber SDOF system was glued (using commercial grade super glue) on a horizontally flat shaker table that is rigidly bolted on top of the shaker armature - the moving end of the shaker (see Figure 6.1). The vertical motion is produced using Derritron VP85 shaker and base motion on the shaker table was controlled by a specialist electronic module via a PC. The specifications of the shaker, accelerometers and other experimental equipment were described in Sections 3.2.1 to 3.2.2 in Chapter 3.
6.2.2 Stimuli

The sprung mass-silicone rubber system was exposed to vertical (z-axis) band limited random vibration, nominally flat from 2 to 80 Hz, at four vibration magnitudes, nominally 0.5, 1.0, 1.5 and 2.0 ms$^{-2}$ r.m.s. (Figure 6.2). The time history of the input base acceleration and the response accelerations on top of the sprung mass has same duration of 80 seconds. The time histories of input in z- axis and response in z, y- and x- axis accelerations are shown as an example in Figure 6.2 - two magnitudes 0.5 ms$^{-2}$ r.m.s. and 2.0 ms$^{-2}$ r.m.s. were chosen to show respective time histories.
Figure 6.2 Example time histories of the input base excitation acceleration and the three orthogonal axes of output accelerations using a flat sprung mass (2.5 kg) and a silicone rubber specimen of a 50 mm diameter and 10 mm thickness, random excitation frequency band of 2 to 80 Hz at 0.5 ms$^{-2}$ r.m.s. (SS1 in Table 6.2).

The power spectral density functions (PSDs) of above shown time histories and three coherence functions between, (1) base excitation acceleration in vertical z-axis and the response vertical z-axis acceleration, (2) base excitation acceleration in z-axis and the response horizontal y-axis acceleration, and (3) base excitation acceleration in z-axis and the response x-axis acceleration were calculated and shown in Figure 6.3. It is evident that the cross-axis motion is negligibly low (see Figure 6.2 and 6.3). In the result section, only vertical z-axis response is considered.
Figure 6.3  Example of PSDs (upper) of the input base excitation acceleration (flat solid line —) and three orthogonal axes of output accelerations (— vertical z-axis; ---- horizontal y-axis; · · · horizontal x-axis) and coherence (lower) between base excitation acceleration and vertical response acceleration (——); base excitation acceleration and horizontal y-axis response acceleration (-----), base excitation acceleration and horizontal x-axis response acceleration (· · ·). Time histories displayed in Figure 6.2 were used to produce above PSD and coherence.
6.2.3 Specimen

Silicone rubber specimens were prepared in different sizes using commercial grade silicone rubber Ecoflex 0010 (Smooth-On Inc., 2014). The procedure involved in specimen preparations was described in Section 3.1.1 in Chapter 3.

6.2.4 Analysis

The four channels of force and acceleration time histories were sampled simultaneously at 204.8 Hz with duration of 80 second with a working frequency range of 2 to 80 Hz. The time histories were then transformed to cross spectral density functions (CSDs), power spectral density functions (PSDs) and ordinary FRFs by applying a Fast Fourier Transform (FFT). The CSDs and PSDs were estimated via Welch’s method at frequencies between 2 and 80 Hz. The FRFs for each of 80-second continuous random signals used a FFT windowing length of 512 samples, a Hamming window with 50% overlap, a sampling rate of 204.8 Hz and an ensuing frequency resolution of 0.4 Hz (Table 6.1). The 0.4-Hz procedure was used to give a higher confidence level with 128 degrees of freedom at each frequency.

Table 6.1  Signal processing procedure used to calculate the transmissibilities between base and the sprung mass top of the sprung mass-silicone rubber system.

<table>
<thead>
<tr>
<th>Excitation frequency (Hz)</th>
<th>Duration (s)</th>
<th>Sample per second</th>
<th>FFT length</th>
<th>Degrees of freedom</th>
<th>Windowing overlap</th>
<th>Frequency resolution (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 to 80</td>
<td>80</td>
<td>204.8</td>
<td>512</td>
<td>128</td>
<td>Hamming 50%</td>
<td>0.4</td>
</tr>
</tbody>
</table>

The complex transmissibility was first computed from the time histories using CSD method:

\[
H_T(f) = \frac{G_{\ddot{Z}_a}(f)}{G_{\ddot{Z}_0}(f)}
\]  \hspace{1cm} (6.1)

where \(H_T(f)\) is the transmissibility, \(f\) is the excitation frequency in Hz, \(G_{\ddot{Z}_a}(f)\) is the CSD between the resultant acceleration of the sprung mass (\(\ddot{Z}\)) in m/s\(^2\) and the input
base excitation acceleration \( \ddot{Z}_0 \) in m/s\(^2\), \( G_{Z_0Z_0}(f) \) is the PSD of the input base excitation acceleration.

The silicone rubber-sprung mass was modelled as SDOF mass-spring-damper analogy with base excitation. By applying linear viscoelastic formulations in Equation G6 and G7 of Appendix G, it is possible to compute the frequency dependent dynamic stiffness \( k \) (N/m) and damping \( c \) (Ns/m).

The mathematical nonlinearity, the amount of the output motion that is not linearly correlated or not entirely caused by, the input motion in the calculated frequency response functions was investigated using the coherency:

\[
\gamma^2(f) = \frac{|G_{Z_0Z_0}(f)|^2}{G_{Z_0Z_0}(f) G_{Z_0Z_0}(f)} \quad (6.2)
\]

where \( G_{Z_0Z_0}(f) \) is the PSD of the sprung mass acceleration and \( \gamma^2(f) \) is the coherency of the system with a value between 0 and 1. The coherency has a maximum value of 1 in a linear single-input system with no noise which indicates that the output motion is entirely caused by, and linearly correlated with, the input motion.

### 6.3 Results

A total of 17 silicone rubber specimens were tested. First, 12 specimens of 50-mm diameter were tested with four different excitation magnitudes for intra- and inter-specimen variability and effect of magnitude. Then 3 specimens of 75-mm diameter and 2 of 100-mm diameter were examined. The specimens were tested for 3 different thicknesses, 3 diameters, 3 sprung masses, 2 sprung mass contours and 4 different base excitation magnitudes. The dependent variables were resonance frequency of transmissibility, dynamic stiffness at resonance, damping constant at resonance. The transmissibility between the vertical z-axis acceleration at base and inline z-axis resultant acceleration on top of sprung mass showed inter-specimen, intra-specimen variability (Figure 6.4) and effect of excitation magnitude (Figure 6.5).

With the twelve 50-mm diameter specimens, there were 3 thicknesses and each with 4 nominally identical specimens. First each of the twelve specimens was subjected to three repeated tests showing their intra-specimen variability. Inter-specimen variability was then tested using the 4 nominally identical specimens. Parametric statistical tests were employed using analysis of variance (ANOVA) method.
For intra-specimen variability, one-factor analysis of variance (ANOVA) was used with main effect as the three repeated tests. Alternatively, the standard deviation of the means obtained from the three repeated measurements with each of the four specimens of the same thickness was used. For inter-specimen variability, one-factor analysis of variance was used with main effect as four identical specimens. For effect of vibration magnitude, one-factor ANOVA was used with main effect as four different excitation magnitudes. A total of 12 specimens were divided into 3 groups for 3 thicknesses and each group were tested individually for intra-, inter-specimen variability and effect of excitation magnitude – a total of 81 tests. This is presented in Sections 6.3.1 to 6.3.2.

Specimens with 75 and 100 mm diameters were investigated with four different excitation magnitudes in comparison to those of 50 mm diameter but with only one specimen for each specification. This allowed examination of effects of excitation magnitude, specimen diameter, thickness, sprung mass and sprung mass contour. Five-factor ANOVA was used to determine whether there was a difference between different level of measures or dependent variables with different experimental conditions or independent variables (Cole et al., 1994). The interaction between two experimental conditions was investigated using Multiple Comparison Method. The ‘interaction’ indicates the combined effect of two different experimental conditions on the dependent variables. If there is an interaction, the effect of one experimental condition depends on the level or change in independent variable of the other conditions. This is presented in Sections 6.3.3 to 6.3.5.

6.3.1 Inter-specimen and intra-specimen variability

Twelve specimens of 50-mm diameter, i.e. three thicknesses and each with four nominally identical samples, were tested using flat 2.5-kg sprung mass at four different excitation magnitudes of broadband vibration.

With vertical z-axis excitation, the inline z-axis transmissibility exhibited a single resonance at around 33 Hz for 10-mm, 26 Hz for 15-mm and 20 Hz for 20-mm thick specimens (Figure 6.4). With each specimen subjected to three repeated tests, the resonance frequency seemed to be the same for different repeats of the same specimen and the same for different specimens of the same thickness. This was consistent for all four excitation magnitudes. The standard deviation of the mean of
the different repeated measurements of the same specimen tends to be similar for the entire specimen range and it is less than 1% for all cases (Table 6.2 A and B). No significant difference was found in resonance frequency, stiffness or damping constant across the three repeats using the same specimen ($p>0.05$, one-factor ANOVA for all three groups with four magnitudes). Amongst the four nominally identical specimens, no significant difference was found in resonance frequency, stiffness or damping constant ($p>0.05$).

![Figure 6.4](image)

*Figure 6.4* Vertical z-axis inline transmissibility – modulus (upper) and phases (lower) – of the 12 specimen (SS1 to SS12 each with three repeated measurements); each specimen was exposed to broadband (2 to 80 Hz) random excitation 1.0 ms$^{-2}$ r.m.s.
6.3.2 Effect of vibration magnitude

With the four excitation magnitudes at 0.5, 1.0, 1.5 and 2.0 ms$^{-2}$ r.m.s., twelve specimens of 50-mm diameter with three thicknesses were tested using flat 2.5-kg sprung mass. There was no significant change in resonance frequency as the magnitude of excitation acceleration increased from 0.5 to 1.0, 1.5 and 2.0 ms$^{-2}$ r.m.s. ($p>0.05$, one-factor ANOVA for all three groups; Figure 6.5). However there was significant change in the stiffness and damping constant calculated at resonance as the magnitude increased ($p<0.05$, one-factor ANOVA for both damping and stiffness with all three groups; Table 6.2 A and B).

Figure 6.5  Vertical z-axis inline transmissibility - modulus (upper) and phases (lower) – of 12 specimens (SS1 to SS12), each specimen was exposed to broadband (2 to 80 Hz) random excitation at four different excitation magnitudes 0.5 (——), 1.0 (----), 1.5 (•••), 2.0 (· · ·) ms$^{-2}$ r.m.s.
Table 6.2A  Mean dynamic stiffness $k$ and damping constant $c$ (± standard deviation of the mean) over three repeats extracted at the first principal peak frequency of transmissibility for two excitation magnitudes of 0.5 and 1.0 ms$^{-2}$ r.m.s. A flat 2.5 kg sprung mass was used with specimens SS1 to SS12.

<table>
<thead>
<tr>
<th>Specimen number</th>
<th>Diameter (mm)</th>
<th>Thickness (mm)</th>
<th>0.5 ms$^{-2}$ r.m.s.</th>
<th>1.0 ms$^{-2}$ r.m.s.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>First peak (Hz)</td>
<td>k (kN/m)</td>
</tr>
<tr>
<td>SS1</td>
<td>50</td>
<td>10</td>
<td>33.2</td>
<td>112 ± 0.3</td>
</tr>
<tr>
<td>SS2</td>
<td>50</td>
<td>10</td>
<td>33.2</td>
<td>112 ± 0.3</td>
</tr>
<tr>
<td>SS3</td>
<td>50</td>
<td>10</td>
<td>33.6</td>
<td>113 ± 0.5</td>
</tr>
<tr>
<td>SS4</td>
<td>50</td>
<td>10</td>
<td>33.6</td>
<td>114 ± 0.3</td>
</tr>
<tr>
<td>SS5</td>
<td>50</td>
<td>15</td>
<td>26.0</td>
<td>67.3 ± 0.4</td>
</tr>
<tr>
<td>SS6</td>
<td>50</td>
<td>15</td>
<td>26.0</td>
<td>69.0 ± 0.5</td>
</tr>
<tr>
<td>SS7</td>
<td>50</td>
<td>15</td>
<td>26.0</td>
<td>67.3 ± 0.5</td>
</tr>
<tr>
<td>SS8</td>
<td>50</td>
<td>15</td>
<td>26.4</td>
<td>69.2 ± 0.4</td>
</tr>
<tr>
<td>SS9</td>
<td>50</td>
<td>20</td>
<td>20.4</td>
<td>43.6 ± 0.3</td>
</tr>
<tr>
<td>SS10</td>
<td>50</td>
<td>20</td>
<td>20.8</td>
<td>44.5 ± 0.5</td>
</tr>
<tr>
<td>SS11</td>
<td>50</td>
<td>20</td>
<td>21.2</td>
<td>45.5 ± 0.4</td>
</tr>
<tr>
<td>SS12</td>
<td>50</td>
<td>20</td>
<td>20.8</td>
<td>44.2 ± 0.3</td>
</tr>
</tbody>
</table>
Table 6.2B  Mean dynamic stiffness $k$ and damping constant $c$ ($\pm$ standard deviation of the mean) over three repeats extracted at the first principal peak frequency of transmissibility for two excitation magnitudes of 1.5 and 2.0 ms$^{-2}$ r.m.s. A flat 2.5 kg sprung mass was used with specimens SS1 to SS12.

<table>
<thead>
<tr>
<th>Specimen number</th>
<th>Diameter (mm)</th>
<th>Thickness (mm)</th>
<th>1.5 ms$^{-2}$ r.m.s.</th>
<th>2.0 ms$^{-2}$ r.m.s.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>First peak (Hz)</td>
<td>$k$ (kN/m)</td>
</tr>
<tr>
<td>SS1</td>
<td>50</td>
<td>10</td>
<td>32.4</td>
<td>$107 \pm 0.2$</td>
</tr>
<tr>
<td>SS2</td>
<td>50</td>
<td>10</td>
<td>32.8</td>
<td>$109 \pm 0.3$</td>
</tr>
<tr>
<td>SS3</td>
<td>50</td>
<td>10</td>
<td>32.4</td>
<td>$103 \pm 0.4$</td>
</tr>
<tr>
<td>SS4</td>
<td>50</td>
<td>10</td>
<td>33.2</td>
<td>$108 \pm 0.5$</td>
</tr>
<tr>
<td>SS5</td>
<td>50</td>
<td>15</td>
<td>25.6</td>
<td>$65.0 \pm 0.3$</td>
</tr>
<tr>
<td>SS6</td>
<td>50</td>
<td>15</td>
<td>26.0</td>
<td>$66.9 \pm 0.5$</td>
</tr>
<tr>
<td>SS7</td>
<td>50</td>
<td>15</td>
<td>25.6</td>
<td>$65.6 \pm 0.4$</td>
</tr>
<tr>
<td>SS8</td>
<td>50</td>
<td>15</td>
<td>26.0</td>
<td>$66.9 \pm 0.3$</td>
</tr>
<tr>
<td>SS9</td>
<td>50</td>
<td>20</td>
<td>20.4</td>
<td>$42.8 \pm 0.5$</td>
</tr>
<tr>
<td>SS10</td>
<td>50</td>
<td>20</td>
<td>21.2</td>
<td>$45.8 \pm 0.3$</td>
</tr>
<tr>
<td>SS11</td>
<td>50</td>
<td>20</td>
<td>21.6</td>
<td>$47.0 \pm 0.5$</td>
</tr>
<tr>
<td>SS12</td>
<td>50</td>
<td>20</td>
<td>21.6</td>
<td>$46.5 \pm 0.5$</td>
</tr>
</tbody>
</table>
Variation in transmissibility, damping constant and stiffness were shown with three repeated measurements using vertical z-axis broadband 2-80 Hz random excitation at 0.5 ms$^{-2}$ r.m.s. (Figure 6.6). The stiffness increased with frequency in the range 20 to 80 Hz where the principal peak transmissibility occurred. The damping constant decreased slightly as frequency was increased from 20 to 80 Hz. The vertical dashed lines indicate the peak frequency of the transmissibility, stiffness and damping constant at corresponding peak frequencies.

**Figure 6.6** Vertical z-axis broadband (2 to 80 Hz) random excitation at 0.5 ms$^{-2}$ r.m.s. with 2.5 kg flat sprung mass: column 1 for specimen with 10-mm thickness, column 2 for 15-mm thickness, and column 3 for 20-mm thickness (each specimen has same diameter of 50-mm). For each specimen the dynamic stiffness $k$, damping $c$, and transmissibility are shown. Frequency of interest: 10 to 40 Hz, with vertical dashed line indicating the peak around 33 Hz for 10-mm, 26 Hz for 15-mm and 20 Hz for 20-mm thick specimen. Refer to Table 6.2 A and B for values.

### 6.3.3 Effect of specimen size

In addition to the 50-mm diameter specimens tested, three specimens of 75-mm diameter and two of 100-mm diameter were tested. Specimens were compared for 3 different thicknesses, 3 diameters, 3 sprung masses, and 2 sprung mass contours. Each specimen was exposed to the same random vibration at the four magnitudes.
Effect of thickness

The resonance frequency decreased as the thickness increased from 10 to 15 and 20 mm. This was consistent for all magnitudes with flat and hip-borne contours and three sprung masses (Figure 6.7; Table 6.3A and B). Over the three thicknesses 10, 15 and 20 mm, there were significant differences in resonance frequencies \((p=0.000\), five-factor ANOVA). There was interaction between thickness and diameter \((p=0.000\), five-factor ANOVA), thickness and sprung mass \((p=0.000\), and thickness and sprung mass contour \((p=0.000\). They were significant \((p<0.05\) for all three interactions). However there was no interaction found between thickness and the excitation magnitude \((p>0.05\), five-factor ANOVA).

![Figure 6.7](image)

**Figure 6.7** The variation in the resonance frequency of the specimen with three different thicknesses: specimen was exposed to broadband (2 to 80 Hz) random excitation at 0.5 (1st column), 1.0 (2nd column), 1.5 (3rd column) and 2.0 (4th column) \(\text{ms}^{-2}\) r.m.s. with flat sprung mass (hollow diamond - 50 mm, hollow square - 75 mm and hollow triangle - 100 mm diameter) and hip-borne sprung mass (solid diamond - 50 mm, solid square - 75 mm and solid triangle 100 mm diameter). Refer to Table 6.3 A and B for values.

