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A scenario for inflationary magnetogenesis without strong coupling problem

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Abstract. Cosmological magnetic fields pervade the entire universe, from small to large scales. Since they apparently extend into the intergalactic medium, it is tantalizing to believe that they have a primordial origin, possibly being produced during inflation. However, finding consistent scenarios for inflationary magnetogenesis is a challenging theoretical problem. The requirements to avoid an excessive production of electromagnetic energy, and to avoid entering a strong coupling regime characterized by large values for the electromagnetic coupling constant, typically allow one to generate only a tiny amplitude of magnetic field during inflation. We propose a scenario for building gauge-invariant models of inflationary magnetogenesis potentially free from these issues. The idea is to derivatively couple a dynamical scalar, not necessarily the inflaton, to fermionic and electromagnetic fields during the inflationary era. Such couplings give additional freedom to control the time-dependence of the electromagnetic coupling constant during inflation. This fact allows us to find conditions to avoid the strong coupling problems that affect many of the existing models of magnetogenesis. We do not need to rely on a particular inflationary set-up for developing our scenario, that might be applied to different realizations of inflation. On the other hand, specific requirements have to be imposed on the dynamics of the scalar derivatively coupled to fermions and electromagnetism, that we are able to satisfy in an explicit realization of our proposal.

Keywords: primordial magnetic fields, inflation

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1 Introduction

Cosmological magnetic fields seem to pervade the entire universe, from large to small scales. The existence of magnetic fields at intergalactic distances has been inferred by the lack of GeV γ -rays detection from blazars, astrophysical objects that are known to produce photons with energies in the TeV range. Interactions with the intergalactic medium should convert at least part of these high energy TeV γ -rays into lower energy secondary charged particles, which then should decay into GeV photons. The latter are however not detected, and the simplest explanation for this fact is the presence of an intergalactic magnetic field, that deflects the secondary charged particles [1]. The required amplitude for such magnetic field is found in [2] to be at least 10^{-15} Gauss. In addition, magnetic fields are also measured within galaxies. They are thought to be amplified by dynamo effects starting from a seed magnetic field, that should have an amplitude of at least 10^{-20} Gauss to render the dynamo mechanism efficient. See [3] for a recent review.

Generating cosmological magnetic fields of such strengths is a challenging theoretical problem. Since they seem to pervade the intergalactic medium, it is tantalizing to believe that they have a primordial origin, possibly being produced during inflation. On the other hand, Maxwell theory of electromagnetism is conformally invariant. This implies that it does not lead by itself to the production of a coherent background of electromagnetic modes in a Friedmann-Robertson-Walker cosmology, since the latter can be expressed in conformally flat coordinates. To generate a sizeable magnetic field during inflation, the conformal invariance of Maxwell action has to be broken. The first attempts in this direction have been pushed forward by Turner and Widrow in [4], by coupling Maxwell electromagnetism to space-time curvature. However, the most promising models in their set-up are plagued by ghosts [5, 6], and are therefore inconsistent. Another interesting scenario has been proposed by Ratra [7] by kinetically coupling the inflaton scalar field to electromagnetism, in such a way to break conformal invariance yet preserving gauge invariance. A consequence of this approach is that the electromagnetic coupling constant is time-dependent, changing rapidly during inflation. This scenario, although appealing in its simplicity, has nevertheless to face serious theoretical problems. The requirements to avoid an excessive production of electromagnetic energy

(strong backreaction problem) and to avoid very large values for the electromagnetic coupling constant (strong coupling problem) allow one to produce only a tiny amplitude of magnetic field during inflation [8]. In fact, despite the efforts of many different groups to improve on Ratra's scenario, due to these and other problems there are very few models of primordial magnetogenesis that can be regarded as fully convincing from a theoretical perspective. See [3, 9, 10] for reviews that also provide detailed surveys of existing models.

In this work we propose a scenario for building gauge-invariant models of primordial magnetogenesis, potentially free from the aftermentioned backreaction and strong coupling issues. The idea is to *derivatively couple* a dynamical scalar field to fermions and electromagnetism during inflation. Derivative scalar-vector couplings that preserve gauge invariance, although explored in modified gravity scenarios, to the best of our knowledge have not been investigated in the context of primordial magnetogenesis. We show that their inclusion induces a time-dependent sound speed for electromagnetic and fermionic modes. Moreover, and most importantly, derivative couplings give additional freedom to control the time-dependence of the electromagnetic coupling constant during inflation: this quantity now depends also on the *time derivative* of the scalar background profile. This fact allows us to find conditions to avoid the serious strong coupling issue that, as pointed out in [8], affects many of the realizations of Ratra's proposal.

We do not need to rely on explicit inflationary models for developing our ideas, that might be applied to different set-ups. On the other hand, specific conditions have to be imposed on the dynamics of the scalar derivatively coupled to the electromagnetic and fermionic actions. The simplest realization of our mechanism requires a homogenous profile for the scalar field with a large time derivative in the early stages of the inflationary era. We will be able to discuss an explicit example that satisfies our requirements in a consistent way.

We start presenting the action for our system, that includes derivative couplings between a scalar and the electromagnetic and fermion fields. We then explain in general terms how this framework allows us to avoid the strong coupling problem of inflationary magnetogenesis, provided that conditions are imposed on the dynamics of the scalar. After a section of conclusions, various technical appendixes explore novel phenomenological aspects of this scenario, and discuss a concrete realization able to satisfy our requirements.

2 The set-up

We consider a gauge invariant theory for electromagnetism during inflation, that includes derivative couplings between a scalar φ and electromagnetic and fermion fields during inflation. For definiteness, we can think to the fermion field as the electron. The scalar φ has a time-dependent homogeneous profile during inflation, and is not necessarily the inflaton. We will specify the requirements we impose on the dynamics of φ once we develop our scenario.