The variation in stiffness showed similar trend as the resonance frequency – the stiffness decreased as the thickness increased from 10 to 15 and 20 mm. This was consistent for all magnitudes with flat and hip-borne contours of three sprung masses (Figure 6.8; Table 6.4 A and B, \(p=0.000\), five-factor ANOVA). There was interaction between thickness and diameter \((p=0.000\), five-factor ANOVA),
thickness and sprung mass ($p=0.000$), thickness and sprung mass contour ($p=0.000$) and thickness and excitation magnitude ($p=0.000$).

Figure 6.8  The variation in the stiffness $k$ (upper) and damping constant $c$ (lower) of the specimen with three different thicknesses: specimen was exposed to broadband (2 to 80 Hz) random excitation at 0.5 (1st column), 1.0 (2nd column), 1.5 (3rd column) and 2.0 (4th column) $\text{m}^2\text{s}^{-2}$ r.m.s. with flat sprung mass (hollow diamond - 50 mm, hollow square - 75 mm and hollow triangle - 100 mm diameter) and hip-bone sprung mass (solid diamond - 50 mm, solid square - 75 mm and solid triangle 100 mm diameter). Refer to Table 6.4 A and B for values.

The damping constant decreased with increasing thickness for all magnitudes with flat and hip-borne contacts of three sprung masses (Figure 6.8; Table 6.4 A and B). Changing thickness did significantly affect damping constant ($p=0.000$, five-factor ANOVA). The interactions between thickness and diameter ($p=0.000$, five-factor ANOVA), thickness and sprung mass ($p=0.000$), thickness and sprung mass contour ($p=0.000$) and thickness and excitation magnitude ($p=0.000$) were significant ($p<0.05$ for all four interaction).
Effect of specimen diameter

The resonance frequency increased as the diameter increased from 50 to 75 and 100 mm for all magnitudes with both contours and all sprung masses (Figure 6.9; Table 6.3 A and B). Over the three diameters, there were significant differences in resonance frequencies ($p=0.000$, five-factor ANOVA). The interaction between diameter and sprung mass ($p=0.000$, five-factor ANOVA), diameter and sprung mass contour ($p=0.000$) were significant ($p<0.05$ for both two interactions). However there was no interaction between diameter and magnitude ($p>0.05$, five-factor ANOVA).

![Figure 6.9](image)

**Figure 6.9** The variation in the resonance frequency of the specimen with three different diameters: specimen was exposed to broadband (2 to 80 Hz) random excitation at 0.5 (1$^{st}$ column), 1.0 (2$^{nd}$ column), 1.5 (3$^{rd}$ column) and 2.0 (4$^{th}$ column) $\text{ms}^{-2}$ r.m.s. with flat sprung mass (hollow diamond - 10 mm, hollow square - 15 mm and hollow triangle - 20 mm thickness) and hip-borne sprung mass (solid diamond - 10 mm and solid square - 15 mm diameter). Refer to Table 6.3 A and B for values.

The variation in the stiffness showed similar trend as the resonance frequency - the stiffness also increased as the diameter increased from 50 to 75 and 100 mm. This was consistent for all the magnitudes of excitation tested with flat and hip-borne contacts of three sprung masses (Figure 6.10; Table 6.4 A and B). Such effect was found to be significant ($p=0.000$, five-factor ANOVA). The interaction between diameter and sprung mass ($p=0.000$, five-factor ANOVA), diameter and sprung
mass contour (p=0.000) and diameter and excitation magnitude were significant (p<0.05 for all three interactions).

Figure 6.10  The variation in the stiffness k (upper) and damping constant c (lower) of the specimen with three different diameters: specimen was exposed to broadband (2 to 80 Hz) random excitation at 0.5 (1\textsuperscript{st} column), 1.0 (2\textsuperscript{nd} column), 1.5 (3\textsuperscript{rd} column) and 2.0 (4\textsuperscript{th} column) ms\(^{-2}\) r.m.s. with flat sprung mass (hollow diamond - 10 mm, hollow square - 15 mm and hollow triangle - 20 mm thickness) and hip-borne sprung mass (solid diamond - 10 mm and solid square - 15 mm thickness). Refer to Table 6.4 A and B for values.

The damping constant increased as the diameter increased from 10 to 15 and 20 mm. This was consistent for all the magnitudes of excitation tested with flat and hip-borne contacts of three sprung masses (Figure 6.10; Table 6.4 A and B). Changing diameter did significantly affect damping constant (p=0.000, five-factor ANOVA). The interactions between diameter and sprung mass (p=0.000, five-factor ANOVA), diameter and sprung mass contour (p=0.000) and diameter and the excitation magnitude (p=0.000) were significant (p<0.05 for all three interaction).
Table 6.3A  Measured first principal peak frequency of transmissibility when the specimens were subjected to vertical z-axis continuous broadband (2 to 80 Hz) random vibration at 0.5 and 1.0 m$^2$ s$^{-2}$ r.m.s. Specimens were pre-loaded with the flat and hip sprung masses (1.5, 2.5 and 5 kg). Vertical inline resonance frequency was rounded to first decimal point.

<table>
<thead>
<tr>
<th>Sprung mass</th>
<th>Diameter (mm)</th>
<th>0.5 m$^2$ s$^{-2}$ r.m.s. First peak (Hz)</th>
<th>1.0 m$^2$ s$^{-2}$ r.m.s. First peak (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contour</td>
<td>Mass (kg)</td>
<td>$f_{10}$</td>
<td>$f_{15}$</td>
</tr>
<tr>
<td>Flat</td>
<td>1.5</td>
<td>50</td>
<td>40.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>75</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>100</td>
<td>-</td>
</tr>
<tr>
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Table 6.3B  Measured first principal peak frequency of transmissibility when the specimens were subjected to vertical z-axis continuous broadband (2 to 80 Hz) random vibration at 1.5 and 2.0 ms$^{-2}$ r.m.s. Specimens were pre-loaded with the flat and hip sprung masses (1.5, 2.5 and 5 kg). Vertical inline resonance frequency was rounded to first decimal point.

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Table 6.4A  Mean dynamic stiffness $k$ and damping constant $c$ over three repeats extracted at the first principal peak frequency of transmissibility. The specimens were subjected to vertical $z$-axis continuous broadband (2 to 80 Hz) random excitation at 0.5 and 1.0 ms$^{-2}$ r.m.s.

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### Table 6.4B  Mean dynamic stiffness k and damping constant c over three repeats extracted at the first principal peak frequency of transmissibility. The specimens were subjected to vertical z-axis continuous broadband (2 to 80 Hz) random excitation at 1.5 and 2.0 ms\(^{-2}\) r.m.s.

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6.3.4 Effect of sprung mass

The resonance frequency decreased as the sprung mass increased from 1.5 to 2.5 and 5.0 kg. This was consistent for all the magnitude of excitation tested with the flat contact of three sprung masses (Figure 6.11; Table 6.3 A and B). When the hip-borne contact of two sprung masses were used, the resonance frequency increased with increasing sprung mass from 1.5 to 2.5 kg for all the magnitude of excitation tested. Over the three sprung masses 1.5, 2.5 and 5.0 kg, there was significant differences in resonance frequencies ($p=0.000$, five-factor ANOVA). The interaction between sprung mass and sprung mass contour was significant ($p<0.05$, five-factor ANOVA). However there was no interaction found between sprung mass and excitation magnitude ($p>0.05$, five-factor ANOVA). In Figure 6.11, resonance frequency corresponds to three different sprung masses were represented by three different markers at each thickness of the specimen in x-axis.

![Figure 6.11](image)

**Figure 6.11** The variation in the resonance frequency of the specimen with three different sprung mass: specimen was exposed to broadband (2 to 80 Hz) random excitation at 0.5 (1st column), 1.0 (2nd column), 1.5 (3rd column) and 2.0 (4th column) ms$^{-2}$ r.m.s. with flat sprung mass (hollow diamond - 1.5 kg, hollow square - 2.5 kg and hollow triangle - 5.0 kg) and hip-borne sprung mass (solid diamond - 1.5 kg and solid square - 2.5 kg). Refer to Table 6.3 A and B for values.

The variation in the stiffness showed opposite trend as the resonance frequency - the stiffness also increased as the sprung mass increased from 1.5 to 2.5 and 5.0 kg (Figure 6.12; Table 6.4 A and B). This was consistent for both all the magnitude of
excitation tested with flat and hip-borne contact of three sprung masses. Such effect was found to be significant ($p=0.000$, five-factor ANOVA). The interaction between sprung mass and sprung mass contour, and sprung mass and excitation magnitude on the stiffness was significant ($p<0.05$, five-factor ANOVA for both cases). In Figure 6.12, $k$ and $c$ correspond to three different sprung masses were represented by three different markers at each thickness of the specimen in x-axis.

Figure 6.12 The variation in the stiffness $k$ (upper) and damping constant $c$ (lower) of the specimen with three different sprung mass: specimen was exposed to broadband (2 to 80 Hz) random excitation at 0.5 ($1^{st}$ column), 1.0 ($2^{nd}$ column), 1.5 ($3^{rd}$ column) and 2.0 ($4^{th}$ column) ms$^{-2}$ r.m.s. with flat sprung mass (hollow diamond - 1.5 kg, hollow square - 2.5 kg and hollow triangle - 5.0 kg) and hip-borne sprung mass (solid diamond - 1.5 kg and solid square - 2.5 kg). Refer to Table 6.4 A and B for values.

The damping constant increased as the sprung mass increased from 1.5 to 2.5 and 5.0 kg. This was consistent for all the magnitude of excitation tested with flat and hip-borne contact of three sprung masses. Changing sprung mass did significantly affect damping constant ($p=0.000$, five-factor ANOVA). The interactions between sprung mass and sprung mass contour ($p=0.000$, five-factor ANOVA) and sprung
mass and the excitation magnitude ($p=0.000$) were significant ($p<0.05$ for both cases).

### 6.3.5 Effect of sprung mass contact contour

Specimen with 50 mm diameter and three thicknesses of 10, 15 and 20 mm and test using flat and hip-borne contact of 2.5 kg sprung mass, are selected to show the effect of the hip-borne contact of sprung mass. At the lowest magnitude of vibration, the transmissibility with the hip-borne contact of sprung mass showed distinctive lower frequency of the single peak than those of the flat contact of sprung mass in the frequency range 15 to 45 Hz (Figure 6.13). This trend was less clear with the 20 mm thick specimen. This effect was significant in resonance frequency ($p=0.000$, five-factor ANOVA). However there was no interaction found between sprung mass contour and the excitation magnitude ($p<0.05$, five-factor ANOVA).

![Figure 6.13](image.png)

**Figure 6.13** Specimens with 50 mm diameter and three thicknesses of 10, 15 and 20 mm tested using broadband (2 to 80 Hz) random excitation at 0.5 ms$^{-2}$ r.m.s. with two different indenter surfaces of the sprung mass: 10 mm thick (1st column), 15 mm thick (2nd column) and 20 mm thick (3rd column) specimen with 2.5 kg flat sprung mass (black dashed line) and 2.5 kg hip-borne sprung mass (blue solid line). Refer to Table 6.4 A for values at peak. For each specimen the dynamic stiffness $k$, damping $c$, and transmissibility are shown.
The variation in the stiffness showed similar softening trend as seen in the resonance frequency – the stiffness decreased as changing sprung mass contour from flat to hip-borne. Such effect on stiffness was found to be significant ($p=0.000$, five-factor ANOVA). The interaction between sprung mass contour and the excitation magnitude was significant ($p<0.05$, five-factor ANOVA). There tends to be an apparent reduction in damping also while changing the sprung mass contour from flat to hip-borne. The effect of sprung mass contour on damping constant was significant ($p=0.000$, five-factor ANOVA). The interaction between sprung mass contour and the excitation magnitude was significant ($p<0.05$, five-factor ANOVA).

6.4 Discussion

The silicone rubber has been widely adopted as human soft tissue substitute, but the ways in which the dynamic characteristics of these materials were investigated vary considerably (e.g. Saraf et al., 2007; Ozcan et al., 2011; Sparks et al., 2015). The obvious advantage of silicone rubber is being more configurable and repeatable (e.g. Sparks et al., 2015). The present study focuses on biomechanical parameters that are closely related to the ‘macro’ biomechanical behaviour seen in whole-body vibration. These include the frequency dependent dynamic stiffness and damping, and the frequency responses to base excitation with the sprung mass free to move.

6.4.1 Resonance frequency

A dominant single resonance was observed for the base-excited mass-silicone rubber system in the present study and also in most whole-body vibration studies with vertical vibration (Huang and Griffin, 2006, 2008a). Although the frequency range at which the resonance occurred were different: 20 to 33 Hz for current study and below 10 Hz for WBV studies, the mode of sprung mass might be similar in both scenarios. The fundamental difference between the human buttocks and the silicone rubber was the amount of the fluid content and the packaging of fluid in the tissue. The ‘micro’ structural arrangements in the silicone rubber and the buttock tissue are different (Sparks et al., 2015). The study from (Sharafi and Blemker, 2010) showed that variation in the microstructure of the muscle fibre affect the macroscopic biomechanical behaviour.
Cross axis responses

The cross-axis responses in the present study were much less apparent (see Figure 6.3) comparing to WBV studies. The WBV has internal upper body coupling largely from inhomogeneous upper body mass. Whereas in the present study, the custom made rigid mass was designed with low centre of mass (maximum total height was 60 mm, see Figure 3.6 in Chapter 3) - eliminate cross-axis responses.

### 6.4.2 Effect of vibration magnitude

The expected excitation magnitude dependency in resonance frequency was absent in the present study. Although the stiffness and damping changed (refer to Table 6.4) with excitation magnitude, these parameters were derived from linear viscoelastic model. If thixotropic or memory effect of the human soft tissue at the excitation-subject interface primarily contributed to the magnitude dependency as suggested by Huang and Griffin (2008a, b), the absence of the magnitude dependency implies silicone rubber does not hold such characteristic. One way of testing this theory is to subject the rigid mass-silicone rubber system to impact force of different magnitudes exerted from an instrumented impact hammer on the sprung mass. The impact hammer test with rigid mass-soft tissue system will be presented in the next chapter, Chapter 7.

The frequency dependent stiffness and the damping behaviour was observed in this study - dynamic stiffness increased as the frequency increased from 20 to 80 Hz whereas damping decreased slightly as the frequency increased from 20 to 80 Hz. The dynamic property of porcine skeletal muscle subjected to base excitation loaded with a sprung mass as an analogue based excited SDOF was investigated by Aimedieu et al. (2003). The authors extracted frequency dependent dynamic stiffness and damping parameters from transmissibility measurement, similar to this study. The current study has reported similar observation for dynamic stiffness as reported in Aimedieu et al. (2003). However damping showed opposite trend to the current study. Aimedieu et al. (2003) has reported that the damping constant was around 300 Ns/m from 5 to 20 Hz and around 556 Ns/m at 30 Hz. The average stiffness ranged from 8.5 kN/m at 5 Hz to 347 kN/m at 30 Hz. The stiffness and the damping constant show similar order to the current study while using the similar size specimen (50 mm diameter and 20 mm thickness) and the sprung mass (5 kg).
6.4.3 Effect of thickness

The resonance frequency decreased with increasing thickness from 10 to 15 and 20 mm. The stiffness and the damping constant for different thicknesses also showed similar trend as the resonance frequency. The stiffness value obtained from lumped parameter models of seated subjects (49 kN/m at 0.25 ms\(^{-2}\) r.m.s. and 32 kN/m at 1.0 ms\(^{-2}\) r.m.s., Huang and Griffin, 2006) and recumbent subjects (60 kN/m at 0.25 ms\(^{-2}\) r.m.s. and 52 kN/m at 1.0 ms\(^{-2}\) r.m.s., Huang and Griffin, 2008a) during vertical base-excited WBV also show similar order to the one obtained from current study. However, how the resonance frequency, stiffness and damping vary with different thicknesses of buttock tissue of different human subject are missing in WBV.

6.4.4 Effect of diameter

As the diameter of the silicone specimen increased from 50 to 75 and 100 mm, the resonance frequency increased. The effect of diameter on stiffness and damping constant showed similar trends as resonance frequency. The resonance frequency measured at the different parts of the human body showed different values (e.g. at front thighs 8 to 10 Hz and at the ischial tuberosities around 5 Hz; Liu et al., 2015a). The author has speculated that the factors causing this difference could be caused by the changes in the tissue thickness, contact area and the pressure of the tissue under things and ischial tuberosities. The finding from this study support the speculation drawn by Liu et al. 2015a.

6.4.5 Effect of sprung mass

The resonance frequency of the silicone specimen decreased with increasing sprung mass from 1.5 to 2.5 and 5.0 kg. However the stiffness and the damping constant of the silicone specimen increased as the sprung mass increased from 1.5 to 2.5 and 5.0 kg. The sprung mass governs the static pressure at the excitation subject interface. Finding from this study show how resonance frequency, stiffness and damping changes with the change in the sprung mass.

6.4.6 Effect of sprung mass contour

The hip-borne like sprung mass was used in this study to mimic the bony structure of tuberosities at the bottom of pelvis. The resonance frequency reduced when changing the sprung mass contact contour from flat to hip-borne. This was
consistently observed in both base-excited and impact hammer tests. There was around 50% reduction in stiffness and around 40% reduction in damping when changing the sprung mass contour to hip-borne. This finding has never been reported before.

6.5 Conclusions

An experimental protocol was developed to characterise the excitation-subject interface of the WBV using the scaled rigid mass-silicone rubber system. A linear SDOF viscoelastic model was utilised to extract stiffness and damping from the measured transmissibility frequency response functions. A dominant single resonance was observed for the base-excited mass-silicone rubber system in the present study and also in most whole-body vibration studies with vertical vibration (Huang and Griffin, 2006, 2008a). Although the frequency range at which the resonance occurred were different: 20 to 33 Hz for current study and below 10 Hz for WBV studies, the mode of sprung mass might be similar in both scenarios. The effect of thickness, diameter, sprung mass and sprung mass contact contour on resonance frequency and stiffness is found to be significant. However the absence of change in the resonance frequency using different magnitudes of base excitation seems to suggest the thixotropic or memory effect of the human body by Huang and Griffin (2008 a, b) is missing in the SDOF rigid mass-silicone rubber system.
CHAPTER 7

Impact hammer test on rigid mass-soft tissue system

7.1 Introduction

Dynamic behaviour of soft tissue at the excitation subject interface of whole-body vibration (WBV) governs the motion transmitted to and through the human body during vibration. Such interface has been systematically studied using base-excited scaled rigid mass-silicone rubber system (in Chapter 6). The magnitude dependant biomechanical responses of WBV were found to be absent in such scaled rigid mass-silicone rubber system. Another way of testing this theory is to subject the rigid mass-silicone rubber system to impact force of different magnitudes exerted from an instrumented impact hammer on the sprung mass.