The gauge-invariant, ghost free action that we examine couples the scalar φ to the electromagnetic and fermion fields as

$$\mathcal{S}_{\text{tot}} = \mathcal{S}_{\text{em}} + \mathcal{S}_{\psi}, \quad (2.1)$$

with

$$\mathcal{S}_{\text{em}} = \int d^4x \sqrt{-g} \left[-\frac{1}{4} \mathcal{B}(\varphi) F_{\mu\nu} F^{\mu\nu} - \mathcal{C}(\varphi) [\partial_\mu \varphi F^\mu_\rho F^{\rho\nu} \partial_\nu \varphi] \right], \quad (2.2)$$

and

$$\mathcal{S}_{\psi} = \frac{1}{2} \int d^4x \sqrt{-g} \{ i \bar{\psi} \gamma^\mu \mathcal{D}_\mu \psi - i k_0 \partial^\mu \varphi \partial_\nu \varphi (\bar{\psi} \gamma^\nu \mathcal{D}_\mu \psi) + h.c. \}. \quad (2.3)$$

The new ingredients that make a difference with respect to standard scenarios as [7] are gauge-invariant *derivative interactions* between the scalar and electromagnetic and fermion fields: these are the dimension-8 operators proportional respectively to $\mathcal{C}(\varphi)$ and k_0 in eqs (2.2) and (2.3). The derivative couplings between scalar and gauge fields, proportional to the function $\mathcal{C}(\varphi)$, have interesting phenomenological consequences that we will explore. But the essential role to build a successful magnetogenesis scenario is played by the derivative operator proportional to k_0 in the fermionic action (2.3).¹ Gauge-invariant derivative couplings with this structure, although occasionally explored in modified gravity scenarios, to the best of our knowledge have not been investigated in the context of primordial magnetogenesis.

We denote with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ the field strength for a gauge field A_μ . Being the electromagnetic action (2.2) written in terms of $F_{\mu\nu}$, it is invariant under the abelian symmetry $A_\mu \rightarrow A_\mu + \partial_\mu \xi$ for arbitrary function ξ . The second operator proportional to $\mathcal{C}(\varphi)$ is of dimension 8, but it is allowed by the symmetries of the system, and as we shall see it can become important if the scalar acquires a non-trivial time-dependent profile.

The fermionic action (2.3) is also gauge invariant. We denote with $\mathcal{D}_\mu = \nabla_\mu - ieA_\mu$ the covariant derivative, with ∇_μ the space-time derivative in curved space. Under a U(1) transformation of the gauge field, $A_\mu \rightarrow A_\mu + \partial_\mu \xi$, the fermionic field ψ and its covariant derivative transform under the U(1) gauge symmetry as

$$\psi \rightarrow e^{ie\xi} \psi, \tag{2.4}$$

$$\bar{\psi} \rightarrow e^{-ie\xi} \bar{\psi}, \tag{2.5}$$

$$\mathcal{D}_\mu \psi \rightarrow e^{ie\xi} \mathcal{D}_\mu \psi, \tag{2.6}$$

where $\xi(x)$ is an arbitrary function controlling the gauge transformation. Hence S_{el} is manifestly gauge invariant, including the derivative dimension-8 operator proportional to k_0 . It is also not difficult to show that the complete action \mathcal{S}_{tot} is ghost free, since A_0 remains a constraint and does not propagate.

Being interested on inflationary cosmology, we work with a conformally flat FRW metric,

$$ds^2 = a^2(\eta) [-d\eta^2 + d\vec{x}^2]. \tag{2.7}$$

For simplicity, we focus our attention to the pure de Sitter case, where the scale factor is $a(\eta) = -1/(H\eta)$, and we start discussing a case in which we allow only for a homogeneous profile φ for the scalar field. We choose units in which at the end of inflation the scale factor is equal to one, $a_{\text{end}} = 1$. Consequently, in these units the scale factor is very small at the beginning of inflation, $a_{\text{in}} = \exp[-N_{\text{ef}}]$ with N_{ef} the e-fold number. We assume that the vector does not backreact on the metric and on the inflationary dynamics; we will critically assess these hypothesis in due course.

Let us start discussing the electromagnetic part of the action in eq (2.2). We introduce the combinations (the prime denotes derivative along conformal time)

$$f^2(\varphi) = \mathcal{B}(\varphi) + \frac{2\mathcal{C}(\varphi)\varphi'^2}{a^2}, \tag{2.8}$$

$$g^2(\varphi) = \mathcal{B}(\varphi). \tag{2.9}$$

We express the vector components as

$$A_\mu = (A_0, A_i + \partial_i \chi), \tag{2.10}$$

¹We could promote k_0 to a function of the scalar φ , without qualitatively changing our arguments. We will comment on this possibility towards the end of section 3.

with χ being the vector longitudinal polarization and A_i the transverse vector components satisfying $\partial^i A_i = 0$. We plug this decomposition into the action (2.2):

$$S_{\text{em}} = \int d^3x d\eta \left[\frac{f^2(\varphi)}{2} (A'_i)^2 + \frac{g^2(\varphi)}{2} A_j \nabla^2 A_j \right] + \int d^3x d\eta \frac{f^2(\varphi)}{2} [2A_0 \nabla^2 \chi' - A_0 \nabla^2 A_0 - \chi' \nabla^2 \chi'] , \quad (2.11)$$

where we neglect contributions arising from space-dependent scalar fluctuations. Hence we learn that the derivative contribution to the action (2.2), proportional to the function \mathcal{C} , can change the effective sound speed of the electromagnetic field, since it makes the functions f and g different. See also [11] for a similar framework, but motivated by different physical principles. We will analyze in detail the rich phenomenology of this scenario in appendix A.

The fermionic action (2.3), in this homogeneous background configuration, can be re-assembled as

$$\mathcal{S}_\psi = \frac{1}{2} \int d^4x \sqrt{-g} \left[i h^2(\varphi) \bar{\psi} \gamma^0 \nabla_0 \psi + e h^2(\varphi) \bar{\psi} \gamma^0 A_0 \psi + i \bar{\psi} \gamma^i \nabla_i \psi + e \bar{\psi} \gamma^i (A_i + \partial_i \chi) \psi + h.c. \right] , \quad (2.12)$$

with

$$h^2(\varphi) = 1 + k_0 \frac{\varphi'^2}{a^2} . \quad (2.13)$$

The essential feature that we need is the fact that the coefficients in front of time and spatial derivatives of the fermion fields can be different. We will make use of this fact in section 3 when addressing the strong coupling problem for our scenario of magnetogenesis. Combining together the fermionic Lagrangian of eq. (2.12) with the electromagnetic Lagrangian (2.11), we find the following constraint equation for A_0

$$A_0 = \chi' - \frac{e h^2 a^4}{f^2} \vec{\nabla}^{-2} (\bar{\psi} \gamma^0 \psi) . \quad (2.14)$$

Plugging this condition in the total action, we find

$$S_{\text{tot}} = \int d^3x d\eta a^4 \left[\frac{f^2}{2a^4} (A'_i)^2 + \frac{g^2}{2a^4} A_j \nabla^2 A_j + i h^2 \bar{\psi} \gamma^0 \overleftrightarrow{\nabla}_0 \psi + i \bar{\psi} \gamma^i \overleftrightarrow{\nabla}_i \psi + e \bar{\psi} \gamma^i A_i \psi \right] + \int d^3x d\eta e a^4 \left[h^2 \bar{\psi} \gamma^0 \psi \chi' + \bar{\psi} \gamma^i \psi \partial_i \chi + \dots \right] , \quad (2.15)$$

with $\bar{\psi} \gamma^a \overleftrightarrow{\nabla}_a \psi \equiv \bar{\psi} \gamma^a \nabla_a \psi - \nabla_a \bar{\psi} \gamma^a \psi$, while the dots in the second line contain dimension-6, gauge invariant four fermion operators (not including gauge field) with structure $(\bar{\psi} \psi)^2$. The consequences of such operators depend on the specific UV completion for our set-up. Indeed, it is always possible to include additional gauge invariant four fermion operators — weighted by an appropriate function of the scalar field φ — with the same structure of the ones discussed, and that compensate for their effects. Such additional operators are gauge invariant and do not involve gauge fields, hence do not affect the magnetogenesis scenario that we are going to develop.

The previous action (2.15) is still gauge invariant. For simplicity, we can choose a unitary gauge $\chi = 0$, that makes the last line of eq. (2.15) vanishing. In this gauge, the total action to consider is then

$$S_{\text{tot}} = \int d^3x d\eta a^4 \left[\frac{f^2}{2a^4} (A'_i)^2 + \frac{g^2}{2a^4} A_j \nabla^2 A_j + i h^2 \bar{\psi} \gamma^0 \overleftrightarrow{\nabla}_0 \psi + i \bar{\psi} \gamma^i \overleftrightarrow{\nabla}_i \psi + e \bar{\psi} \gamma^i A_i \psi \right]. \quad (2.16)$$

We will suppose that at the end of inflation the scalar φ ceases to roll and stabilizes on some minimum of its potential, so that $h = 1$ and the quantities f and g become identical. Such a scenario has interesting phenomenological consequences. In appendix A we show that it is possible to generate a scale invariant spectrum for a magnetic field without causing large backreaction on the inflationary energy density. In particular, we generalize the kinetically coupled model of [7] to include the effect of a time dependent sound speed for the electromagnetic waves. In the next section, instead, we focus on the main point of this work: by exploiting our particular form of the fermionic action, it is possible to avoid the so-called strong coupling problem for primordial magnetogenesis, that has been first pointed out in [8].

3 Avoiding the strong coupling problem during inflation

In the standard kinetically coupled scenario for magnetogenesis introduced by Ratra in [7], it is possible to generate a sufficiently large amplitude of scale invariant magnetic field without encountering backreaction problems during inflation. In the simplest realizations of Ratra's scenario, one finds the condition that the function f appearing in eq (2.16) is an increasing function of the scale factor, $f = a^2$. See e.g. [3, 10] for reviews.

In our set-up, we have the two independent functions f and g to exploit for generating a scale invariant spectrum for the magnetic field. We show in appendix A that a scale invariant magnetic field can be generated if

$$f = a^\alpha \quad \text{with} \quad \alpha \geq 2, \quad (3.1)$$

provided that

$$g = a^\beta \quad \text{with} \quad \beta = \frac{3\alpha + 4}{5}. \quad (3.2)$$

This implies that we have more parameter space available for obtaining a scale invariant spectrum for the magnetic field, than in the standard scenario developed by Ratra [7] (recovered for the special choice $\alpha = \beta = 2$). Recall that we choose units in which the scale factor starts very small at the beginning of inflation, $a_{\text{in}} = \exp[-N_{\text{ef}}]$ with N_{ef} the e-fold number, while at the end of inflation $a_{\text{end}} = 1$. Starting from a_{end} the functions f and g coincide since, as explained in the previous section, we make the hypothesis that the scalar φ stops rolling at the end of inflation.

Let us consider the consequences of these facts for the coupling between the electromagnetic field and fermions during inflation, thinking for definiteness to ψ as the electron field. The fermionic action that we investigate can be read from eq. (2.16) (where we select the gauge $\chi = 0$):

$$S_\psi = \int d^3x d\eta a^4 \left[i h^2 \bar{\psi} \gamma^0 \overleftrightarrow{\nabla}_0 \psi + i \bar{\psi} \gamma^i \overleftrightarrow{\nabla}_i \psi + e \bar{\psi} \gamma^i A_i \psi \right], \quad (3.3)$$

where h is

$$h^2(\varphi) = 1 + k_0 \frac{\varphi'^2}{a^2}. \quad (3.4)$$

In a standard scenario, without the derivative couplings proportional to k_0 in the fermionic action (2.3), one encounters a strong coupling problem for primordial magnetogenesis [8].

For setting the stage, let us explicitly discuss this problem as a special case of our discussion, by choosing $k_0 = 0$ and so (by eq (3.4)) $h = 1$. After canonically normalize the electromagnetic gauge potential (see eq. (2.16)) and the fermionic electron field (see eq. (3.3)) as

$$A_\mu \rightarrow \frac{\hat{A}_\mu}{f}, \quad \psi \rightarrow \frac{\hat{\psi}}{a^2}, \quad (3.5)$$

one obtains the following effective coupling between the fields involved

$$\mathcal{L}_{\mathcal{A}\psi\psi} = \frac{e}{f} \bar{\hat{\psi}} \gamma^i \hat{\psi} \hat{A}_i. \quad (3.6)$$

So the effective coupling scales as e/f . But we know that f is a monotonic increasing function during inflation, see eq (3.1): hence at early times f is very small. This makes the effective coupling e/f extremely large at early inflationary stages, thus leading to a strong coupling regime in which the theory cannot be trusted [8]. This is a general feature of conformally coupled models, although possible ways-out might be found for particular, non-monotonic profiles of the conformal functions [12, 13], or adding helicity to the electromagnetic field [14]. For other examples not involving scalar fields see [15, 16].