Impact hammer test has been a standardised procedure to extract frequency response functions of mechanical structures by applying an impact force using hand held hammer. It offers a quicker and simpler setup comparing to a base-excited shaker test (Figure 7.1a). However, the impact test could only produce a frequency response function with a rigid foundation. Impact hammer test has been used to measure frequency dependent viscoelastic properties, i.e. damping and stiffness, of both natural and artificial materials (Lin et al., 2005; Ooi and Ripin, 2011; Ozcan et al., 2011). The boundary conditions of such method need to have a rigid base and a sprung mass free to move after the application of the impulsive force (similar to Figure 7.1b). Frequency dependant viscoelastic properties can be readily estimated by formulating the equation of motion using elastic modulus and loss factor (Jones, 2001). Impact hammer test offers a simpler and quicker setup comparing to base-excited shaker test. If the specimen were homogeneous, linear and isotropic, the estimated viscoelastic properties would be the same for both experimental setups. None of the above assumption applies to real material. Nonetheless, experimental studies provide a basis for nonlinear analytical and constitutive modelling.

Dynamic loading of fresh porcine skeletal muscle, using drop impact compression, examined viscoelastic properties in loading rates only relevant to injuries caused by automotive crashes (Loocke et al., 2009; Takaza et al., 2013a). Loading rate or strain rate expressed in constitutive formulae can be interpreted as a combination of
frequency and magnitude in whole-body vibration. But these formulations were not directly ready to offer analytical solutions for prediction of global responses of WBV. For soft tissue, Fung (1993) showed that response to an excitation with steady-state frequency spectrum was required to describe its dynamic behaviour. Most studies used transient drop test or cyclic loading at discrete frequencies.

![Diagram of a single degree of freedom (SDOF) mass-spring-damper system with base motion (a) and rigid foundation (b):](image)

- $m$, $k$, and $c$ are the sprung mass (in kg), dynamic stiffness (in N/m) and damping constant (in Ns/m) respectively; $\ddot{z}_0(t)$ is the time history of input base excitation acceleration that could be provided by a shaker; $\ddot{z}(t)$ the time history of the response acceleration (in $\text{m/s}^2$), and $f(t)$ the time history of the input excitation force (in N) that could be exerted by an impact hammer.

The present study intends to characterise dynamic property of a set of scaled silicone rubber and porcine skeletal muscles using FRFs generated from impact hammer tests. This scaled rigid mass-soft tissue system is considered a close representation of the in vivo loading condition of the soft tissue at the excitation-subject interface during WBV. It is hypothesised that primary resonance frequencies of FRFs, stiffness and damping constants of both artificial and porcine soft tissues change with specimen thickness, diameter, sprung mass and contact contour of the sprung mass. The study will also draw on the similarity in dynamic behaviour between the artificial and the porcine soft tissue. It was suspected that silicone rubber would not be able to represent the biomechanical characteristics of biological tissues during WBV.
7.2 Method

The experimental study is conducted using a rigid mass-soft specimen analogue SDOF system to examine the dynamic properties of silicone rubber and porcine muscle tissue specimen. An impulsive force was exerted in the vertical (z-axis) and horizontal (x-axis) direction of the sprung mass on top of the soft specimen with three different diameters and thicknesses, three sprung masses, and two sprung mass contact contours (see Figure 3.16 in Chapter 3). The measured time histories, i.e. the excitation impact force and the resultant tri-axial accelerations measured on top of the sprung mass were processed and transformed into FRFs, first accelerance and then receptance. By applying linear viscoelastic theories, the dynamic stiffness and damping were derived from receptance in the frequency domain (Lin et al., 2005; Ozcan et al., 2011). See Appendix H for procedure.

7.2.1 Apparatus

The sprung mass-soft tissue SDOF system was placed on a horizontally flat test bench that is rigidly attached to the ground (Figure 3.16 in Chapter 3). The vertical impact force was applied nominally at the centre of the sprung mass shaft (Figure 3.17a in Chapter 3) and the horizontal impact force at the centre edge of the sprung mass using a Dytran 5800B4 impact hammer (Figure 3.17b in Chapter 3). Refer to sections 3.3.2 to 3.3.3 in Chapter 3 for more details about experimental setup and the equipment used in this study.

7.2.2 Stimuli

The impact force exhibited a short impulse with its duration and subsequently frequency governed by the hammer head mass and the tip stiffness. The frequency spectrum of this force was designed to be flat up to about 100 Hz, i.e. the roll off frequency. The impulse excitation is equivalent to a broadband random excitation but with limited sampling duration due to the short event. A sample time history of the input impact force and the response accelerations are shown in Figure 7.2. The frequency range of interest in this study is up to 80 Hz (see Figure 7.3). A hard plastic tip was used for the hammer to attenuate higher frequency content.
Figure 7.2  Example time histories of the impact excitation force and the three orthogonal axes of accelerations by applying vertical z-axis impact force (a) and horizontal x-axis impact force (b) using a flat sprung mass of 1.5 kg and a silicone rubber specimen 50 mm in diameter and 10 mm thick (SS1 in Table 7.1).

7.2.3 Specimen

The silicone rubber specimens were prepared with different diameter (50, 75 and 100 mm) and thicknesses (10, 15 and 20 mm) using Ecoflex 0010 silicone rubber. The fresh porcine specimens were prepared with a nominal dimension of 50 ±2 mm and three different nominal thicknesses 10, 15 and 20 mm. Refer to sections 3.1.1 and 3.3.1 in Chapter 3 for the procedure.

7.2.4 Analysis

The four channels of force and acceleration time histories were sampled simultaneously at 107520 Hz with duration of 1.219 second with a working frequency range of 80 Hz. They were then transformed to PSDs and FRFs by applying a fast Fourier transform (FFT) at full length with a single rectangular window giving a frequency resolution of 0.8203 Hz. The averages of 3 to 5 repeated measurements were presented in the analysis.

ABACUS data acquisition module uses a combination of a single analogue anti-aliasing filter at 49 kHz and many digital anti-aliasing filters (for each of its 83 frequency range) executed in the digital signal processors (DSPs) to sample
continuously low-pass filtered data. The digital anti-aliasing filter cut-off frequency is selected automatically by the controller software according to the chosen 'frequency span', i.e. frequency range of interest 0 to 80 Hz for the present study.

The complex FRF accelerance was first computed from the time histories using power spectral density and cross spectral density functions:

$$ H_A(\omega) = \frac{G_{ZF}(\omega)}{G_{FF}(\omega)} $$

(7.1)

where $H_A$ is the accelerance, $\omega$ is the excitation frequency in Hz, $G_{ZF}(\omega)$ is the cross spectral density function between the resultant acceleration of the sprung mass ($\ddot{Z}$) in m/s$^2$ and the excitation impact force ($F$) in N, $G_{FF}(\omega)$ is the power spectral density function of the impact force.

By applying ‘omega arithmetic’ and the SDOF mass-spring-mass model with rigid foundation, it is possible to compute the receptance:

$$ H_R(\omega) = \frac{G_{ZF}(\omega)}{G_{FF}(\omega)} $$

(7.2)

where $G_{ZF}(\omega)$ is the cross spectral density function between the resultant displacement $Z$ in m derived from acceleration and impact force $F$ in N. See Equation H6 and H7 of Appendix H.

By applying linear viscoelastic formulations in Equation H3 and H10 of Appendix H, it is possible to compute the frequency dependent dynamic stiffness $k$ (N/m) and damping $c$ (Ns/m)

The mathematical nonlinearity, the amount of the output motion that is not linearly correlated or not entirely caused by, the input motion in the calculated frequency response functions was investigated using the coherency:

$$ \gamma^2(\omega) = \frac{|G_{ZF}(\omega)|^2}{G_{FF}(\omega) G_{\ddot{Z}\ddot{Z}}(\omega)} $$

(7.3)

where $G_{\ddot{Z}\ddot{Z}}(\omega)$ is the PSD of the sprung mass acceleration and $\gamma^2(\omega)$ is the coherency of the system with a value between 0 and 1. The coherency has a maximum value of 1 in a linear single-input system with no noise which indicates that the output motion is entirely caused by, and linearly correlated with, the input motion. In the present study, the coherence function between inline input impact force and the response sprung mass acceleration is flat at around 1 through the frequency range of interest of 0 to 80 Hz.
7.3 Results

7.3.1 Silicone rubber specimen

A total of 17 silicone rubber specimens were tested. First, 12 specimens of 50-mm diameter were tested for intra- and inter-specimen variability. Then 3 specimens of 75-mm diameter and 2 of 100-mm diameter were examined. The specimens were tested for 3 different thicknesses, 3 diameters, 3 sprung masses, and 2 sprung mass contours. The dependent variables or effects for the tests were resonance frequency of accelerance, dynamic stiffness at resonance, damping constant at resonance. The power spectral density functions (PSDs) of the impact force and inline resultant acceleration provided a visual of inter-specimen and intra-specimen variability for vertical (Figure 7.3) and horizontal impact (Figure 7.4).

With the twelve 50-mm diameter specimens, there were 3 thicknesses and each with 4 nominally identical specimens. First each of the twelve specimens was subjected to five repeated measurements so as to test their intra-specimen variability. Inter-specimen variability was then tested between the 4 nominally identical specimens. Parametric statistical tests were employed using analysis of variance (ANOVA) method.

For intra-specimen variability, one-factor analysis of variance (ANOVA) was used with main effect as the five repeats. Alternatively, the standard deviation of the means obtained from the five repeated measurements on each one of the four specimens of the same thickness was used to evaluate the variability between repeated measurements. For inter-specimen variability, one-factor analysis of variance was used but with main effect as four identical specimens. Total of 12 specimens were divided into 3 groups according to the thickness of the specimen and each group were tested individually for intra and inter-specimen variability in resonance frequency, stiffness and damping constant (i.e., 36 tests were needed).

Specimens with 75 and 100 mm diameters were investigated in comparison to those of 50 mm diameter but with only one specimen for each specification. This allowed examination of effects of specimen diameter, thickness, sprung mass and two sprung mass contour. Four-factor ANOVA was used to determine whether there was a difference between different level of measures or dependent variables with different level of experimental conditions or independent variables (Cole et al., 1994).
The interaction between two experimental conditions was investigated using Multiple Comparison Method. ‘Interaction’ refers to combined effects of the two different experimental conditions on the dependent variables. If there is an interaction, the effect of one experimental condition depends on the level or change in independent variable of the other conditions. See Sections 7.3.1.2 to 7.3.1.4.

7.3.1.1 Inter-specimen and intra-specimen variability

Twelve specimens of 50-mm diameter, i.e. three thicknesses and each with four nominally identical samples, were tested using flat 2.5-kg sprung mass for intra- and inter-specimen variability.

With impact force exerted in the vertical z-axis of the sprung mass, the PSDs of the inline z-axis acceleration and accelerance exhibited a single resonance at around 26 Hz for 10-mm, 22 Hz for 15-mm and 19 Hz for 20-mm thick specimens (Figure 7.3). With horizontal x-axis impacts, the PSDs of the inline x-axis acceleration and inline accelerance showed a single resonance at 4.1 Hz for 10-mm, 3.3 Hz for 15-mm and 2.5 Hz for 20-mm thick specimens with their peak magnitudes less than half of the peak magnitudes obtained from the vertical impact (Figure 7.4). With each specimen subjected to five repeated impact tests, the resultant acceleration at peak (non-flat lines of the PSDs in Figure 7.3 and 7.4) increases with increasing impact force magnitude (flat lines of the PSDs in Figure 7.3 and 7.4). The resonance frequency seemed to be the same for different repeats of the same specimen and the same for different specimens with the same thickness. The standard deviation of the mean of the different repeated measurement of the same specimen tends to be similar for the entire specimen tested and it is less than 2% for all cases tested (Table 7.1). No significant difference was found in resonance frequency, across the five repeats using the same specimen ($\rho > 0.05$, one-factor ANOVA for all three groups). However there was a significant difference in stiffness and damping constant, across the five repeats using the same specimen ($\rho < 0.05$, one-factor ANOVA for all three groups except the one individual test of stiffness of the 10 mm thickness of horizontal impact which showed no significant different, $\rho=0.24$). Between the four nominally identical specimens, no significant difference was found in resonance frequency, stiffness and damping constant ($\rho > 0.05$, one-factor ANOVA for all three groups).
Figure 7.3  Vertical z-axis impact with flat sprung mass: PSDs of impact force (flat lines, $N^2/Hz$) and resultant acceleration (non-flat lines, $(m/s^2)^2/Hz$) of 12 specimens (SS1 to SS12 each with five repeat measurements).
Figure 7.4  Horizontal x-axis impact with flat sprung mass: PSDs of impact force (flat lines, N²/Hz) and resultant acceleration (non-flat lines, (m/s²)²/Hz) of 12 specimens (SS1 to SS12 each with five repeat measurements).
Table 7.1  Mean dynamic stiffness $k$ and damping constant $c$ (± standard deviation of the mean) over five repeats extracted at the first principal peak frequency of accelerance for both vertical $z$-axis and horizontal $x$-axis impacts. A flat 2.5 kg sprung mass was used with specimen ID SS1 to SS12.

<table>
<thead>
<tr>
<th>ID</th>
<th>Diameter (mm)</th>
<th>Thickness (mm)</th>
<th>Vertical z-axis impact</th>
<th>Horizontal x-axis impact</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>First peak (Hz)</td>
<td>$k$ (kN/m)</td>
</tr>
<tr>
<td>SS1</td>
<td>50</td>
<td>10</td>
<td>26.2</td>
<td>79.2 ± 0.2</td>
</tr>
<tr>
<td>SS2</td>
<td>50</td>
<td>10</td>
<td>26.2</td>
<td>79.5 ± 0.2</td>
</tr>
<tr>
<td>SS3</td>
<td>50</td>
<td>10</td>
<td>26.2</td>
<td>78.9 ± 0.2</td>
</tr>
<tr>
<td>SS4</td>
<td>50</td>
<td>10</td>
<td>26.2</td>
<td>79.2 ± 0.2</td>
</tr>
<tr>
<td>SS5</td>
<td>50</td>
<td>15</td>
<td>22.2</td>
<td>43.1 ± 0.5</td>
</tr>
<tr>
<td>SS6</td>
<td>50</td>
<td>15</td>
<td>22.2</td>
<td>43.2 ± 0.4</td>
</tr>
<tr>
<td>SS7</td>
<td>50</td>
<td>15</td>
<td>22.2</td>
<td>43.4 ± 0.4</td>
</tr>
<tr>
<td>SS8</td>
<td>50</td>
<td>15</td>
<td>22.2</td>
<td>43.9 ± 0.4</td>
</tr>
<tr>
<td>SS9</td>
<td>50</td>
<td>20</td>
<td>18.8</td>
<td>38.2 ± 0.3</td>
</tr>
<tr>
<td>SS10</td>
<td>50</td>
<td>20</td>
<td>19.2</td>
<td>38.5 ± 0.3</td>
</tr>
<tr>
<td>SS11</td>
<td>50</td>
<td>20</td>
<td>18.8</td>
<td>38.5 ± 0.3</td>
</tr>
<tr>
<td>SS12</td>
<td>50</td>
<td>20</td>
<td>19.4</td>
<td>38.7 ± 0.3</td>
</tr>
</tbody>
</table>
Variation in accelerance, receptance, damping constant and stiffness were shown with five vertical (Figure 7.5) and five horizontal impacts (Figure 7.6). For vertical impacts, the stiffness increased with frequency in the range 15 to 30 Hz where the principal peak accelerance occurred; damping constant decreased slightly with frequency in the range 15 to 30 Hz. For horizontal impacts, the stiffness gradually increased and the damping constant gradually decreased with frequency from 1 to 10 Hz where the first peak occurred.

Figure 7.5  Vertical z-axis impact on 50-mm diameter specimen with 2.5 kg flat sprung mass: column 1 for 10-mm thick specimen, column 2 for 15-mm, and column 3 for 20-mm. For each specimen the dynamic stiffness $k$, damping $c$, receptance ($s^2$/kg), and accelerance (1/kg) are shown. Frequency of interest: 10 to 30 Hz, with vertical dashed line indicating the peak around 26 Hz for 10-mm, 22 Hz for 15-mm and 19 Hz for 20-mm thick specimen. Refer to Table 7.1 for values.
Figure 7.6  Horizontal x-axis impact on 50 mm diameter specimen with 2.5 kg flat sprung mass: column 1 for 10-mm thick specimen, column 2 for 15-mm, and column 3 for 20-mm. For each specimen the dynamic stiffness k, damping c, receptance (s^2/kg), and accelerance (1/kg) are shown. Frequency of interest: 1 to 10 Hz, with vertical dashed line indicating the peak around 4.1 Hz for 10-mm, 3.3 Hz for 15-mm and 2.2 Hz for 20-mm thick specimen. Refer to Table 7.1 for values.

7.3.1.2  Effect of specimen size

In addition to the 50-mm diameter specimens tested, three specimens of 75-mm diameter and two of 100-mm diameter were tested. Specimens were compared for 3 different thicknesses, 3 diameters, 3 sprung masses, and 2 sprung mass contours.

Effect of thickness

The resonance frequency decreased as the thickness increased from 50 to 75 and 100 mm. This was consistent for both vertical and horizontal impact with flat and hip-borne contacts of three sprung masses (Figure 7.7; Table 7.2). Over the three thicknesses 10, 15 and 20 mm, there was significant differences in resonance frequencies for both vertical (p=0.000, four-factor ANOVA) and horizontal impacts (p=0.000). The interaction between thickness and diameter (p=0.000, four-factor ANOVA), thickness and sprung mass (p=0.000) and thickness and sprung mass contour (p=0.000) were significant for both vertical and horizontal impacts (p<0.05 for all three interactions).
Figure 7.7  The variation in the resonance frequency of the specimen with three different thicknesses: vertical (left column) and horizontal (right column) with flat sprung mass (hollow diamond - 50 mm, hollow square - 75 mm and hollow triangle - 100 mm diameter) and hip-borne sprung mass (solid diamond - 50 mm, solid square - 75 mm and solid triangle 100 mm diameter). Refer to Table 7.2 for values.