In our case, including the derivative couplings proportional to the quantity k_0 , we have the additional function h at our disposal. The canonical normalization for vector and fermion fields now explicitly depends on h :

$$A_\mu \rightarrow \frac{\hat{A}_\mu}{f}, \quad \psi \rightarrow \frac{\hat{\psi}}{a^2 h}. \quad (3.7)$$

Hence the coupling between vector and fermion fields in eq (2.16) is controlled by the effective Lagrangian

$$\mathcal{L}_{\mathcal{A}\psi\psi} = \frac{e}{f h^2} \bar{\hat{\psi}} \gamma^i \hat{\psi} \hat{A}_i \quad (3.8)$$

so we gain a factor $1/h^2$ with respect to eq. (3.6). We can now investigate situations where the time dependence of the quantity $1/h^2$ compensates the function f in the denominator of the previous formula. As the simplest example, we can demand that

$$\frac{1}{f h^2} = 1, \quad (3.9)$$

that is, the function h compensates exactly for the smallness of f in the early stages of inflation. Making this choice, eq. (3.8) rewrites

$$\mathcal{L}_{\mathcal{A}\psi\psi} = e \bar{\hat{\psi}} \gamma^i \hat{\psi} \hat{A}_i. \quad (3.10)$$

Hence the effective coupling between fermion and gauge boson is now constant during inflation. So it does not suffer from the strong coupling problem that affects the standard conformally coupled, $h = 1$ scenario.

Using the definition for h and for f , the condition (3.9) can be expressed as

$$1 + k_0 \frac{\varphi'^2}{a^2} = a^{-\alpha} \quad \text{with } \alpha \geq 2. \quad (3.11)$$

We can think of two different ways to satisfy this condition:

- The first possibility is to make the time derivative φ' of the scalar field *very large* at the beginning of inflation, when $a_{\text{in}} = \exp[-N_{\text{ef}}]$. In this regime, the condition of the previous formula can be re-expressed as (recall we are working in conformal time)

$$k_0 (\partial_a \varphi)^2 \frac{a'^2}{a^2} \simeq a^{-\alpha} \quad \Rightarrow \quad \varphi \simeq \frac{2}{\alpha \sqrt{k_0} H} a^{-\frac{\alpha}{2}} \quad (3.12)$$

during the early epoch of the inflationary quasi-de Sitter era. So the scalar φ is in a *fast-rolling* regime during these initial stages of inflation, changing with a rate depending on a parameter $\alpha \geq 2$, and does not correspond to the usual slowly-rolling inflaton field. At the same time, we have to demand that its total energy density does not dangerously backreact on the geometry. Possible realizations of these conditions can be found; an explicit one will be analyzed in appendix B by identifying our field φ with an auxiliary scalar during inflation, whose kinetic and potential energies are modulated by a suitable function of the inflaton field.

- The second possibility is to promote the parameter k_0 to a function of the field profile φ , able to satisfy eq (3.11) without demanding a large time derivative for φ . Following this route, one can embed this mechanism in a standard, slow-roll inflationary model with φ' small during inflation. For example, in single field slow-roll inflation, one has the approximate equality

$$\frac{\varphi'^2}{a^2} \simeq 2\epsilon H^2 \quad (3.13)$$

with H the Hubble parameter during inflation, and $\epsilon = -H'/(aH^2) < 1$ is a slow-roll parameter (recall we are taking derivatives along conformal time). In this case, in order to satisfy the requirement (3.11), we can demand that the function k_0 has a scalar-dependent profile

$$k_0 \simeq \frac{a^{-\alpha}}{2\epsilon H^2} \quad (3.14)$$

during the first stage of inflation.

We have shown that, provided that gauge-invariant derivative interactions are included, we can avoid a strong coupling problem between fermionic and electromagnetic fields during the early stages of inflation. Further important requirements have to be taken into account for obtaining a fully satisfactory scenario of primordial magnetogenesis. For example, one should analyze in details the dynamics of fluctuations of the scalar φ and the geometry, and ensure that no additional strong coupling issues emerge when coupling such fluctuations with the electromagnetic and fermionic fields.² For example, one might worry about dimension-8 operators involving fluctuations $\delta\varphi$ of our field φ coupled to fermions and gauge fields. Schematically such operators have the form $\partial_\mu \delta\varphi \partial^\nu \delta\varphi (\bar{\psi} \gamma^\mu A_\nu \psi)$, and they originate from second order perturbations of the operator weighted by k_0 in eq (2.3). Once all

²We thank Marco Peloso, Ricardo Ferreira and Jonathan Ganc for discussions on this subject.

the fields are canonically normalized, such operators are weighted by a factor c_0 that scales as $c_0 = 1/(f E_{\text{kin}})$ during the early stages of inflation, with E_{kin} the scalar kinetic energy. In scenarios where the scalar kinetic energy does not backreact on the inflationary expansion, so to satisfy the inequality $E_{\text{kin}} \ll H^2 M_{\text{pl}}^2$, c_0 can be well larger than $1/H^4$ during inflation (unless the scale of inflation is low), leading to a strong coupling issue. Whether or not this is a problem depends on the explicit realization of our scenario, and on its possible UV completions including additional higher dimensional operators. For example, one can consider set-ups in which the scalar kinetic energy is not necessarily much smaller than the scale of inflation, so that a more careful, model dependent study is needed to evaluate whether such operators lead to strong coupling. (Since such analysis involves backreaction of scalar fields, it would nevertheless be easier to perform with respect to a case in which it is the electromagnetic field that backreacts on the geometry.) Alternatively, one could consider the effect of additional gauge invariant, higher dimensional operators coupling the scalar to fermions and gauge fields; one example are dimension-12 operators of the schematic form $(\partial\varphi)^2 (\bar{\psi} \gamma A \psi)$, that once expanded in terms of scalar perturbations contain (among others) operators of the right structure to compensate for the effects of the aforementioned ones.

Of course, a detailed analysis of these and other issues will be important to really understand whether it is possible to build explicit models for primordial magnetogenesis along these lines, that are under control when examined order by order in perturbation theory. The general proposal discussed in this paper, if on the right track, might point towards set-ups with a non-trivial structure, possibly determined by some symmetry principle, in which the role of higher dimensional derivative operators is crucial in defining a viable magnetogenesis model during inflation.

4 Outlook

We proposed a framework for building gauge invariant models of primordial magnetogenesis, that are potentially free from strong coupling problems. The main idea is to derivatively couple electromagnetic and fermion fields to a scalar field φ during inflation. This scalar is not necessarily the inflaton. Gauge-invariant derivative scalar-vector couplings, although explored in modified gravity scenarios, to the best of our knowledge have not been investigated in the context of primordial magnetogenesis. We show that their inclusion induces a non-trivial sound speed for electromagnetic and fermionic modes. Moreover, and most importantly, derivative couplings give additional freedom to control the time-dependence of the electromagnetic coupling constant during inflation, by making this quantity depending also on the *time derivative* of the scalar background profile. This fact allows us to find conditions to avoid the serious strong coupling issue that, as first pointed out in [8], affects the simplest realizations of magnetogenesis scenarios.

Besides potentially avoiding the strong coupling problem, our scenario allows us to generate a large scale magnetic field of sufficient amplitude, that does not suffer from large backreaction issues. We do not need to rely on explicit inflationary models for developing our ideas, that might be applied to different cases. On the other hand, specific conditions have to be imposed on our system of a scalar derivatively coupled to the electromagnetic and fermionic actions. A first realization of our mechanism requires that the homogenous profile for scalar field has a large time derivative during the early stages of the inflationary era. We shown in an explicit concrete example how to satisfy such conditions in a consistent

way. A second realization instead imposes that some of the functions of the scalar field, that multiply our derivative operators, vary considerably during inflation.

It would be interesting to apply more broadly these ideas to specific models of inflation, to study at which extent the requirements of obtaining theoretically convincing magnetogenesis set-ups can constrain a given inflationary model. When analyzing explicit inflationary models, it will be important to carefully study the dynamics of scalar perturbations, and how the derivative couplings to the electromagnetic field can affect their behaviour. Indeed, in recent years there have been many investigations of possible observational consequences of primordial magnetogenesis for the production of CMB anisotropies and non-Gaussianities (see e.g. [17–20]), and more in general on constraints on magnetogenesis scenarios from CMB observations [21–24]. These studies have been generally done taking the kinetically coupled magnetogenesis model of [7] as reference scenario. It would be interesting to generalize these results to the set-up presented in this work including a time-dependent sound speed for the electromagnetic potential, and taking into account the full dynamics of the fluctuations for the fields involved.

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A Phenomenology of this scenario

A.1 Generating a scale invariant spectrum for the magnetic field

We now investigate how our set-up can lead to the production of a homogeneous magnetic field with scale invariant spectrum. To start with, we focus on the consequences of the scalar-vector action (2.2), neglecting the effects of fermion production during inflation, that we assume to be negligible.³

Our purposes is to study cosmology in a conformally flat de Sitter background (see eq (2.7)). Hence we focus on the scalar-vector part of the action (2.16), that is

$$S_{\text{EM}} = \int d^3x d\eta \left[\frac{f^2(\varphi)}{2} (A'_i)^2 + \frac{g^2(\varphi)}{2} A_j \nabla^2 A_j \right]. \quad (\text{A.1})$$

³Let us point out that the derivative couplings we considered are not the most general ones, and others could be examined, for example motivated by Galileon symmetries [27, 28]. On the other hand, for illustrative purposes, the minimal structure of our actions (2.2) and (2.3) will be sufficient.

We relate the derivatives of the gauge potential to the electric and magnetic field via the following formulae

$$\begin{aligned} F_{0i} &= a^2 E_i, \\ F_{ij} &= a^2 \epsilon_{ijk} B_k, \end{aligned} \quad (\text{A.2})$$

that allow us to rewrite our action (A.1) as

$$S = \frac{1}{4} \int d^4x \sqrt{-g} \mathcal{L} = \int d^4x \left[\frac{f^2}{4} F_{0i} F_{0i} - \frac{g^2}{4} F_{ij} F_{ij} \right]. \quad (\text{A.3})$$

This action differs from the usual ‘kinetic coupled’ model [7] since our construction allows us for a time-dependent sound speed for the electromagnetic potential when $f \neq g$. We have learned in section 2 that (neglecting fermions) the constraint equation eliminates the longitudinal polarization χ of the vector, hence $\partial_i A_i = 0$. We therefore decompose

$$\begin{aligned} \vec{A} &= \sum_{\lambda=\pm} \int \frac{d^3k}{(2\pi)^{3/2}} \vec{e}_\lambda(\vec{k}) e^{i\vec{k}\cdot\vec{x}} \frac{\hat{V}_\lambda(k)}{f}, \\ &= \sum_{\lambda=\pm} \int \frac{d^3k}{(2\pi)^{3/2}} \vec{e}_\lambda(\vec{k}) e^{i\vec{k}\cdot\vec{x}} \left[a_\lambda(\vec{k}) \frac{V_\lambda(k)}{f} + a_\lambda^\dagger(-\vec{k}) \frac{V_\lambda^*(k)}{f} \right], \end{aligned} \quad (\text{A.4})$$

where the circular polarization operators satisfy $\vec{k} \cdot \vec{e}_\pm(\vec{k}) = 0$, $\vec{k} \times \vec{e}_\pm(\vec{k}) = \mp ik \vec{e}_\pm(\vec{k})$, $\vec{e}_\pm(-\vec{k}) = \vec{e}_\pm^*(\vec{k})$, and are normalized according to $\vec{e}_\lambda^*(\vec{k}) \cdot \vec{e}_{\lambda'}(\vec{k}) = \delta_{\lambda\lambda'}$. The annihilation and creation operators appearing in (A.4) satisfy the commutation relations

$$\left[a_\lambda(\vec{k}), a_{\lambda'}^\dagger(\vec{k}') \right] = \delta_{\lambda\lambda'} \delta^{(3)}(\vec{k} - \vec{k}'). \quad (\text{A.5})$$