The variation in stiffness showed similar trend as the resonance frequency - the stiffness decreased as thickness increased, (Figure 7.8). This was consistent for both vertical and horizontal impacts with flat and hip-borne contacts of all three sprung masses. Such effect was significant for both vertical ($p=0.000$, four-factor ANOVA) and horizontal ($p=0.000$) impacts. The interaction between thickness and diameter ($p=0.000$, four-factor ANOVA), thickness and sprung mass ($p=0.000$) and thickness and sprung mass contour ($p=0.000$) were significant for both vertical and horizontal impacts ($p<0.05$ for all three interactions).
Figure 7.8  The variation in the stiffness $k$ and damping constant $c$ of the specimen with three different thicknesses: vertical stiffness (first column), vertical damping constant (second column), horizontal stiffness (third column) and horizontal damping constant (fourth column) with flat sprung mass (hollow diamond - 50 mm, hollow square - 75 mm and hollow triangle - 100 mm diameter) and hip-borne sprung mass (solid diamond - 50 mm, solid square - 75 mm and solid triangle 100 mm diameter). Refer to Tables 7.3 A and B for values.

There was no clear trend for the damping constant at different thicknesses for vertical impacts. However for horizontal impacts, the damping constant decreased slightly as the thickness increased from 10 to 15 and 20 mm. Changing thickness did affect damping constant for both vertical ($p=0.004$, four-factor ANOVA) and horizontal impacts ($p=0.004$). The interaction between thickness and diameter ($p=0.000$, four-factor ANOVA), thickness and sprung mass ($p=0.000$) and thickness and sprung mass contour ($p=0.000$) were significant for both vertical and horizontal impacts ($p<0.05$ for all three interaction).

Effect of diameter

The resonance frequency increased (Figure 7.9; Table 7.2) as the diameter increased from 50 to 75 and 100 mm. This was consistent for both vertical and horizontal impacts with flat and hip-borne contact of three sprung masses. Over the three diameters 50, 75 and 100 mm, there were significant differences in resonance frequencies for both vertical ($p=0.000$, four-factor ANOVA) and horizontal ($p=0.000$) impacts. The interaction between diameter and sprung mass ($p=0.000$, four-factor ANOVA) diameter and sprung mass contour ($p=0.000$) were significant for both vertical and horizontal impacts ($p<0.05$ for both two interactions).
Figure 7.9  The variation in the resonance frequency of the specimen with three different diameters: vertical (left column) and horizontal (right column) with flat sprung mass (hollow diamond - 10 mm, hollow square - 15 mm and hollow triangle - 20 mm thickness) and hip-borne sprung mass (solid diamond - 10 mm and solid square - 15 mm thickness). Refer to Table 7.2 for values.

The variation in the stiffness showed similar trend as the resonance frequency - the stiffness increased (Figure 7.10) as the diameter increased from 50 to 75 and 100 mm. This was consistent for both vertical and horizontal impacts with flat and hip-borne contact of three sprung masses. Such effect was found to be significant for both vertical ($p=0.000$, four-factor ANOVA) and horizontal ($p=0.000$) impacts. The interaction between diameter and sprung mass ($p=0.000$, four-factor ANOVA) diameter and sprung mass contour ($p=0.000$) were significant for both vertical and horizontal impacts ($p<0.05$ for both two interactions).
Figure 7.10 The variation in the stiffness $k$ and damping constant $c$ of the specimen with three different diameters: vertical stiffness (first column), vertical damping constant (second column), horizontal stiffness (third column) and horizontal damping constant (fourth column) with flat sprung mass (hollow diamond - 10 mm, hollow square - 15 mm and hollow triangle - 20 mm diameter) and hip-borne sprung mass (solid diamond - 10 mm and solid square - 15 mm diameter). Refer to Table 7.3 A and B for values.

There was no clear trend for the damping constant at different diameter for vertical impacts. However for horizontal impacts, the damping constant increased slightly as the diameter increased from 50 to 75 and 100 mm. Changing diameter did affect damping constant for both vertical ($p=0.004$, four-factor ANOVA) and horizontal impacts ($p=0.004$). The interaction between diameter and sprung mass ($p=0.000$, four-factor ANOVA) and diameter and sprung mass contour were significant for both vertical and horizontal impacts ($p<0.05$ for both two interaction).
Table 7.2  Measured first principal peak frequency of the accelerance when the specimens were subjected to vertical z-axis and horizontal x-axis impacts. Specimens were pre-loaded with flat and hip sprung masses (1.5, 2.5 and 5 kg). Vertical and the horizontal resonance frequencies were rounded to the first significant digit.

<table>
<thead>
<tr>
<th>Sprung mass</th>
<th>Diameter (mm)</th>
<th>Vertical z-axis impact First peak (Hz)</th>
<th>Horizontal x-axis impact First peak (Hz)</th>
</tr>
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<tbody>
<tr>
<td></td>
<td></td>
<td>$f_{10mm}$ $f_{15mm}$ $f_{20mm}$</td>
<td>$f_{10mm}$ $f_{15mm}$ $f_{20mm}$</td>
</tr>
<tr>
<td>Contour</td>
<td>Mass (kg)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flat</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td>50</td>
<td>34.2 26.4 21.4</td>
<td>5.7 4.1 3.3</td>
</tr>
<tr>
<td></td>
<td>75</td>
<td>42.2 35.2 30.4</td>
<td>9.8 7.4 5.7</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>- 42.2 39.2</td>
<td>- 11 9.0</td>
</tr>
<tr>
<td>Flat</td>
<td>2.5</td>
<td>26.4 22.4 20.2</td>
<td>4.1 3.3 2.5</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>35.2 29.2 25.4</td>
<td>8.2 5.7 4.9</td>
</tr>
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<td>- 35.2 30.2</td>
<td>- 8.2 6.6</td>
</tr>
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<td></td>
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<tr>
<td>Flat</td>
<td>5.0</td>
<td>21.2 21.2 19.4</td>
<td>3.3 2.5 2.5</td>
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<td>50</td>
<td>27.4 22.2 19.4</td>
<td>4.9 4.1 3.3</td>
</tr>
<tr>
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<td>- 26.2 21.2</td>
<td>- 5.7 4.9</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hip</td>
<td>1.5</td>
<td>20.2 18.2 17.2</td>
<td>3.3 2.5 2.5</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>23.4 20.2 19.4</td>
<td>4.1 3.3 3.3</td>
</tr>
<tr>
<td></td>
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</tr>
<tr>
<td></td>
<td>100</td>
<td></td>
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<tr>
<td>Hip</td>
<td>2.5</td>
<td>21.2 18.4 17.4</td>
<td>3.3 3.3 2.5</td>
</tr>
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<td></td>
<td>50</td>
<td>25.4 23.4 23.4</td>
<td>5.7 4.9 4.1</td>
</tr>
<tr>
<td></td>
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<td></td>
<td>100</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 7.3A  Mean dynamic stiffness k and damping constant c (± standard deviation of the mean) over five repeats extracted at the first principal peak frequency of accelerance for vertical z-axis impact.

<table>
<thead>
<tr>
<th>Sprung mass</th>
<th>Diameter</th>
<th>Mass (kg)</th>
<th>k (kN/m) and c (Ns/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>k10mm c10mm</td>
</tr>
<tr>
<td>Contour</td>
<td>Mass (mm)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flat</td>
<td>1.5</td>
<td>50 75 100</td>
<td>71.1 ± 0.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>72.3 ± 0.2</td>
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<td></td>
<td></td>
<td></td>
<td>110.4 ± 0.2</td>
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<tr>
<td></td>
<td></td>
<td>1.5 75 100</td>
<td>56.5 ± 0.3</td>
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<tr>
<td></td>
<td></td>
<td>2.5 75 100</td>
<td>- 108.4 ± 0.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5.0 75 100</td>
<td>- 75.3 ± 0.7</td>
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<tr>
<td>Flat</td>
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<td>50 75 100</td>
<td>79.2 ± 0.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.5 75 100</td>
<td>91.1 ± 0.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.5 75 100</td>
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<tr>
<td></td>
<td></td>
<td>5.0 75 100</td>
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<td></td>
<td>100</td>
<td></td>
<td>- 130.4 ± 0.3</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>- 110.2 ± 1.9</td>
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<tr>
<td>Hip</td>
<td>1.5</td>
<td>50 75 100</td>
<td>95.8 ± 0.7</td>
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<td></td>
<td></td>
<td>1.5 75 100</td>
<td>110.3 ± 0.2</td>
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<tr>
<td></td>
<td></td>
<td>2.5 75 100</td>
<td>146.2 ± 0.6</td>
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<td></td>
<td></td>
<td>5.0 75 100</td>
<td>133.3 ± 0.9</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td></td>
<td>- 155.2 ± 0.8</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>- 162.3 ± 3.5</td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td>50 75 100</td>
<td>51.9 ± 0.6</td>
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<td></td>
<td></td>
<td>33.9 ± 0.4</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>63.8 ± 0.6</td>
</tr>
<tr>
<td></td>
<td>110.1 ± 2.5</td>
<td>91.9 ± 1.1</td>
<td>105.2 ± 1.6</td>
</tr>
</tbody>
</table>
Table 7.3B  Mean dynamic stiffness \( k \) and damping constant \( c \) (± standard deviation of the mean) over five repeats extracted at the first principal peak frequency of accelerance for horizontal x-axis impact.

<table>
<thead>
<tr>
<th>Sprung mass</th>
<th>Diameter (mm)</th>
<th>k (kN/m) and c (Ns/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Contour</td>
<td>Mass (kg)</td>
</tr>
<tr>
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<tr>
<td>Flat</td>
<td>1.5</td>
<td>50</td>
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<tr>
<td></td>
<td></td>
<td>75</td>
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<tr>
<td>Flat</td>
<td>2.5</td>
<td>50</td>
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<tr>
<td>Flat</td>
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<td>50</td>
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<td>Hip</td>
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</tbody>
</table>
7.3.1.3 Effect of sprung mass

The resonance frequency decreased as the sprung mass increased from 1.5 to 2.5 and 5.0 kg (Figure 7.11; Table 7.2). This was consistent for both vertical and horizontal impacts with the flat sprung mass contour. With the hip-borne contour of two sprung masses, the resonance frequency increased with increasing sprung mass from 1.5 to 2.5 kg for both vertical and horizontal impacts. Over the three sprung masses 1.5, 2.5 and 5.0 kg, there were significant differences in resonance frequencies for both vertical ($p=0.000$, four-factor ANOVA) and horizontal ($p=0.000$) impacts. The interaction between sprung mass and sprung mass contour was significant for both vertical and horizontal impacts ($p<0.05$, four-factor ANOVA for both cases). In Figure 7.11, resonance frequency corresponds to three different sprung masses were represented by three different markers at each thickness of the specimen in x-axis.

![Figure 7.11](image)

Figure 7.11 The variation in the resonance frequency of the specimen with three different sprung masses: vertical (left column) and horizontal (right column) with flat sprung mass (hollow diamond - 1.5 kg, hollow square - 2.5 kg and hollow triangle - 5.0 kg) and hip-borne sprung mass (solid diamond - 1.5 kg and solid square - 2.5 kg). Refer to Table 7.2 for values.
The stiffness increased as the sprung mass increased from 1.5 to 2.5 and 5.0 kg (Figure 7.12; Table 7.3). This was consistent for both vertical and horizontal impacts with flat and hip-borne sprung masses. Such effect was found in both vertical ($p=0.000$, four-factor ANOVA) and horizontal impacts ($p=0.000$). The interaction between sprung mass and sprung mass contour on the stiffness was significant for both vertical and horizontal impacts ($p<0.05$, four-factor ANOVA for both cases). In Figure 7.12, $k$ and $c$ corresponds to three different sprung masses were represented by three different markers at each thickness of the specimen in x-axis.

The damping constant increased for the vertical impact with both flat and hip-borne sprung masses as the sprung mass increased from 1.5 to 2.5 and 5.0 kg. For horizontal impacts, the same trend in damping was observed with hip-borne sprung mass but not with the flat sprung mass. Changing sprung mass did affect damping constant for both vertical ($p=0.004$, four-factor ANOVA) and horizontal impacts ($p=0.004$). The interaction between sprung mass and sprung mass contour was
significant for both vertical and horizontal impacts ($p<0.05$, four-factor ANOVA for both cases).

### 7.3.1.4 Effect of sprung mass contact contour

Specimen with 75-mm diameter and 15-mm thickness and test using flat and hip-borne contact contour of 2.5 kg sprung mass are selected to show the effect of sprung mass contact contour. For vertical impacts, the accelerance and receptance with the hip-borne contact of sprung mass showed distinctive lower frequency of a single peak than those of the flat sprung mass in the frequency 20 to 40 Hz (Figure 7.13 left). This trend was less clear with horizontal impacts but still appreciable in the frequency 1 to 10 Hz (Figure 7.13 right). Changing sprung mass contour from flat to hip-borne did affect resonance frequency for both vertical ($p=0.000$, four-factor ANOVA) and horizontal impacts ($p=0.000$).

![Figure 7.13](image)

Figure 7.13 Specimen with 75 mm diameter and 15 mm thickness tested with two different indenter surface of the sprung mass: Vertical z-axis impact (left column) and horizontal x-axis impact (right column) with 2.5 kg flat sprung mass (blue solid line) and 2.5 kg hip-borne sprung mass (black dashed line). Refer to Tables 7.3 A and B for values.

The stiffness decreased when changing the sprung mass from flat to hip-borne for both vertical ($p=0.000$, four-factor ANOVA) and horizontal impacts ($p=0.000$). There was reduction in damping when changing the sprung mass contour from flat to hip-borne for both vertical ($p=0.000$, four-factor ANOVA) and horizontal impacts ($p=0.000$).
7.3.2 Porcine muscle specimen

Nine specimens were tested within two hours of animal death (Table 4). With the limited number of specimens and window of time, it is still possible to depict the behaviour of the biological tissue against the artificial tissue. The results below show: the inter-specimen and intra-specimen variation (Figure 7.14 and 7.15), effect of specimen thickness (Figure 7.16 and 7.17), and effect of the flat and hip-borne sprung masses (Figure 7.18). Guided by the timeline in Table 7.4, one could inspect the rigor mortis effect which is commonly observed in biological tissues.

7.3.2.1 Inter-specimen and intra-specimen variation

With impact force exerted in the vertical z-axis on the sprung mass, the PSDs of the inline z-axis acceleration and accelerance exhibited a principle resonance frequency around 25 Hz and a secondary one around 40 Hz (Figure 7.14 and Figure 7.16). With horizontal x-axis impacts, the PSDs of the inline x-axis acceleration and inline accelerance showed a single resonance around 3 Hz with its peak magnitude almost half of the peak magnitude obtained from the vertical impact (Figure 7.15 and Figure 7.17). With each specimen subjected to about 2 to 3 repeated hits, the resultant acceleration at peak (non-flat lines of the PSDs in Figure 7.14 and 7.15) increases with increasing impact force magnitude (flat lines of the PSDs in Figure 7.14 and 7.15). The frequencies at which the peak resultant acceleration occurred seemed to be the same for different repeats of the same specimen and the same for different specimens with or without the same thickness despite the experiment timeline.

7.3.2.2 Effect of specimen thickness

Specimens of different thicknesses had similar stiffnesses and damping constants at the peak frequency around 25 Hz for vertical impacts (Figure 7.16) and at around 2.5 Hz for horizontal impacts (Figure 7.17). See PS1, PS2, PS3 in Table 7.4. For vertical impacts, both stiffness and damping tended to increase with frequency from 20 to 30 Hz where the first peak of accelerance occurred. For horizontal impacts, the stiffness gradually increased and the damping slightly decreased with frequency from 1 to 5 Hz where the first peak occurred. The stiffness \( k \) did not vary consistently as the thickness increased from 10 to 15 and 20 mm with both vertical and horizontal impacts. Values of stiffness \( k \) and damping \( c \) of the first peaks in Table 7.4 were
shown in Figure 7.16 to illustrate their variation at different frequencies. There was a slight reduction in damping as the thickness increased for vertical impacts. The uncertainty in the different repeated measurement (standard deviation of the mean) tends to be similar for the entire sample tested and value is less than 1 % of the mean for all cases tested (refer to Table 7.4 for values).

Figure 7.14  Vertical z-axis impact with flat sprung mass: PSDs of impact force (blue flat lines, N²/Hz) and resultant acceleration (green non-flat lines, (m/s²)²/Hz) of 10 specimens (PS1 to PS10 each with several repeat runs) showing time of test from animal death.
Table 7.4  Mean dynamic stiffness $k$ and damping constant $c$ (± standard deviation of the mean) over 2 to 4 repeats extracted at the first principal peak frequency of accelerance for both vertical $z$-axis and horizontal $x$-axis impacts. A flat 2.5 kg sprung mass was used with specimens PS1 to PS10 and a hip-borne 2.5 kg sprung mass was used with specimens PS3hp and PS10hp.

<table>
<thead>
<tr>
<th>Time after death (min)</th>
<th>Specimen number</th>
<th>Thickness (mm)</th>
<th>Vertical $z$-axis impact</th>
<th>Horizontal $x$-axis impact</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>First peak (Hz)</td>
<td>$k$ (kN/m)</td>
</tr>
<tr>
<td>48</td>
<td>PS1</td>
<td>10</td>
<td>25.2</td>
<td>54.6 ± 0.9</td>
</tr>
<tr>
<td>60</td>
<td>PS2</td>
<td>15</td>
<td>25.2</td>
<td>68.8 ± 1.4</td>
</tr>
<tr>
<td>68</td>
<td>PS3</td>
<td>20</td>
<td>25.2</td>
<td>56.1 ± 0.6</td>
</tr>
<tr>
<td>76</td>
<td>PS4</td>
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<td>25.2</td>
<td>66.7 ± 0.6</td>
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<td>62.8 ± 1.0</td>
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<td>25.2</td>
<td>59.2 ± 0.2</td>
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<tr>
<td>97</td>
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<td>15</td>
<td>25.4</td>
<td>53.2 ± 0.6</td>
</tr>
<tr>
<td>112</td>
<td>PS6</td>
<td>20</td>
<td>24.4</td>
<td>52.4 ± 0.3</td>
</tr>
<tr>
<td>118</td>
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<td>20</td>
<td>25.4</td>
<td>65.1 ± 0.6</td>
</tr>
<tr>
<td>153</td>
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<td>25.2</td>
<td>68.0 ± 0.6</td>
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<tr>
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<td>24.2</td>
<td>58.1 ± 0.1</td>
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<td>PS3hp</td>
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<td>23.4</td>
<td>54.4 ± 0.1</td>
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<tr>
<td>172</td>
<td>PS11hp</td>
<td>10</td>
<td>25.2</td>
<td>57.1 ± 1.0</td>
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</tbody>
</table>
Figure 7.15  Horizontal x-axis impact with flat sprung mass: PSDs of impact force (blue flat lines, N^2/Hz) and resultant acceleration (green non-flat lines, (m/s^2)^2/Hz) of 7 of the 10 specimens (PS1 to PS10 each with several repeat runs) showing time of test from animal death.
Figure 7.16  Vertical z-axis impact with 2.5 kg flat sprung mass: column 1 for PS1 with thickness and time after death, column 2 for PS2, and column 3 for PS3. For each specimen (PS1 to PS3) the dynamic stiffness $k$, damping $c$, receptance ($s^2$/kg), and accelerance (1/kg) are shown. Frequency of interest: 20 to 30 Hz, with vertical dashed line indicating the peak around 25 Hz. Refer to Table 7.4 for values.

Figure 7.17  Horizontal x-axis impact with 2.5 kg flat sprung mass: column 1 for PS1 with thickness and time after death, column 2 for PS2, and column 3 for PS3. For each specimen (PS1 to PS3) the dynamic stiffness $k$, damping $c$, receptance ($s^2$/kg), and accelerance (1/kg) are shown. Frequency of interest: 1 to 5 Hz, with vertical dashed line indicating the peak around 2.5 Hz. Refer to Table 7.4 for values.
7.3.2.3  **Effect of sprung mass contact contour**

Tests PS10 and PS10hp in Table 7.4 showed the effect of sprung mass contact contour. For vertical impacts, the accelerance and receptance with the hip-borne sprung mass show distinctive higher magnitude and lower frequency of the first peak than those of the flat sprung mass in the frequency range 20 to 30 Hz ([Figure 7.18 left](#)). This is less clear with horizontal impacts but still appreciable in the frequency range 1 to 5 Hz ([Figure 7.18 right](#)).

![Figure 7.18](#) Specimen 10 tested approximately 153 minutes after death: Vertical z-axis impact (left column) and horizontal x-axis impact (right column) with 2.5 kg flat sprung mass (PS10, blue solid line) and 2.5 kg hip-borne sprung mass (PS10hp, black dashed line). Refer to Table 7.4 for values at peak.

The stiffness values in Table 7.4 show the softening effect (reduction in stiffness) of the hip-borne contact of sprung mass was most obvious with the thinnest specimen of 10-mm thickness in both vertical and horizontal impacts. There was a reduction in damping using the hip-borne sprung mass during vertical impact, but not so with horizontal impacts.
7.4 Discussion

The rigid mass-soft tissue analogy was considered a close representation of the *in vivo* loading scenario of the soft tissue at the excitation-subject interface during Whole-Body Vibration. The silicone rubber and porcine skeletal muscle have both been widely adopted as human soft tissue substitutes, but the ways in which the dynamic characteristics of these materials were investigated vary considerably (e.g. Aimiedieu et al., 2003; Loocke et al., 2009; Ozcan et al., 2011; Takaza et al., 2013a). Despite the obvious advantage of silicone rubber being more configurable and repeatable (e.g. Sparks et al., 2015), freshly extracted porcine skeletal muscle is still considered a closer estimate of human soft tissue (e.g. Loocke et al., 2009). The present study focuses on biomechanical parameters that are closely related to the 'macro' biomechanical behaviour seen in whole-body vibration. These include the frequency dependent dynamic stiffness and damping, and the frequency responses to impact forces in the two orthogonal axes.

7.4.1 The resonance frequency of silicone rubber and porcine muscle

7.4.1.1 Vertical z-axis impact

The PSDs and accelerance of the different silicone rubber specimens exhibited a single resonance at 20 Hz for 20-mm, 22 Hz for 15-mm and 26 Hz for 20-mm thick specimens. With different porcine muscle specimens, the PSDs and accelerance showed two main resonances at around 25 and 40 Hz. The primary resonance frequencies of similarly sized silicone rubber and porcine muscle specimens were comparable. However, the bimodal resonance observed in the porcine muscle was absent from the silicone rubber.

The bimodal resonance frequency observation with the porcine muscle deviated from the single resonance observation of silicone rubber and WBV. This could be discussed firstly from the view of structural arrangement of muscle tissue in fibre level. The muscle fibers in vivo are invested in a network of blood and lymphatic vessels. Some contents of these vessels may have been expelled during the specimen preparation or during initial loading of the rigid mass-porcine muscle system. The highest velocity of the sprung mass, compressing the muscle tissue, after impact was less than 0.2 m/s comparing to the drop impact velocity of up to 3 m/s in the study conducted by Takaza et al. (2013a), who reported an average of
8% fluid mass loss. The variation in the microstructure of the soft tissue affects the macroscopic behaviour (Sharafi and Blemker, 2010).

Secondly, FRFs of whole-body vibration transmit motion in axes orthogonal to the axis of excitation and are highly corrected to the inline response (Huang and Furguson, 2012). The second mode shown in the vertical mode could be caused by the rotational mode of the sprung mass arising from the structural irregularity of the porcine muscle specimen when compressed. However after a careful inspection of cross-axis response, there was no appreciable cross-axis response around the second resonance frequency. A new trial with multiple vertical measurements at different location of the sprung mass would be required which has not been done.

A single resonance observation with the silicon specimen was comparable to that the single resonance observed in apparent mass of vertical WBV. However the resonance frequency of the silicone specimen largely differ from that observed in apparent mass of vertical WBV - 4 to 6 Hz for seated (Huang and Griffin, 2006), and 8 to 10 Hz for recumbent (Huang and Griffin, 2008a).

7.4.1.2 Horizontal x-axis impact

The PSDs and accelerance of different silicone rubber specimens repeatedly showed a main resonance at around 2 to 4 Hz and the different porcine muscle specimens repeatedly showed a main resonance at around 2 to 3 Hz. The frequency is similar to that observed in apparent mass of recumbent human subjects during horizontal longitudinal WBV (Huang and Griffin, 2008b). It is plausible that when the passive muscles are subjected to shear 'horizontal' load, the present in vitro setup using silicone rubber and the porcine muscle specimens shares a similar motion transmission mechanism to the back of a recumbent human.

Resonance frequency measured using five repeated measurements with the same silicone specimen showed no significant variation in both vertical and horizontal impacts. However there was a significant difference for the stiffness and damping with different repeats. This variability could be mainly caused by the magnitude of the impact force exerted in different repeats. The variation in the stiffness and damping due to different magnitude of impact force imply that stiffness and damping constant of the silicone rubber behave nonlinearly with different magnitudes impact force. The absence of change in resonance frequency using different magnitudes
of the impacts force seems to suggest the thixotropic or memory effect of the human body by Huang and Griffin, 2008a and b, is missing in the silicone and the porcine experiments. This begs further investigation using base excitation with fresh porcine specimens.

7.4.2 Effect of thickness

The resonance frequency decreased with the increasing thickness from 10 to 15 and 20 mm for both vertical and horizontal impacts in silicone rubber specimens tested, but no systematic change was found in the fresh porcine skeletal muscle specimens. With the vertical impact hammer test using the silicone rubber specimen, Ozcan et al. (2011) reported that the resonance frequency decreased with the increasing thickness from 40 to 180 mm.

The stiffness of the silicone rubber specimen of different thicknesses showed similar trend as the resonance frequency, however there was no obvious trend with damping constant. The stiffness and damping values obtained from silicone rubber and porcine muscle specimens are comparable and they show similar order of magnitude in both vertical and horizontal impacts (see Figure 7.19). The stiffness value obtained from lumped parameter models of seated subjects (49 kN/m at 0.25 ms\(^2\) r.m.s. and 32 kN/m at 1.0 ms\(^2\) r.m.s., Huang and Griffin, 2006) and recumbent subjects (60 kN/m at 0.25 ms\(^2\) r.m.s. and 52 kN/m at 1.0 ms\(^2\) r.m.s., Huang and Griffin, 2008a) during vertical base-excited WBV also show similar order to the current study. However the damping for vertical excitation and both stiffness and damping for horizontal excitation of recumbent subjects (Huang and Griffin, 2008b) is far higher than those observed in this study. The stiffness obtained from the vertical impact test using silicone specimens showed slightly higher values than the one obtained with the shaker test in Chapter 6 (Figure 7.20). The damping also showed similar observation. This implies that the stiffness and the damping values are sensitive to excitation levels.
Figure 7.19 Dynamic stiffness (first row) and damping constant (second row) at the first peak frequency for vertical (left column) and horizontal (right column) using three thicknesses of specimens (10, 15 and 20 mm) with flat (silicone specimen - hollow diamond and triangle; porcine specimen - hollow square and circle) and hip-borne sprung mass (silicone specimen - solid diamond and triangle; porcine specimen - solid square and circle), with values in Table 7.3 A and B and 7.4.

Figure 7.20 Dynamic stiffness (left) and damping constant (right) at the first peak frequency using three thicknesses of specimens (10, 15 and 20 mm): base excited test with flat (0.5 ms$^{-2}$ r.m.s. - hollow square and 2.0 ms$^{-2}$ r.m.s. - hollow diamond) and hip-borne sprung mass (0.5 ms$^{-2}$ r.m.s. - solid square and 2.0 ms$^{-2}$ r.m.s. - solid diamond); vertical impact hammer test using porcine specimen - hollow circle; vertical impact hammer test using silicone rubber with flat sprung mass - hollow triangle and hip-borne sprung mass - solid triangle.
7.4.3 Effect of diameter

As the diameter of the silicone specimen increased from 50 to 75 and 100 mm, the resonance frequency increased for both vertical and horizontal impacts. The effect of diameter stiffness showed similar trends as resonance frequency for both vertical and horizontal impacts. The changes in the diameter of the silicone specimen did change the damping constant, however there was no clear trend observed. The resonance frequency measured at the different parts of the human body showed different values (e.g. at front thighs 8 to 10 Hz and at the ischial tuberosities around 5 Hz; Liu et al., 2015a, b). The author has speculated that the factors causing this difference could be caused by the changes in the tissue thickness, contact area and the pressure of the tissue under things and ischial tuberosities. The finding from this study support the speculation drawn by Liu et al., 2015a, b.

7.7.4 Effect of sprung mass

The resonance frequency of the silicone specimen decreased with increasing sprung mass from 1.5 to 2.5 and 5.0 kg for both vertical and horizontal impacts. The stiffness of the silicone specimen increased as the sprung mass increased from 1.5 to 2.5 and 5.0 kg for both vertical and horizontal impacts. The damping constant increased as the sprung mass increased from 1.5 to 2.5 and 5.0 kg for vertical impact, but no clear trend was found with horizontal impacts.

7.4.5 Effect of sprung mass contact contour

Changing sprung mass contact contour from flat to hip-borne reduced the resonance frequency of both silicone rubber and porcine specimens for both vertical and horizontal impacts. There is a reduction of 50% in stiffness of silicone rubber specimens when changing from hip-borne to flat sprung mass for both vertical and horizontal impacts. This reduction in stiffness was observed only with the thinnest 10-mm thick porcine specimen for both vertical and horizontal impacts. The reduction in damping constant of silicone rubber specimen was observed for both vertical and horizontal impacts. However with porcine specimen, the reduction was found only with vertical impacts. The contact contour partly alters the behaviour of the soft tissue in both base excited shaker test (in Chapter 6) and the impact hammer in this study. Along with the micro structure of buttocks, they would primarily contribute to the difference that observed in this study comparison to WBV
studies. In the structural dynamics studies, the frequency dependent dynamic stiffness and damping was extracted using the similar receptance technique (i.e. Lin et al., 2005; Ozcan et al., 2011; Ooi and Ripin, 2011; Koblar et al., 2014) to characterise the rubber bearings. However the frequency range of interest in their studies is much higher than of this study and most of the biomechanics study. However this parameter extraction method mainly relies on the receptance that is derived from the accelerance in the frequency domain with inherited integral errors. The integral errors specially occur at lower frequencies, e.g. up to 1 to 2 Hz - the range relevant to the horizontal WBV. This could be one drawback of this method especially when the low frequency responses are of interest.

7.5 Conclusions

Impulse responses of the scaled rigid mass-soft tissue system which mimic the excitation-subject interface of the WBV are presented for the first time using silicone rubber and freshly harvested porcine muscle. A linear SDOF viscoelastic model was utilised to extract stiffness and damping from the measured accelerance and receptance frequency response functions. With a porcine muscle bimodal, a repeatable accelerance is observed for the (cross fibre) vertical impact tests at around 25 and 40 Hz and the similar size silicone rubber specimen showed a single resonance at 20 Hz for 20 mm, 22 Hz for 15 mm and 26 Hz for 20 mm thick specimens - a higher frequency range than those observed in vertical WBV. The second resonance appeared above 40 Hz could be caused by either the irregularities in the specimen shape or microstructure and the fluid-structure interaction of the muscle fibre and fascicle. A repeatable peak accelerance is observed for the horizontal impact tests at around 3 Hz in both silicone rubber and porcine muscle specimens - similar to that observed in horizontal WBV. The effect of thickness, diameter, sprung mass and sprung mass contact contour on resonance frequency and stiffness of the silicon specimens is found to be significant in both vertical and horizontal impacts. However the porcine specimen thickness has no clear effect on the parameters extracted. The absence of change in the resonance frequency using different magnitudes of impact force seems to suggest the thixotropic or memory effect of the human body is missing in the SDOF rigid mass-soft tissue system.
CHAPTER 8

General discussion

The previous chapters have considered how SDOF linear and nonlinear models can be utilised in time domain to predict the biomechanical responses of WBV (a preliminary study; Chapter 4); whether standard uni-axial cyclic compression test using silicone rubber specimens can reproduce the thixotropic or memory effect of the human body (Chapter 5). The dynamic responses of scaled rigid mass-soft tissue system has been examined: first using base-excited broadband random vibration at varying magnitudes (Chapter 6) and then using impact force exerted by impact hammer (Chapter 7). The main findings of these chapters are summarised and discussed in this section.

8.1 A preliminary study on modelling of biomechanical responses to whole-body vibration in time domain

The numerous viscoelastic models for biological soft tissues were primarily based on quasi-static and cyclic loading at limited magnitudes of excitations. There has been no reported analytical study on the base-excited nonlinear magnitude dependence of driving point response dynamic force of viscoelastic materials. The preliminary study presented in Chapter 4 focussed on time domain modelling using SDOF linear and nonlinear viscoelastic models. The result obtained from the simulated data using base-fixed configuration showed that model with base-fixed configuration accurately estimates the response. However the response estimates, from the similar model with base-excitation configuration, suffered from the integration error during optimisation. This may be partly from the integration technique used in this study (i.e. Runge-Kutta). Several studies in structural dynamics have acknowledged the integration error during the time domain optimisation (i.e. Kyprianou and Worden, 2001; Worden and Manson, 2012). However there was lack of literature on this issue. The models trained with the experimental data failed to predict the responses at multiple magnitudes.
8.2 Uni-axial cyclic compressive responses of silicone rubber

The uni-axial cyclic compression test was designed to examine the biomechanical properties, i.e. stiffness and elastic modulus, of silicone rubber: first to compare these properties with the published data and then to examine whether they can produce any thixotropic or memory effect observed in human response. The test showed that the stiffness of the silicone rubber increased (stiffening) with increasing excitation load and the frequency. Similar stiffening effect was reported in the compression tests of skin and adipose tissue (Wu et al., 2007). However this observation deviated from the thixotropic effect or softening effect of excitation-subject interface of WBV where the stiffness of the soft tissue at such interface decrease with increasing excitation magnitude and frequency. Biomechanical properties largely depend on excitation levels and boundary conditions. In the cyclic compression test, the specimen is placed between two clamps, the excitation is applied on one end and other end is fixed. This is not the realistic boundary condition of the soft tissue at the excitation-subject interface of the WBV. This suggests that the conventional cyclic compression test would unable to reproduce the magnitude dependency observed in WBV.

8.3 Dynamic behaviour of silicone rubber and porcine muscle

A dominant single resonance was observed for the base-excited rigid mass-silicone rubber system in the present study (Chapter 6) and also in most whole-body vibration studies with vertical vibration (Huang and Griffin, 2006, 2008a). Although the frequency range at which the resonance occurred were different: 20 to 33 Hz for current study and below 10 Hz for WBV studies, the mode of sprung mass might be similar in both scenarios. Similar single resonance mode was observed for impact hammer tests on mass-silicone rubber system with lower resonance frequency (20 to 26 Hz) than that of base-excited one (Chapter 7). Two main resonances at around 25 and 40 Hz were observed for impact hammer tests of rigid mass-porcine muscle system (Chapter 7). Although the porcine muscle was considered a closer representation of human soft tissue than silicone rubber, the bimodal response in porcine muscle seemed to disagree. Effects of thickness and diameter of the specimen, sprung mass, sprung mass contour and excitation magnitude have been systematically discussed in Chapters 6 and 7. The discussion in this section focuses
on the effect of vibration magnitude and sprung mass contact contour, and the bimodal response observed in porcine muscles.