In terms of the Fourier quantum field \hat{V}_λ , our action (A.3) results

$$\begin{aligned} S &= \frac{1}{2} \sum_\lambda \int dt d^3k \left[f^2 \left| \left(\frac{\hat{V}_\lambda(k)}{f} \right)' \right|^2 - \frac{g^2}{f^2} k^2 \left| \hat{V}_\lambda(k) \right|^2 \right], \\ &\equiv \frac{1}{2} \sum_\lambda \int dt d^3k \left[\left| \hat{V}_\lambda' \right|^2 - \left(\frac{g^2}{f^2} k^2 - \frac{f''}{f} \right) \left| \hat{V}_\lambda \right|^2 \right], \end{aligned} \quad (\text{A.6})$$

and the equation of motion for the mode function is

$$V_\lambda'' + \left(\frac{g^2}{f^2} k^2 - \frac{f''}{f} \right) V_\lambda = 0. \quad (\text{A.7})$$

This variable V_λ is canonically normalized. We decompose the electric and magnetic fields as

$$\begin{aligned} \vec{E} &= \sum_{\lambda=\pm} \int \frac{d^3k}{(2\pi)^{3/2}} \left[\vec{e}_\lambda(\vec{k}) e^{i\vec{k}\cdot\vec{x}} a_\lambda(\vec{k}) E_\lambda(k) + h.c. \right], \\ \vec{B} &= \sum_{\lambda=\pm} \int \frac{d^3k}{(2\pi)^{3/2}} \left[\vec{e}_\lambda(\vec{k}) e^{i\vec{k}\cdot\vec{x}} a_\lambda(\vec{k}) B_\lambda(k) + h.c. \right], \end{aligned} \quad (\text{A.8})$$

and their mode functions are related to V_λ by ($\lambda = \pm$)

$$E_\lambda = -\frac{1}{a^2} \left(\frac{V_\lambda}{f} \right)', \quad B_\lambda = \frac{\lambda k V_\lambda}{a^2 f}. \quad (\text{A.9})$$

We proceed computing the energy density associated with this system. We have

$$\rho = -T_0^0 = \frac{f^2}{2a^4} F_{0i} F_{0i} + \frac{g^2}{4a^4} F_{ij} F_{ij} = \frac{1}{2} [f^2 E^2 + g^2 B^2], \quad (\text{A.10})$$

and so

$$\langle \rho \rangle = \sum_\lambda \int \frac{d^3 k}{(2\pi)^3} \left\{ \frac{f^2}{2} |E_\lambda(k)|^2 + \frac{g^2}{2} |B_\lambda(k)|^2 \right\} \quad (\text{A.11})$$

$$\equiv \langle \rho_E \rangle + \langle \rho_B \rangle. \quad (\text{A.12})$$

The total energy is given by a sum of magnetic and electric contributions, with energy densities

$$\begin{aligned} \frac{d\langle \rho_E \rangle}{d \ln k} &= \sum_\lambda \frac{k^3}{4\pi^2} f^2 |E_\lambda(k)|^2 = \sum_\lambda \frac{k^3}{4\pi^2 a^4} f^2 \left| \left(\frac{V_\lambda}{f} \right)' \right|^2, \\ \frac{d\langle \rho_B \rangle}{d \ln k} &= \sum_\lambda \frac{k^3}{4\pi^2} g^2 |B_\lambda(k)|^2 = \sum_\lambda \frac{k^5}{4\pi^2 a^4} \frac{g^2}{f^2} |V_\lambda|^2. \end{aligned} \quad (\text{A.13})$$

For definiteness, we now adopt a power-law Ansatz for the time-dependence of the functions f and g :

$$f(\eta) = a^\alpha = (-H\eta)^{-\alpha}, \quad g(\eta) = a^\beta = (-H\eta)^{-\beta}, \quad (\text{A.14})$$

and we set $\eta_{\text{end}} = -1/H$ as the time at which inflation ends, when $f = g = a_{\text{end}} = 1$. This choice leads to the following equation for V_λ

$$V_\lambda'' + \left[(-H\eta)^{2\alpha-2\beta} k^2 - \frac{\alpha(\alpha+1)}{\eta^2} \right] V_\lambda = 0. \quad (\text{A.15})$$

As mentioned above, the sound speed (more appropriately, the speed of light) of photons at small scales is given by $c_{\text{eff}}^2 = g^2/f^2$, so that

$$c_{\text{eff}} = (-H\tau)^{\alpha-\beta} = a^{\beta-\alpha}. \quad (\text{A.16})$$

To avoid superluminal propagation (which, in turns, may create problems for finding an UV completion for the model [29]), we require that $g^2 \leq f^2$. This implies that the speed of light starts very small at the beginning of inflation (i.e. much smaller than the speed of gravitons) and approaches the value $c_{\text{eff}} = 1$ at the end of inflation. Therefore, we impose that $\beta > \alpha$.