Excitation magnitude

The expected excitation magnitude dependency in resonance frequency was absent in the present study – neither within base-excited shaker test nor within impact hammer test. This may be partly due to the excitation levels used in these studies that were not distinctive enough to produce adequate relative deformation in the soft specimens. With both broadband spectrum input excitations (base acceleration for base-excited rigid mass-silicone rubber system and impact force on sprung mass for the same rigid mass-silicone rubber system), the impact hammer test produced an evidently lower resonance frequency (around 26 Hz, Figure 8.1b) than that produced by the based-excited shaker test (around 35 Hz, Figure 8.1a). One could examine the sprung mass vertical motion in these two different excitation techniques - relative motion of the sprung mass with base excitation (Figure 8.1a) and absolute motion with impact excitation (Figure 8.1b). The impact force exerted in silicone rubber produce less sprung mass motion than the one with base excitation with higher magnitude (2.0 ms⁻² r.m.s.). This suggests that rigid mass-silicone rubber system may produce magnitude dependency in resonance frequency if the magnitudes of excitations are substantially different.

Figure 8.1  PSDs of: (a) - relative acceleration between shaker table and the sprung mass (—— 0.5, ---- 1.0, · · · 1.5, · · · · · · 2.0 ms⁻² r.m.s.); (b) - sprung mass acceleration during impact hammer test using silicone specimen (five repeated measurements are shown); (c) - sprung mass acceleration during impact hammer test using porcine specimen (three repeated measurements are shown). Tests using similar size silicone specimen of 50-mm diameter and 10-mm thickness and the porcine specimen with 50-mm diameter and 10-mm thickness with flat sprung mass of 2.5 kg are chosen to show here.
Sprung mass contact contour

The protruded ‘hip-borne’ surface was designed to mimic the bony structure of the tuberosities at the bottom of pelvis. The resonance frequency reduced when changing the sprung mass contact contour from flat to hip-borne. This was consistently observed in both base-excited and impact hammer tests. There was around 50% reduction in stiffness and around 40% reduction in damping when changing the sprung mass contour to hip-borne.

With same rigid mass-silicone rubber system using the hip-borne like sprung mass, base-excited shaker test produce lower resonance frequency (around 22 Hz, Figure 8.2 a) than that produced by the hammer test (around 26 Hz, Figure 8.2 b). This effect is less clear compared with the one observed with the flat sprung mass in the previous section (Figure 8.1) but still appreciable. The contact contour partly alters the behaviour of the soft tissue in both base excited shaker test and the impact hammer test. Along with the micro structure of buttocks, contact contour would primarily contribute to the difference that observed in this study comparison to WBV studies.

Figure 8.2  PSDs of: (a) - relative acceleration between shaker table and the sprung mass (0.5, 1.0, 1.5, 2.0 ms$^{-2}$ r.m.s.); (b) - sprung mass acceleration during impact hammer test using silicone specimen (five repeated measurements are shown); (c) - sprung mass acceleration during impact hammer test using porcine specimen (three repeated measurements are shown). Tests using similar size silicone specimen of 50-mm diameter and 10-mm thickness and the porcine specimen with 50-mm diameter and 10-mm thickness with hip-borne sprung mass of 2.5 kg are chosen to show here.
**Bimodal response of mass-porcine muscle system**

The bimodal resonances observed with the porcine muscle (Figure 8.1 c and 8.2 c) deviated from a single resonance observed in apparent mass of human body during vertical WBV (Huang and Griffin, 2006; Huang and Griffin, 2008a). Several possible factors may have contributed to this discrepancy. Firstly, fluid contents and the way they are contained and coupled in skeletal muscles will affect the dynamic behaviour of the tissue especially when compressed. Some fluid contents of muscle fibers may be expelled during specimen preparation or during initial static compression from the rigid sprung mass. Using a drop impact velocity of up to 3 m/s, Takaza et al. (2013a) reported an average of 8% fluid mass loss. Secondly, variation in microstructure of muscle fibre affects macroscopic biomechanical behaviour. Two fiber-level models were able to predict similar macroscopic shear moduli within 13.5% difference, however, two fascicle-level models were able to predict considerably wider variation in macroscopic shear moduli – up to 161% difference (Sharafi and Blemker, 2010). It is unclear so far how shear moduli affect the macroscopic dynamic behaviour of a mass-soft tissue system. It is also unclear how the shear moduli change with different magnitude and waveform of excitation.

From a structural dynamics point of view, a bimodal response could arise from 1) two vertical modes of the sprung mass, and 2) a vertical mode and a rotational mode of the sprung mass. The latter seemed to be unlikely per the low level of cross-axis response reported in this study. The accelerometer arrangement in the present study was difficult to measure any rotational movement of the sprung mass. Two spaced vertical accelerometers would be required.

Alternatively, the bimodal frequency response could be perceived as a trough related to an anti-resonance between the two modes of vibration in the mass-porcine muscle system. Anti-resonances usually occur in lightly damped coupled systems (Wahl et al., 1999). The 'micro' and 'macro' structural vibration may be coupled in the rigid mass-porcine muscle system to produce anti-resonances. Human body and the rigid mass-porcine muscle system are well damped. It would be difficult to separate the anti-resonance from the resonance peak if the system is highly damped.
CHAPTER 9
General conclusions, limitations and recommendations

9.1 General conclusions

The study described in this thesis attempts to characterise the biomechanical responses of the excitation-subject interface of WBV using scaled rigid mass-soft tissue system. An in depth experimental investigation has been conducted performing uni-axial cyclic compression, base-excited shaker test and impact hammer test. A preliminary study on time domain modelling of biodynamic responses to WBV has also been investigated.

The time domain SDOF linear and nonlinear models calibrated in this study failed to predict the biomechanical responses at different magnitudes. The base-excited system suffered highly with integration error compared with the base-fixed configuration.

With the uni-axial cyclic compression test, silicone rubber showed the hardening effect: stiffness of the silicone rubber increased with the excitation magnitude and the frequency. It seemed to suggest that thixotropic or memory dependent observation in WBV is missing in the silicone rubber with cyclic compression test.

A dominant single resonance was observed for both base-excited and impact hammer rigid mass-silicone rubber system. Although the frequency range at which resonance occurred were different in the current study and the WBV studies, the mode of sprung mass might be similar. The bimodal resonance frequencies at frequency range higher than the WBV were observed with rigid mass-porcine muscle system. Although the porcine muscle was considered a closer representation of human soft tissue than silicone rubber, the bimodal response in porcine muscle seemed to disagree. Effects of thickness and diameter of the specimen, sprung mass and sprung mass contour on resonance frequency, dynamic stiffness and damping were found to be significant. Effect of excitation magnitude on dynamic stiffness and damping was also found to be significant. However the effect of excitation magnitude on resonance frequency was not significant. This seems to suggest that magnitude
dependent resonance frequency observation in WBV is absent in the scaled rigid mass-soft tissue system.

The sprung mass contact contour partly alters the behaviour of the soft tissue in both base excited shaker test and the impact hammer test. Along with the micro structure of buttocks, contact contour would primarily contribute to the difference that is observed in this study in comparison to WBV studies.

Concluding, based on absence of expected magnitude dependency in resonance frequency, neither within base-excited shaker test nor within impact hammer, present study suggests that the thixotropic or memory effect of the human body by Huang and Griffin (2008a, b) is missing in the SDOF rigid mass-soft tissue system.

9.2 Limitations

The silicone rubber and the porcine muscle were considered to be most close representation to the human soft tissue. However, the experimental tests on human soft tissue may lead to a different outcome. The fundamental difference between the human buttocks and the silicone rubber was the amount of the fluid content and the packaging of fluid in the tissue. This difference should be acknowledged when examining the mechanical behaviour of silicone rubber to WBV.

To mimic the bony structure of the pelvis, four hip-borne like protruded surface indenter knobs were designed and used. Four knobs were used mainly to maintain the stability of the sprung mass during the motion. The results may be altered when accurate replicate of hip-borne geometrics are in use.

Finally, the biomechanical properties (i.e. stiffness and damping) of the silicone rubber and the porcine muscle were extracted using linear viscoelastic model. The result may vary when the nonlinear models are in place.

9.3 Recommendations

It was suspected that the absence of magnitude dependency in rigid mass-soft tissue system may be partly due to the fact that excitation levels used in this study were not distinctive enough to produce adequate relative deformation in the soft specimens. Further investigation is needed to verify this claim using base-excited shaker test with substantially higher magnitudes of excitation. The current experimental rig would not be capable of testing the rigid mass-soft tissue system
using substantially higher base excitation levels. This is partly because of the unsupported sprung mass in the present study. By analysing all the drawbacks and the limitations in the current experimental rig, the new experimental rig has been proposed and designed for further studies (see Figure 9.1). Biological fluid chamber could also be used with the rigid mass-soft tissue system to mimic the fluid packing in the tissue at buttocks. This will help to examine the effect of fluid packing in the tissue.

![Proposed experimental rig design: a - end stopper, b - weight sliding bar, c - sprung mass, d - specimen, e - bottom end stopper, f - force plate and g - shaker table (see Figure I1; Appendix I for detailed engineering drawings of the design).](image)

The contact contour of the hip-borne sprung mass was found to alter the biomechanical responses of the rigid mass-soft tissue system. However the current design of the hip-borne sprung mass did not accurately represent all the geometric features of the hip. The new design with the accurate features of the hip would be of interest in future studies.
It was suspect that the bimodal observation in the rigid mass-porcine system may be caused by the rotational movement of the sprung mass. However it was difficult to measure such movement using current accelerometer arrangement. Two spaced vertical accelerometers should be in place in future studies to encounter such movement.

The shear moduli of the silicone rubber and the porcine muscle have not been examined in the current study. There is lack of literature on how shear moduli of both biological and artificial soft tissue vary with different magnitude and waveform of excitation. It is unclear so far how shear moduli affect the macroscopic dynamic behaviour of a mass-soft tissue system. It is also unclear how shear moduli change with different magnitude and waveform of excitation. The shear response of rigid mass-soft tissue system has to be investigated.

The experimental studies in this research used linear viscoelastic formulation 'omega arithmetic' to quantify the biomechanical properties of silicone rubber and the porcine muscle tissue. As the highly nonlinear nature of the materials used in this study, the nonlinear hyper-elastic models (e.g., Mooney-Rivlin and Fung's anisotropic soft tissue models) would be realistic to represent the biomechanical parameters. This has to be done to examine the deviation between the parameters obtained from the linear and the nonlinear models.

Time domain mathematical formulations of the restoring forces at the excitation-subject interface suffer from the integration error during the optimisation process. This is partly because of the Runge-Kutta method employed for the integration. It would be plausible to investigate other integration techniques (i.e. a ramp invariant digital recursive filtering relationship) to compare and quantify this integration error. This will help to develop new time domain mathematical formulation for WBV.
Appendix A

Engineering drawings of the experimental rig and other equipment used in this study

(Chapter 3)

Figure A1  Sprung mass assembly design (materials used: Aluminium and Lead)
**Figure A2**  Shaker table design (material used: Aluminium)
Figure A3  Specimen’s base platen design (material used: Aluminium)
Figure A4  Flat indenter plate design (material used: Plexiglas)
Figure A5  Hip-borne indenter plate design - isometric 3D-view

Figure A6  Hip-borne indenter plate design (material used: Aluminium)
Figure A7  Specimen mold design (for 50mm diameter specimen; material used: Plexiglas)
Figure A8  Specimen mold design (for 75mm diameter specimen; material used: Plexiglas)
Figure A9  Specimen mold design (for 100mm diameter specimen; material used: Plexiglas)
Appendix B

Silicone rubber material specification

(Chapter 3)

The silicone rubber material used in this study is Ecoflex® 00-10. It is a platinum-catalysed silicone containing a two parts mixture. To prepare the silicone ready for testing a ratio of part A and B needs to be decided. For harder and less damped specimens the mix will need to contain more of ‘Part A’, for a softer and more damped specimen the mix will need to contain more from ‘Part B’. The data sheet which is supplied by the manufacturer only contains the information for the mixing ratio of 1:1. The user required more trials and tests to validate the mechanical properties of the specimens which are prepared using the different mixing ratio.

Figure B1  Product technical data sheet of the silicone rubber material (Smooth-On Inc., 2014).
Appendix C:

Linear and nonlinear viscoelastic models in Simulink, including equations of motion.

(Chapter 4)

The SDOF linear system in time domain

The system shown in Figure 3.21 (Chapter 3) consists of masses, m₀ and m₁, supported by a parallel combination of a spring, kₐ, and an elastically attached viscous damper, cₓ. This system is commonly referred to as the linear SDOF model, and its equations of motion are given by:

\[ m₁ \ddot{z}_1 + cₓ \dot{z}_0₁ + kₐ z₀₁ = 0 \]  
\[ m₀ \ddot{z}_0 - cₓ \dot{z}_0₁ - kₐ z₀₁ = F₁ \]  

\( z₀₁ \) and \( \dot{z}_0₁ \) are the relative displacement and relative velocity between the sprung mass \( m \) and the base (un-sprung mass), \( z₀₁ = z₁ - z₀ \) and \( \dot{z}_0₁ = \dot{z}_1 - \dot{z}_0 \).

Above described SDOF linear model was implemented in Simulink (see Figure C1).

![Figure C1](image_url)  
Schematic diagram of SDOF linear model in simulink.
As the inputs for the Simulink model, $\dot{z}_0$ and $z_0$ were obtained by integrating the $\ddot{z}_0$ twice in time as described in Appendix E. All the variables and the parameters in the Simulink model have the same meaning as described in the SDOF linear model. The Equation C1 was integrated twice forward in time using the Matlab function ode4 (4\textsuperscript{th} order Runge-kutta method) for initial value problems with fixed step. The four model parameters ($m_0, m_1, k_L$ and $c_L$) were unknown and determined using the optimization procedure described in the Appendix D.

**The SDOF system with cubic stiffness in time domain**

The system shown in Figure 3.22(a) (Chapter 3) consists of masses, $m_0$ and $m_1$, supported by a parallel combination of two springs, $k_L$ and $k_{NL}$, and an elastically attached viscous damper, $c_L$. This system is commonly referred to as the SDOF cubic stiffness model, and its equations of motion are given by:

$$m_1 \ddot{z}_1 + c_L \dot{z}_0 + k_L z_0 + k_{NL} z_0^3 = 0$$

$$m_0 \ddot{z}_0 - c_L \dot{z}_0 - k_L z_0 - k_{NL} z_0^3 = F_{TF}$$

These two equations were modelled using Simulink (see Figure C2).

![Figure C2](image)

Figure C2  Schematic diagram of SDOF system with cubic stiffness in simulink.

As the inputs for the Simulink model, $\dot{z}_0$ and $z_0$ were obtained by integrating the $\ddot{z}_0$ twice in time as described in Appendix E. All the variables and the parameters in the
Simulink model have the same meaning as described in the SDOF linear model. The Equation C3 was integrated twice forward in time using the Matlab function ode4 (4th order Runge-kutta method) for initial value problems with fixed step. The five model parameters \((m_0, m_1, k_L, k_{NL} \text{and } c_L)\) were unknown and determined using the optimization procedure described in Appendix D.

**The SDOF Zener model in time domain**

The SDOF system shown in Figure 3.22(b) (Chapter 3) consists of masses, \(m_0\) and \(m_1\), supported by a parallel combination of a spring, \(k_{L1}\), and an elastically attached viscous damper, \(c_L\), where the connecting spring has stiffness \(k_{L2}\). The spring in series with the dashpot is referred to as the secondary spring as opposed to the primary spring which is the one in parallel with the damper. An intermediate point ‘0’ was chosen in-between the viscous damper and the secondary spring to formulate the equation of motion. This system is commonly referred to as the ‘Zener’ model, and its equations of motion are given by:

\[
\begin{align*}
    m_1 \ddot{z}_1 + c_L \dot{z}_1 + k_{L1} z_{01} &= 0 \quad \text{(C5)} \\
    c_L \dot{z}_1 + k_{L2} z_{10} &= 0 \quad \text{(C6)}
\end{align*}
\]

\(z_{10}\) is the relative displacement between the base \(m_0\) and the intermediate point I, \(z_{01}\) is the relative displacement between the sprung mass \(m_1\) and the base and and \(\dot{z}_1\) is the relative velocity between the sprung mass \(m_1\) and the base (un-sprung mass \(m_0\)), \(z_{10} = z_0 - z_1\), \(z_{01} = z_1 - z_0\) and \(\dot{z}_1 = \dot{z}_1 - \dot{z}_1\).

Using simple arithmetic manipulation, Equation C5 and Equation C6 are combined as one equation as follows:

\[
\begin{align*}
    m_1 \ddot{z}_1 + (m_1 k_{L1}/c_L) \dot{z}_1 + (k_{L1} k_{L2} c_L) z_{01} + (k_{L1} k_{L2} c_L) z_{01} &= 0 \quad \text{(C7)}
\end{align*}
\]

And the total force measured at the base is given by:

\[
\begin{align*}
    m_0 \ddot{z}_0 + m_1 \ddot{z}_1 &= F_{TF} \quad \text{(C8)}
\end{align*}
\]

These equations of motion were modelled using Simulink (see Figure C3).
Figure C3  Schematic diagram of SDOF Zener model in simulink.

As the inputs for the Simulink model, \( z_0 \) and \( z_0 \) were obtained by integrating the \( z_0 \) twice in time as described in Appendix D. All the variables and the parameters in the Simulink model have the same meaning as described in the SDOF linear model. The Equation C7 was integrated twice forward in time using the Matlab function ode4 (4\(^{th}\) order Runge-kutta method) for initial value problems with fixed step. The five model parameters \( (m_0, m_1, k_{L1}, k_{L2} \) and \( c_L \)) were unknown and determined using the optimization procedure described in Appendix E.

**The SDOF Bouc-Wen system in time domain**

A nonlinear Bouc-Wen formulation assuming the exponential term ‘n’ to be 1 was used in this time domain modelling. The Bouc-Wen formulation describes the hysteresis relationship between relative displacement \( (z_{01}(t)) \) and the restoring force in addition to the linear (constant) components of stiffness and damping (Figure 22(c)) (Chapter 3). It is usually defined in the form of its first time derivative:

\[
\dot{F}_{BW} = (k-k_L)z_{01} - \alpha|z_{01}|F_{BW}^n - \beta|z_{01}|F_{BW}^n
\]

where \( n = 1 \); \( k \) and \( k_L \) are stiffness constants – the difference of these two stiffness constants is usually represented by a single parameter ‘A’ in the literature; \( \alpha \) and \( \beta \) are coefficients related to the shape of the hysteresis loop; \( z_{01} \) and \( \dot{z}_{01} \) are the relative...
displacement and relative velocity between the sprung mass $m_1$ and the base $m_0$, $z_{01} = z_1 - z_0$ and $\dot{z}_{01} = \dot{z}_1 - \dot{z}_0$.

The equations of motion for the SDOF system (with Bouc-Wen system) in Figure 3.24(c) (Chapter 3) become:

$$m_1 \ddot{z}_1 + c_L \dot{z}_{01} + k_L z_{01} + F_{BW} = 0 \quad (C10)$$

$$m_0 \ddot{z}_0 - c_L \dot{z}_{01} - k_L z_{01} - F_{BW} = 0 \quad (C11)$$

Figure C4  Schematic diagram of SDOF Bouc-Wen model in simulink.

As the inputs for the Simulink model, $\dot{z}_0$ and $z_0$ were obtained by integrating the $\ddot{z}_0$ twice in time as described in Appendix D. All the variables and the parameters in the Simulink model have the same meaning as described in the SDOF linear model. The Equation C10 was integrated twice forward in time and Equation C9 was integrated once using the Matlab function ode4 ($4^{th}$ order Runge-kutta method) for initial value problems with fixed step. The seven model parameters ($m_0, m_1, k, k_L, c_L, \alpha$ and $\beta$) were unknown and were determined using the optimization procedure described in Appendix E.
Appendix D

JADE - Adaptive Differential Evolutionary algorithm

(Chapter 4)

The optimisation procedure used to calculate the model parameters in Chapter 4 was based on an evolutionary scheme, namely Differential Evolution (DE). As in all evolutionary optimisation procedures, a population of possible solutions (here, the vector of parameter estimates) is iterated in such a way that succeeding generations of the population contain better solutions to the problem in accordance with the Darwinian principle of ‘survival of the fittest’. The problem was framed here as a minimisation problem with the cost function defined as a normalised mean-square error between the ‘measured’ data and that predicted using a given parameter estimate, i.e.,

\[ \text{MSE} = \frac{100}{N \sigma^2} \sum \left[ F_{RF} - \hat{F}_{RF}(a) \right]^2 \]  

where, \( a \) is model parameters, \( F_{RF} \) is the measured restoring force and \( \hat{F}_{RF}(a) \) is the calculated restoring force and \( N \) is the number of samples and \( \sigma^2 \) the variance (or standard deviation) of the measured restoring forces. Note that this definition of cost function could quite easily be used with velocity or acceleration data; this means that whatever data is sampled, there will be no need to apply numerical differentiation or integration procedures.

The standard DE algorithm attempts to transform a randomly generated initial population of parameter vectors into an optimal solution through repeated cycles of evolutionary operations, in this case: mutation, crossover and selection. In order to assess the suitability of a certain solution, an objective or cost function relating to the value of this solution must be constructed. The cost function appropriate to the identification problem here is the one given in the Equation D1.
Figure D1 shows the schematic for the procedure for evolving between populations. The following process is repeated with each vector within the current population being taken as a target vector. Each of these vectors has an associated cost taken from Equation D1. Each target vector is pitted against a trial vector in a selection process which results in the vector with lowest cost advancing to the next generation. The process for constructing the trial vector involves variants of the standard evolutionary operators: mutation and crossover.

Figure D1  Schematic for the standard differential evolution algorithm (A and B are two randomly selected vectors combined to form a scaled difference vector and C is third randomly selected vector added to scaled difference vector; adapted from Worden and Manson, 2012).
JADE (Adaptive Differential Evolution with optional external archive) is recently proposed by Zhang and Sanderson, 2009 to improve convergence performance, by implementing a new mutation strategy ‘DE/current-to-p-best with and without external archive’ and adaptively controlling the parameters. JADE adopt the same ‘binary’ crossover and one-to-one selection scheme as many other classic DE variants do as described in Figure 1.1. Main contributions of JADE are summarized as follows:

- ‘DE/current-to-p-best with or without external archive’:
  ‘DE/current-to-p-best’ is a generalization of the classic ‘DE/current-to-best’ mutation strategy, in which not only the best-solution information is utilised, but also the information of the other good solutions is utilised.

In fact any of the top $100p\%$, p $(0,1]$, solution can be randomly chosen in ‘DE/current-to-p-best’ to play the role that the single best solution plays in ‘DE/current-to-best’.

In order to diversify the population and avoid the premature conversion, the difference between the archive inferior solutions and the current population can be incorporated into the mutation operation (Zhang and Sanderson, 2009). This is an optional choice for the JADE. Hence the mutation utilised in JADE is called ‘DE/current-to-p-best with or without external archive’. In this study we only consider the variant of JADE with external archive and still denote it as JADE.
Table 1: Pseudo code of JADE with Archive (adapted from Zhang & Sanderson, 2009)

<table>
<thead>
<tr>
<th>Line</th>
<th>Procedure of Jade with Archive</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td><strong>Start</strong></td>
</tr>
<tr>
<td>2.</td>
<td>Set $\mu_{CR} = 0.5; \mu_{F} = 0.5; A = \phi$</td>
</tr>
<tr>
<td>3.</td>
<td>Create a random initial population ${x_{i}} l = 1, 2, \ldots, NP$</td>
</tr>
<tr>
<td>4.</td>
<td><strong>For</strong> $g = 1$ to $G$</td>
</tr>
<tr>
<td>5.</td>
<td>$S_{F} = \phi; S_{CR} = \phi$;</td>
</tr>
<tr>
<td>6.</td>
<td><strong>For</strong> $i = 1$ to $NP$</td>
</tr>
<tr>
<td>7.</td>
<td>Generate $CR_{i} = \text{rand}<em>{i}(\mu</em>{CR}, 0.1), \text{Fi} = \text{randc}<em>{i}(\mu</em>{F}, 0.1)$</td>
</tr>
<tr>
<td>8.</td>
<td>Randomly selected $x_{p}$ as one of the 100p% best vectors</td>
</tr>
<tr>
<td>9.</td>
<td>Randomly selected $x_{r1} \neq x_{i}$ from current population $P$</td>
</tr>
<tr>
<td>10.</td>
<td>Randomly selected $x_{r2} \neq x_{r1} \neq x_{i}$ from PUA</td>
</tr>
<tr>
<td>11.</td>
<td>$V_{ig} = x_{ig} + f_{i}(x_{best} - x_{ig}) + f_{i}(x_{r1} - x_{r2})$</td>
</tr>
<tr>
<td>12.</td>
<td>Generate $j_{rand} = \text{randint}(1,D)$</td>
</tr>
<tr>
<td>13.</td>
<td><strong>For</strong> $j = 1$ to $D$</td>
</tr>
<tr>
<td>14.</td>
<td>If $j = j_{rand}$ or $\text{rand}(0,1) &lt; C_{ri}$</td>
</tr>
<tr>
<td>15.</td>
<td>$U_{jig} = v_{jig}$</td>
</tr>
<tr>
<td>16.</td>
<td>Else</td>
</tr>
<tr>
<td>17.</td>
<td>$U_{jig} = x_{jig}$</td>
</tr>
<tr>
<td>18.</td>
<td><strong>End if</strong></td>
</tr>
<tr>
<td>19.</td>
<td><strong>End for</strong></td>
</tr>
<tr>
<td>20.</td>
<td>If $f(x_{ig}) \leq f(u_{ig})$</td>
</tr>
<tr>
<td>21.</td>
<td>$X_{ig+1} = x_{ig}$</td>
</tr>
<tr>
<td>22.</td>
<td>Else</td>
</tr>
<tr>
<td>23.</td>
<td>$X_{ig+1} = u_{ig}; x_{ig} \rightarrow A; CR_{i} \rightarrow SCR, F_{i} \rightarrow SF$</td>
</tr>
<tr>
<td>24.</td>
<td><strong>End if</strong></td>
</tr>
<tr>
<td>25.</td>
<td><strong>End for</strong></td>
</tr>
<tr>
<td>26.</td>
<td>Randomly remove solutions from $A$ so that $</td>
</tr>
<tr>
<td>27.</td>
<td>$\mu_{CR} = (1-c). \mu_{CR} + c. \text{mean} A \ (SCR)$</td>
</tr>
<tr>
<td>28.</td>
<td>$\mu_{F} = (1-c). \mu_{F} + c. \text{mean} L \ (SF)$</td>
</tr>
<tr>
<td>29.</td>
<td><strong>End for</strong></td>
</tr>
<tr>
<td>30.</td>
<td><strong>End</strong></td>
</tr>
</tbody>
</table>
Appendix E

Integration of time series data – time domain method

(Chapter 4)

The numerical integration procedures can be used to produce estimated velocity and displacement data from acceleration measurements by a variety of methods. Time domain trapezium integration method was used in the present study. Due to accumulated error at low frequencies, the method is prone to a linear ‘drift’ in the mean level of the integrated signal as the time elapses (Worden and Tomlinson, 2001). Nevertheless, such drawback is resolved by introducing a high-pass filter (with zero phase shift) before and after an integration, to remove the low frequency error. Careful selection of the high-pass cut-off frequency is needed. If the frequency is too low the data remains ‘drifted’ and if the cut-off frequency is too high, the filter removes useful data which should remain in the signal. In the present study, all signals were high-pass filtered at 1.5 Hz using 4-pole Butterworth zero-phase filter before and after the first integration, and low-pass filtered at 30 Hz using 4-pole Butterworth zero-phase filter once before the first integration (Chapter 4). The signals were normalised before and after each integration to remove their means. The velocity and displacement time histories obtained by integrating the measured accelerations for the different magnitudes of excitation were shown in individual chapter (Chapter 4). The power spectral density for the measured accelerations, calculated velocities and displacements for the different magnitudes of excitation were also shown.
Figure E1  Computational procedures for the time domain integration of measured accelerations.
Appendix F

MATLAB script for the JADE algorithm

(Chapter 4)

The optimisation procedure ‘JADE’ described in Appendix D was coded in MATLAB and the script is given below.


The m-file with the name ‘jade_sdoф_linear_general_wrapper.m’ is the main file which calls others during the calculation process.

jade_sdoф_bw_general_wrapper.m

Function [final_min_generation_cost, m0_diff, m1_diff, A_diff, c_diff, alpha_diff, beta_diff, n_diff, best_generation_cost, mean_generation_cost, median_generation_cost, crossover_mean_vector, scaling_location_vector, m0, m1, A, c, alpha, beta, n, time] = jade_sdoф_bw_general_wrapper(true_m0, true_m1, true_A, true_c, true_alpha, true_beta, true_n, rms, flo, fhi, N, population_size, dimension, constraint_matrix, generation_number, pbest_ratio, parameter_memory_factor, archive_switch)

% EMPLOYS JADE ADAPTIVE DIFFERENTIAL EVOLUTION TO CALCULATE THE PARAMETERS OF THE SDOF Bouc-Wen MODEL WITH BASE EXCITATION.
% modified 2015-05-06 TT

[acc_in, vel_in, disp_in, acc_out, vel_out, true_disp_out, restoring_force, total_force, time] = sdoф_bw_data_generate(true_m0, true_m1, true_A, true_c, true_alpha, true_beta, true_n, rms, flo, fhi, N);
% Attempts to find minimum of a function within a specified range by using the JADE adaptive differential evolution algorithm with optional external archive.

% Set initial crossover and scaling factor mean values and calculates the number of pbest vectors to be considered in the mutation strategy.

crossover_mean = 0.5;
crossover_sd = 0.1;
scaling_location_parameter = 0.5;
scaling_scale_parameter = 0.1;
pbest_number = round(pbest_ratio*population_size);

% Initialises vector of mean and best function value for each generation.

best_generation_cost = zeros(1,generation_number);
mean_generation_cost = zeros(1,generation_number);
median_generation_cost = zeros(1,generation_number);
crossover_mean_vector = zeros(1,generation_number);
scaling_location_vector = zeros(1,generation_number);

% Initialise population and sets the archived_pool to be an empty matrix.

gene_pool =
init_sdof_bw(population_size,dimension,constraint_matrix);
archived_pool = [];

% FUNCTION EVALUATION FOR INITIAL GENE POOL
%gene_cost_vector =
cost_bw_general(gene_pool,population_size,disp_true,input_var);
%gene_cost_vector =
cost_sdof_bw_base_general(gene_pool,population_size,true_disp_out,rms,flo,fhi,N);

% Stores mean and best function value for initial generation.

best_generation_cost(1,1) = min(gene_cost_vector);
mean_generation_cost(1,1) = mean(gene_cost_vector);
median_generation_cost(1,1) = median(gene_cost_vector);
crossover_mean_vector(1,1) = crossover_mean;
scaling_location_vector(1,1) = scaling_location_parameter;

for generation = 2 : generation_number
    generation
    % MUTATION
[mutation_pool, scaling_factor_vector] = mutate_jade(gene_pool, gene_cost_vector, archived_pool, population_size, dimension, constraint_matrix, scaling_scale_parameter, scaling_location_parameter, pbest_number);

% CROSSOVER

[child_pool, crossover_ratio_vector] = crossover_jade(gene_pool, mutation_pool, population_size, dimension, crossover_mean, crossover_sd);

% FUNCTION EVALUATION FOR CHILD POOL

child_cost_vector = cost_sdof_bw_base_general(child_pool, population_size, true_disp_out, rms, flo, fhi, N);

% SELECTION

gene_pool, gene_cost_vector, archived_pool, population_size, dimension, crossover_mean, selection_jade(gene_pool, gene_cost_vector, child_pool, child_cost_vector, archived_pool, population_size, dimension, crossover_mean, scaling_location_parameter, crossover_ratio_vector, scaling_factor_vector, parameter_memory_factor, archive_switch);

% Stores mean and best function value for current generation.

best_generation_cost(1, generation) = min(gene_cost_vector);
mean_generation_cost(1, generation) = mean(gene_cost_vector);
median_generation_cost(1, generation) = median(gene_cost_vector);
crossover_mean_vector(1, generation) = crossover_mean;
scaling_location_vector(1, generation) = scaling_location_parameter;
end

figure;
plot(best_generation_cost, 'r'); hold on;
plot(mean_generation_cost, 'b'); hold on;
plot(median_generation_cost, 'g'); hold on;

[final_min_generation_cost, index] = min(gene_cost_vector);
sprintf('The point %s has the lowest cost of %d', num2str(gene_pool(index,:), fliplr), final_min_generation_cost)
m0 = gene_pool(index, 1);
m1 = gene_pool(index, 2);
A = gene_pool(index, 3);
c = gene_pool(index,4);
alpha = gene_pool(index,5);
beta = gene_pool(index,6);
n = gene_pool(index,7);

[acc_in,vel_in,disp_in,acc_out,vel_out,pred_disp_out,restoring_force,total_force,time] = sdof_bw_data_generate(m0,m1,A,c,alpha,beta,n,rms,flo,fhi,N);
figure;
plot(time,true_disp_out,'b');hold on;
plot(time,pred_disp_out,'r');

m0_diff = 100*(m0 - true_m0)/true_m0;
m1_diff = 100*(m1 - true_m1)/true_m1;
A_diff = 100*(A-true_A)/true_A;
c_diff = 100*(c-true_c)/true_c;
alpha_diff = 100*(alpha-true_alpha)/true_alpha;
beta_diff = 100*(beta-true_beta)/true_beta;
n_diff = 100*(n-true_n)/true_n;

sdof_bw_data_generate.m

function[acc_in,vel_in,disp_in,acc_out,vel_out,disp_out,restoring_force,total_force,time] = sdof_bw_data_generate(m0,m1,A,c,alpha,beta,n,rms,flo,fhi,N)

% Generates the SDOF BW model data for given set of parameters (un-sprung mass 'm0', sprung mass 'm1', stiffness parameter 'A', damping 'c', BW parameters 'alpha','beta', exponential coefficient 'n', magnitude of given input base excitation acceleration in root mean square 'rms', highpass cut-off frequency 'flo', lowpass cut-off frequency 'fhi', filter order 'N')

% modified 2015-05-06 TT

fixed_step = 0.005;
time=90;
input_mean = 0;
input_seed = 0;
input_var = rms^2; % m/s^2 input base excitation acceleration variance

Wlo = 2*pi*flo; % rad/s, highpass filter
Whi = 2*pi*fhi; % rad/s, lowpass filter
open_system('sdof_bw_bm_general');
options=simset('Solver','ode4','FixedStep',fixed_step);
set_param('sdof_bw_bm_general/m0','Gain',num2str(m0));
set_param('sdof_bw_bm_general/mlRecip','Gain',num2str(1/m1));
set_param('sdof_bw_bm_general/A','Gain',num2str(A));
set_param('sdof_bw_bm_general/c','Gain',num2str(c));
set_param('sdof_bw_bm_general,alpha','Gain',num2str(alpha));
set_param('sdof_bw_bm_general/beta','Gain',num2str(beta));
set_param('sdof_bw_bm_general/Constant','Value',num2str(n));
set_param('sdof_bw_bm_general/Constant1','Value',num2str(n-1));
set_param('sdof_bw_bm_general/acc_in','Mean',num2str(input_mean));
set_param('sdof_bw_bm_general/acc_in','Variance',num2str(input_var));
set_param('sdof_bw_bm_general/acc_in','Seed',num2str(input_seed));
set_param('sdof_bw_bm_general/Integrator','InitialCondition','0');
set_param('sdof_bw_bm_general/Integrator1','InitialCondition','0');
set_param('sdof_bw_bm_general/Integrator2','InitialCondition','0');
set_param('sdof_bw_bm_general/Integrator3','InitialCondition','0');
set_param('sdof_bw_bm_general/Integrator4','InitialCondition','0');
set_param('sdof_bw_bm_general/afil','Wlo',num2str(Wlo));
set_param('sdof_bw_bm_general/afil','Whi',num2str(Whi));
set_param('sdof_bw_bm_general/afil','N',num2str(N));
[t,x,y] = sim('sdof_bw_bm_general', time-fixed_step,options);

% figure;
% subplot(2,2,1);
% plot(time,acc_in,'r-',time,acc_out,'b--'); ylim([-20 20]);
% xlabel('Time ( s )'); ylabel('Acceleration ( m / s^2 )');
legend('acc\_in','acc\_out',1);
% title([ 'fn = ' num2str(fn) ' Hz , fr = ' num2str(fr) ' Hz ']);
% subplot(2,2,3);
% plot(time,vel_in,'r-',time,vel_out,'b--');
% xlabel('Time ( s )'); ylabel('Velocity ( m / s )');
legend('vel\_in','vel\_out',1);
% subplot(2,2,4);
% plot(time,disp_in,'r-',time,disp_out,'b--');
% xlabel('Time ( s )'); ylabel('Displacement ( m )');
legend('disp\_in','disp\_out',1);
% subplot(2,2,2);
% plot(time,restoring_force,'r-',time,total_force,'b--');
% xlabel('Time ( s )'); ylabel('Force ( N )');
legend('restoring\_force','total\_force',1);

init_sdof_bw.m

function [gene_pool] =
init_sdof_bw(population_size,dimension,constraint_matrix)

% Initialises the gene_pool so that all parameters are drawn randomly from between their permissible lower and upper bounds, as specified by the constraint_matrix.