We also require that the part proportional to k in eq. (A.15) dominates the coefficient of V_λ at early times, i.e. that

$$\alpha - \beta + 1 > 0. \quad (\text{A.17})$$

This condition guarantees the validity of the adiabatic approximation at early times, so that we can assume that modes of fixed momentum k are initially in the adiabatic vacuum

state. The solution of eq. (A.15) that reduces to positive frequency modes at early times is (up to an unphysical phase)

$$\begin{aligned}
 V_\lambda &= \frac{i}{2\sqrt{1+\alpha-\beta}} \sqrt{\frac{\pi}{aH}} H_{\frac{1+2\alpha}{2(1+\alpha-\beta)}}^{(1)} \left(\frac{k}{H} \frac{a^{\beta-\alpha-1}}{\alpha-\beta+1} \right) \\
 \alpha > -\frac{1}{2} : \quad f \left(\frac{V_\lambda}{f} \right)' &= -i \frac{k\sqrt{\pi}}{2\sqrt{H}\sqrt{1+\alpha-\beta}} a^{\beta-\alpha-\frac{1}{2}} H_{-\frac{1-2\beta}{2(1+\alpha-\beta)}}^{(1)} \left(\frac{k}{H} \frac{a^{\beta-\alpha-1}}{\alpha-\beta+1} \right) \quad (\text{A.18})
 \end{aligned}$$

and

$$\begin{aligned}
 V_\lambda &= \frac{i}{2\sqrt{1+\alpha-\beta}} \sqrt{\frac{\pi}{aH}} H_{-\frac{2\alpha+1}{2(1+\alpha-\beta)}}^{(1)} \left(\frac{k}{H} \frac{a^{\beta-\alpha-1}}{\alpha-\beta+1} \right) \\
 \alpha < -\frac{1}{2} : \quad f \left(\frac{V_\lambda}{f} \right)' &= i \frac{k\sqrt{\pi}}{2\sqrt{H}\sqrt{1+\alpha-\beta}} a^{\beta-\alpha-\frac{1}{2}} H_{\frac{1-2\beta}{2(1+\alpha-\beta)}}^{(1)} \left(\frac{k}{H} \frac{a^{\beta-\alpha-1}}{\alpha-\beta+1} \right) \quad (\text{A.19})
 \end{aligned}$$

with $H_\nu^{(1)}(x)$ denoting the Hankel function of the first kind.

These solutions have the correct asymptotic early time limits, and the arbitrary phase is chosen so that the modes associated with V_λ are real and positive at late times.

Let us focus on the simplest situation of particular interest for generating a large-scale magnetic field: the case of a scale invariant magnetic energy density. Plugging eqs. (A.18), (A.19) into eq (A.13), we find the following condition

$$\begin{aligned}
 \left. \frac{d\langle\rho_B\rangle}{d\ln k} \right|_{\text{late}} = \text{scale independent} & \quad \text{for} \quad \alpha > -\frac{1}{2} \quad \text{and} \quad \beta = \frac{3\alpha+4}{5}, \\
 & \quad \text{or} \quad \alpha < -\frac{1}{2} \quad \text{and} \quad \beta = \frac{7\alpha+6}{5}. \quad (\text{A.20})
 \end{aligned}$$

Fixing the parameter β as in the previous formulae allows us to greatly simplify the expressions in equations (A.18), (A.19), since the index of the Hankel function acquires the value $5/2$, and we get the exact solutions (valid at all times)

$$\begin{aligned}
 \alpha > -\frac{1}{2} : \quad V_\lambda &= \frac{3(1+2\alpha)^2 H^2 a^\alpha}{25\sqrt{2}k^{5/2}} \left(1 - iz - \frac{z^2}{3} \right) e^{iz}, \\
 f \left(\frac{V_\lambda}{f} \right)' &= \frac{-(1+2\alpha) H a^{\frac{3+\alpha}{5}}}{5\sqrt{2}k^{1/2}} (1-iz) e^{iz}, \quad (\text{A.21})
 \end{aligned}$$

and

$$\begin{aligned}
 \alpha < -\frac{1}{2} : \quad V_\lambda &= \frac{3(2\alpha+1)^2 H^2 a^{-\alpha-1}}{25\sqrt{2}k^{5/2}} \left(1 - iz - \frac{z^2}{3} \right) e^{iz}, \\
 f \left(\frac{V_\lambda}{f} \right)' &= \frac{3(2\alpha+1)^3 H^3 a^{-\alpha}}{25\sqrt{2}k^{5/2}} \left(1 - iz - \frac{2z^2}{5} + \frac{iz^3}{15} \right) e^{iz}, \quad (\text{A.22})
 \end{aligned}$$

where $z \equiv \frac{5}{|1+2\alpha|} \frac{k}{H} a^{-\frac{|1+2\alpha|}{5}}$. Notice that, for this choices of β , we have

$$1 + \alpha - \beta = \frac{|1+2\alpha|}{5} \quad (\text{A.23})$$

in both regimes, and so we automatically satisfy the $1 + \alpha - \beta > 0$ condition.

From these exact, all-time solutions, we get the late time energy densities (summing over the two photon polarizations)

$$\begin{aligned}
 \alpha > -\frac{1}{2} : \quad \frac{d\langle\rho_B\rangle}{d\ln k}\Big|_{\text{late}} &= \frac{9(1+2\alpha)^4 H^4}{2500\pi^2} a^{-\frac{6(2-\alpha)}{5}}, & \frac{d\langle\rho_E\rangle}{d\ln k}\Big|_{\text{late}} &= \frac{(1+2\alpha)^2 H^4}{100\pi^2} \left(\frac{k}{H}\right)^2 a^{-\frac{2(7-\alpha)}{5}}, \\
 \alpha < -\frac{1}{2} : \quad \frac{d\langle\rho_B\rangle}{d\ln k}\Big|_{\text{late}} &= \frac{9(2\alpha+1)^4 H^4}{2500\pi^2} a^{-\frac{6(\alpha+3)}{5}}, & \frac{d\langle\rho_E\rangle}{d\ln k}\Big|_{\text{late}} &= \frac{9(2\alpha+1)^6 H^4}{2500\pi^2} \left(\frac{H}{k}\right)^2 a^{-2\alpha-4}.
 \end{aligned}
 \tag{A.24}$$

From the previous expressions, one learns that the $\alpha < -\frac{1}{2}$ case leads to a too large electric field energy produced during inflation. Indeed, $d\langle\rho_E\rangle/d\ln k$ is divergent in the infrared limit $k \rightarrow 0$, signalling a strong backreaction of the electric field on the inflating background. Therefore we disregard this choice. Notice instead that in the $\alpha > -\frac{1}{2}$ regime we have

$$\begin{aligned}
 V_\lambda V_\lambda^{*'} - \text{h.c.} &= 1, \\
 V_\lambda V_\lambda^{*'} + \text{h.c.} &= \frac{9\alpha(1+2\alpha)^4}{625} \left(\frac{H}{k} a^{\frac{1+2\alpha}{5}}\right)^5 - \frac{3(1+2\alpha)^2(1-3\alpha)}{125} \left(\frac{H}{k} a^{\frac{1+2\alpha}{5}}\right)^3 - \frac{2-\alpha}{5} \left(\frac{H}{k} a^{\frac{1+2\alpha}{5}}\right).
 \end{aligned}
 \tag{A.25}$$

Since a strongly increases during inflation, and since $1+2\alpha > 0$, the mode function is strongly increasing, and there is classicalization for all $\alpha > -\frac{1}{2}$: the energy associated to the classical field is finite.