% modified 2015-05-06 TT

lower_matrix =
repmat(constraint_matrix(1,:),population_size,1);
range_matrix =
repmat((constraint_matrix(2,:)-constraint_matrix(1,:)),population_size,1);
gene_pool =
lower_matrix +
range_matrix.*rand(population_size,dimension);
function cost_vector =
cost_sdof_bw_base_general(gene_pool,population_size,true_disp_out,rms,flo,fhi,N)

  % Calculates the cost for each of the potential solutions within gene_pool the cost value is simply the mean square error between the predicted output displacement 'pred_disp_out' and the true output displacement 'true_disp_out'.
  % modified 2015-05-06 TT

  cost_vector = zeros(population_size,1);
  for i = 1 : population_size
    m0 = gene_pool(i,1) ;
    m1 = gene_pool(i,2) ;
    A = gene_pool(i,3) ;
    c = gene_pool(i,4) ;
    alpha = gene_pool(i,5) ;
    beta = gene_pool(i,6) ;
    n = gene_pool(i,7) ;

    % If a particular choice of parameters causes an error in Simulink,
    % this returns a cost of 200percent rather than crashing.
    try
      [acc_in,vel_in,disp_in,acc_out,vel_out,pred_disp_out,restoring_force,total_force,time] = sdof_bw_data_generate(m0,
                                                                                                    m1,A,c,alpha,beta,n,rms,flo,fhi,N);

      cost_vector(i,1) = (100/(length(true_disp_out)*std(true_disp_out)^2)) * sum((true_disp_out-pred Disp_out).^2) ;
    catch me
      cost_vector(i,1) = 200 ;
    end
  end
end
mutate_jade.m

function [mutation_pool,scaling_factor_vector] =
mutate_jade(gene_pool,gene_function_value_vector,archived_pool
,population_size,dimension,bounds_matrix,scaling_scale_parameter,scaling_location_parameter,pbest_number,min_max_integer)

% Generates a mutation_pool using the single strategy of JADE, namely the
dE/current-to-pbest/1 strategy with optional archive. This consists of mutating each vector in the
gene_pool with two scaled vector differentials. The first vector differential is from the current vector to one of the
best vectors in the gene_pool and the second differential is from one random vector in the gene_pool to another. If the
archive option is selected, then the second random vector in the second differential is extracted from the union of the
gene_pool and a set of archived inferior solutions. Both of the differentials have the same scaling factor but each member
of the population will have a different scaling factor which is calculated using a truncated (between 0 and 1) Cauchy
distribution with the current location parameter (mode & median) and pre-determined scale parameter (0.1 is suggested).
Calculates a scaling factor vector which contains a scaling factor for each member of the population drawn from a
truncated Cauchy distribution - a vector of length equal to twice the population_size is calculated as negative scaling
factors are discarded. Scaling factors of greater than 1 are fixed to 1.

% modified 2015-05-06 TT

scaling_factor_vector = [] ;
multiplier = 1 ;
while (length(scaling_factor_vector) < population_size)
    multiplier = multiplier*2 ;
    scaling_factor_vector =
    scaling_scale_parameter*tan(pi*(rand(multiplier*population_size,1)-0.5))+scaling_location_parameter ;
    scaling_factor_vector =
    scaling_factor_vector(scaling_factor_vector > 0) ;
    scaling_factor_vector(scaling_factor_vector > 1) = 1 ;
end
scaling_factor_vector =
scaling_factor_vector(1:population_size,1) ;
% Randomly generates a vector of pbest vectors to be used in
the mutation strategy.

[dummy,index] = sort(gene_function_value_vector,1,'ascend')

pbest_vector =
index(ceil(pbest_number*rand(population_size,1)))

% Generates a vector of random integers between 1 and
population_size to act as r1 in mutation strategy. Checks to
see that r1 is distinct from the current vector and
recalculates if not.

r1_vector = ceil(population_size*rand(population_size,1))
index = find((r1_vector - (1:population_size)'==0)

while index
    r1_vector(index) =
    ceil(population_size*rand(length(index),1))
    index = find((r1_vector - (1:population_size)'==0)
end

% Generates a union_pool which contains the current gene_pool
and the archived pool of inferior solutions (this can be empty
in the case of the optional archive not being used). A vector
of random integers between 1 and union_length is generated to
act as r2 in mutation strategy. Checks to see that r2 is
distinct from both the current vector and the r1_vector and
recalculates if not.

union_pool = [gene_pool ; archived_pool]
[union_length,dummy] = size(union_pool)

r2_vector = ceil(union_length*rand(population_size,1))
index1 = find((r2_vector - (1:population_size)'==0)
index2 = find((r2_vector - r1_vector)==0)
index = union(index1,index2)

while index
    r2_vector(index) =
    ceil(population_size*rand(length(index),1))
    index1 = find((r2_vector - (1:population_size)'==0)
    index2 = find((r2_vector - r1_vector)==0)
    index = union(index1,index2)
end

% Calculates the mutation_pool using the single mutation
strategy of DE/current-to-pbest/1. The differential between
the pbest vector and the current vector and the differential
between the r1 vector and r2 vector are calculated, multiplied
by the relevant scaling factor then added to the current
vector.
pbest_matrix = gene_pool(pbest_vector,:) ;
r1_matrix = gene_pool(r1_vector,:) ;
r2_matrix = union_pool(r2_vector,:) ;
pbest_minus_current_matrix = pbest_matrix - gene_pool ;
r1_minus_r2_matrix = r1_matrix - r2_matrix ;
scaling_factor_matrix = repmat(scaling_factor_vector,1,dimension) ;
scaled_pbest_minus_current_matrix = scaling_factor_matrix.*pbest_minus_current_matrix ;
scaled_r1_minus_r2_matrix = scaling_factor_matrix.*r1_minus_r2_matrix ;
mutation_pool = gene_pool + scaled_pbest_minus_current_matrix + scaled_r1_minus_r2_matrix ;

crossover_jade.m

function [child_pool,crossover_ratio_vector] =
crossover_jade(gene_pool,mutation_pool,population_size,dimension,crossover_mean,crossover_sd)

% Combines the gene_pool and mutation_pool using a binomial crossover operation based upon comparing a series of dimension randomly generated numbers with the individual crossover ratio for the vector. Calculates a crossover ratio vector (one for each member of population) from a truncated normal distribution with a specified mean and standard deviation.

% modified 2015-05-06 TT

crossover_ratio_vector =
crossover_sd*randn(population_size,1)+crossover_mean ;
crossover_ratio_vector(crossover_ratio_vector < 0) = 0 ;
crossover_ratio_vector(crossover_ratio_vector > 1) = 1 ;
crossover_ratio_matrix =
repmat(crossover_ratio_vector,1,dimension) ;

% Constructs a matrix of population_size*dimension of uniform random numbers in range (0,1) which are then compared with the crossover ratios. If the value is less than the crossover ratio, the child pool component is equal to the corresponding mutation pool component, else it is made equal to the corresponding gene pool component. In order to ensure that at least one component of each child pool vector is different
from the its corresponding gene pool vector, one randomly chosen component of each vector is made equal to the appropriate mutation pool component.

\[
\text{mutation-choice} = \text{rand(population-size,dimension)} \leq \text{crossover-ratio-matrix} ;
\]

\[
\text{child_pool} = (\text{mutation-choice}.*\text{mutation_pool}) + ((1 - \text{mutation-choice}).*\text{gene_pool}) ;
\]

\[
\text{mutation-switch-index} = (\text{floor(dimension*rand(population-size,1)})*\text{population-size} + (1: \text{population-size})' ;
\]

\[
\text{child_pool(mutation-switch-index)} = \text{mutation_pool(mutation-switch-index)} ;
\]

---

**selection_jade.m**

```matlab
function [gene_pool,gene_function_value_vector,archived_pool,population_size,dimension,crossover_mean,scaling_location_parameter] = selection_jade(gene_pool,gene_function_value_vector,child_pool ,child_function_value_vector,archived_pool,population_size,dimension,crossover_mean,scaling_location_parameter,crossover_ratio_vector,scaling_factor_vector,parameter_memory_factor,archive_switch)

% Updates the gene_pool (and gene_function_value_vector) by selecting the best vector between the gene_pool and child_pool (for all members of the population). The crossover_mean and scaling_location_parameters are then updated to incorporate the successful crossover ratios and scaling factors. If an external archive is being employed, the unsuccessful elements of the gene_pool are added to the current archived_pool – this is then randomly pared down to size so as to avoid bloating. Finds which child vectors are better (lower or higher depending on problem type) than their corresponding parent vectors and updates the gene_pool and gene_function_value_vector.

% modified 2015-05-06 TT

better_vector = child_function_value_vector < gene_function_value_vector ;
better_matrix=repmat(better_vector,1,dimension) ;
```
worse_vector = 1 - better_vector;
worse_matrix = repmat(worse_vector, 1, dimension);
gene_function_value_vector = better_vector.*child_function_value_vector + worse_vector.*gene_function_value_vector;
gene_pool = better_matrix.*child_pool + worse_matrix.*gene_pool;

% Calculates the arithmetic mean of all the successful crossover ratios and the Lehmer mean of all the successful scaling parameters then updates the crossover_mean and scaling_location_parameter taking into account the parameter_memory_factor.

successful_crossover_ratios = better_vector.*crossover_ratio_vector;
successful_crossover_ratios = successful_crossover_ratios(successful_crossover_ratios > 0);
successful_scaling_factors = better_vector.*scaling_factor_vector;
successful_scaling_factors = successful_scaling_factors(successful_scaling_factors > 0);
if isempty(successful_crossover_ratios)
    successful_crossover_mean = crossover_mean;
    successful_scaling_factors_lehmer_mean = scaling_location_parameter;
else
    successful_crossover_mean = mean(successful_crossover_ratios);
    successful_scaling_factors_lehmer_mean = sum(successful_scaling_factors.^2)/sum(successful_scaling_factors);
end
crossover_mean = (1-parameter_memory_factor)*crossover_mean + parameter_memory_factor*successful_crossover_mean;
scaling_location_parameter = (1-parameter_memory_factor)*scaling_location_parameter + parameter_memory_factor*successful_scaling_factors_lehmer_mean;

% If an external archive is being used, adds the unsuccessful gene_pool vectors to the archived_pool. Checks that the length of the new archived_pool to see that it does not exceed the population_size and, if it does, randomly pares the archived_pool.

if archive_switch
    rejected_index = find(worse_vector);
    rejected_gene_pool = worse_matrix.*gene_pool;
    rejected_gene_pool = gene_pool(rejected_index, :);

F-12
archived_pool = [archived_pool ; rejected_gene_pool] ;
[archive_length,dummy] = size(archived_pool) ;
if archive_length > population_size
    random_index = randperm(archive_length);
    random_index = random_index(1:population_size) ;
    archived_pool = archived_pool(random_index,:) ;
end
end
Appendix G

Frequency dependent dynamic stiffness, damping and transmissibility for the SDOF system.

(Chapter 6)

Figure G1  Single degree of freedom (SDOF) mass-spring-damper system with base motion (a) and rigid foundation (b): \( m, k \) and \( c \) are the sprung mass (in kg), dynamic stiffness (in N/m) and damping constant (in Ns/m) respectively; \( z_0(t) \) and \( z(t) \) are the time histories of input base excitation acceleration provided by a shaker and the response acceleration (in m/s\(^2\)) respectively and \( f(t) \) is the time history of the input excitation force (in N) provided by an impact hammer.

Equation of motion (EOM) of the forced vibration of the SDOF (Figure 1a):

\[
 m \ddot{z}(t) + c (\dot{z}(t) - \dot{z}_0(t)) + k (z(t) - z_0(t)) = 0
\]  
(G1)

\( z(t) - z_0(t) \) and \( \dot{z}(t) - \dot{z}_0(t) \) are the relative displacement and relative velocity between the sprung mass \( m \) and the base (un-sprung mass).

Taking the Fourier transform of the above:

\[
 m(-\omega^2)Z(\omega) + k(\omega) Z(\omega) + c(i\omega)Z(\omega) = 0
\]  
(G2)

Complex transmissibility as frequency response function can be written as:

\[
 H_T(\omega) = \frac{Z}{Z_0} = \frac{k + c\omega}{(k - m\omega^2) + i\omega} 
\]  
(G3)

Where \( H_T(\omega) \) is experimentally measured transmissibility (e.g. by shaker test) and \( \omega \) is angular frequency (in rad/s).
The real and the imaginary parts of $H_T(\omega)$ can be used to find $k$ and $c$ from (G3):

$$\text{Re}(H_T(\omega)) = 1 + \frac{\omega^2 (\omega^2 - \omega_0^2)}{(\omega^2 - \omega_0^2)^2 + \frac{c^2 \omega^2}{m^2}}$$  \quad (G4)

$$\text{Im}(H_T(\omega)) = \frac{\frac{c}{m} \omega^3}{(\omega^2 - \omega_0^2)^2 + \frac{c^2 \omega^2}{m^2}}$$  \quad (G5)

Where $\omega_0$ is the natural frequency of the system and can be written as $\omega_0 = \sqrt{\frac{k}{m}}$

Using the mathematical manipulation, stiffness $k$ and the damping $c$ becomes:

$$k(\omega) = m \omega^2 (1 - \frac{(\text{Re}(H_T(\omega)) - 1)}{((\text{Re}(H_T(\omega)) - 1)^2 + \text{Im}(H_T(\omega))^2})$$  \quad (G6)

$$c(\omega) = m \omega \left( \frac{\text{Im}(H_T(\omega))}{((\text{Re}(H_T(\omega)) - 1)^2 + \text{Im}(H_T(\omega))^2)\right}$$  \quad (G7)

The stiffness (in N/m) is a parameter reflecting the elastic properties of the samples and the damping (in Ns/m) is reflecting the internal dissipative forces.
Appendix H

Frequency dependent dynamic stiffness, damping, receptance and accelerance for the SDOF system

(Chapter 7)

Equation of motion (EOM) of the forced vibration of the SDOF (Figure G1(b)):

\[ m \ddot{z} + c \dot{z} + k z = f(t) \]  

(H1)

Rearranged the above using complex dynamic stiffness \( k^* \):

\[ m \ddot{z} + k^* z = f(t) \]  

(H2)

Where \( k^* = k(1 + i \eta) \), and \( \eta \) is the loss factor – the energy dissipation per radian to the peak potential energy in a cycle.

According to similar formulation by Ungar and Kerwin (1962):

\[ c = |k| \frac{\eta}{\omega} \]  

(H3)

where \( \omega \) is the angular frequency in rad/s, \( |k| \) is the modulus of the complex stiffness \( k \) in N/m.

Then,

\[ m \ddot{z}(t) + k(1 + i \eta) z(t) = f(t) \]  

(H4)

Taking the Fourier transform of the above:

\[ m(-\omega^2)Z(\omega) + k(\omega)(1 + i \eta(\omega)) Z(\omega) = F(\omega) \]  

(H5)

Complex receptance as frequency response function can be written as:

\[ H_R(\omega) = \frac{Z(\omega)}{F(\omega)} = \frac{1}{k(\omega) \left[ (1 - r^2) + i \eta(\omega) \right]} \]  

(H6)

Where \( r \) is the frequency ratio, \( r = \frac{\omega}{\omega_n} \) and, \( \omega_n = \sqrt{\frac{k}{m}} \) is the natural frequency in rad/s, \( F(\omega) \) and \( Z(\omega) \) are the frequency domain equivalence of \( f(t) \) and \( z(t) \).

The complex FRF, \( H_R(\omega) \), is calculated from measured accelerance (e.g. by impact hammer) \( H_A(\omega) = \dot{Z}(\omega) / F(\omega) \) using ‘Omega arithmetic’:
\[ H_R(\omega) = \frac{A(\omega)}{-\omega^2} \]  \hspace{1cm} (H7)

The real and the imaginary parts of \( R(\omega) \) can be used to find \( k \) and \( \eta \) from (H6):

\[
\text{Re}(H_R) = \frac{1 - r^2}{k[(1 - r^2)^2 + \eta^2]}  \hspace{1cm} (H8)
\]

\[
\text{Im}(H_R) = \frac{-\eta}{k[(1 - r^2)^2 + \eta^2]}  \hspace{1cm} (H9)
\]

The frequency dependent \( k \) and \( \eta \) can be estimated as:

\[
k(\omega) = \frac{\text{Re}(H_R)}{|H_R|^2 (1 - r^2)} = \frac{\text{Re}(H_R)}{|H_R|^2} + m \omega^2  \hspace{1cm} (H10)
\]

\[
\eta(\omega) = \frac{\text{Im}(R)}{\text{Re}(R)} (r^2 - 1) = \frac{\text{Im}(R)}{\text{Re}(R)} \left( \frac{m \omega^2}{k} - 1 \right)  \hspace{1cm} (H11)
\]

where \(|R|\) is the modulus of the measured complex receptance.

The frequency dependent damping constant \( c(\omega) \) can be then calculated by (H3) above.
Appendix I

Engineering drawings for the proposed experimental rig for further research in this topic

(Chapter 9)

Figure I 1 Proposed experimental rig design for further study (materials recommended: Aluminium and Plexiglas)
References


**Montalvao D, Claudio RALD, Ribeiro AMR and Duarte-Silva J (2013).** Experimental measurement of the complex Young’s modulus on a CFRP laminate considering the constant hysteretic damping model. *Composite Structures*, 97, 91 - 98.


Sharafi B and Blemker SS (2010). A micromechanical model of skeletal muscle to explore the effects of fiber and fascicle geometry. Journal of Biomechanics. 43. 3207 - 3213.