So we focus our attention to a scenario with a scale-invariant magnetic energy density, with parameters chosen in the range

$$\alpha > -\frac{1}{2} \quad \text{and} \quad \beta = \frac{3\alpha + 4}{5}.
 \tag{A.26}$$

The standard case of unit sound speed, $\alpha = \beta$, is obtained by choosing $\alpha = 2$. In our case we have freedom to choose any preferred value $\alpha > -1/2$, although the parameter β has then to be tuned accordingly to (A.26).

Let us calculate the amplitude of magnetic field towards the end of inflation, that occurs at $a = a_f = 1$. This quantity can be estimated by the general formula (see e.g. [8])

$$\delta_B^2 = \frac{d\langle\rho_B\rangle}{d\ln k}\Big|_{a=1}
 \tag{A.27}$$

$$= \frac{9(1+2\alpha)^4 H^4}{2500\pi^2}.
 \tag{A.28}$$

So the spectrum of the magnetic field is scale invariant.

For values of α of order unity, one obtains a magnetic field of amplitude $\delta_B \simeq (H/M_{\text{Pl}})^2 \cdot 10^{58} G$ when inflation ends. After the end of inflation, δ_B^2 decays as $1/a^4$, exactly as radiation: it is straightforward to estimate the amplitude of magnetic field today in the scale invariant case [3, 8, 10]

$$\delta_B \simeq 5 \times 10^{-10} \left(\frac{H}{10^{-5} M_{\text{Pl}}}\right) G.
 \tag{A.29}$$

As discussed in the introduction, observations require an amplitude of at least $10^{-15} G$ at intergalactic scales: these values are not difficult to obtain with a Hubble scale during inflation

larger than $H \geq 10^{-10} M_{\text{Pl}}$, corresponding to a scale of inflation $E_{\text{inf}} \geq 10^{-5} M_{\text{Pl}}$ (with $E_{\text{inf}}^4 \sim 3H^2 M_{\text{Pl}}^2$).

Hence, we shown that allowing for derivative couplings between the scalar clock and the electromagnetic action one can enrich the phenomenology of the conformally coupled scenario [7]. It would be interesting to further study phenomenological ramifications of this scenario. For example investigating how the electromagnetic field affects the curvature perturbation [21, 22]. Or examining the structure of correlation involving curvature perturbation and the electromagnetic field, that have been analyzed in the conformally coupled case (see e.g. [17–20, 23, 24]). Or studying whether new derivative interactions generalizing the ones we considered here can have an impact on the evolution of the magnetic field during radiation and matter dominated era [25]. It is expected that the non-trivial sound speed for the electromagnetic field plays an important role in characterizing the phenomenology of this set-up. We leave these interesting topics for future work, and focus now on ensuring that our scenario avoids to produce too much electromagnetic energy during inflation.

A.2 Conditions to avoid large backreaction from the electromagnetic field

Inflation is a period of quasi-exponential cosmological expansion, that lasts for $N_{\text{tot}} = \ln\left(\frac{a_f}{a_i}\right)$ e-folds, where a_i and a_f are the values of the scale factors at the beginning and end of inflation. In order to solve the basic problems of standard Big Bang cosmology, and generate a scale invariant spectrum of curvature perturbations at large scales, one finds that $N_{\text{tot}} \geq 50$ [30]. To ensure that the electromagnetic field does not dangerously backreact on the quasi-de Sitter inflationary expansion, we require that the energy stored in the electric and magnetic fields is smaller than the inflationary energy density $\rho_{\text{inf}} = (3H^2 M_{\text{Pl}}^2)^{1/4}$ during inflation.

In our set-up, the electromagnetic potential has a time-dependent sound speed. Hence, each electromagnetic mode, characterized by momentum k , typically leaves the horizon at a different time with respect to corresponding mode in the scalar inflationary fluctuations. (A similar behaviour was first pointed out in [31] in a different context of a two-scalar inflationary set-up.) At a given value a of the scale factor, the electromagnetic modes that leave the horizon have comoving momentum $k = aH/c_s$. During inflation, the electromagnetic sound speed typically scales rapidly with the scale factor (since $c_s = a^{\alpha-\beta}$). These facts imply that in order to generate a coherent magnetic field with scale invariant spectrum at scales of the order of present-day horizon, we have to satisfy the inequality

$$\int_{\frac{a_i H}{c_s(a_i)}}^{\frac{a_f H}{c_s(a_f)}} \frac{dk}{k} \geq 50, \quad (\text{A.30})$$

that translates to

$$(1 + \alpha - \beta) N_{\text{tot}} \geq 50. \quad (\text{A.31})$$

Interestingly, if $(1 + \alpha - \beta) \leq 1$, we need more than 50 e-folds of inflation to generate a coherent scale invariant spectrum for the magnetic field up to very large scales. Let us focus again on the case of scale-invariant magnetic fields in the range $\alpha > -1/2$ and $\beta = (3\alpha + 4)/5$ as dictated by eq. (A.26). The total energies stored in the electric and magnetic fields after N_{ef} e-folds since the beginning of inflation (of course $N_{\text{ef}} \leq N_{\text{tot}}$) are given by the following integrals, performed over the classical modes that crossed the horizon during the epoch of

interest. For the total electric energy, using eqs. (A.24) we obtain

$$\begin{aligned}
 \rho_E &= \int_{a_i H/c_s(a_i)}^{a H/c_s(a)} \frac{dk}{k} \frac{d\langle\rho_E\rangle}{d\ln k}, \\
 &= \int_{a_i^{\frac{1+2\alpha}{5}} H}^{a^{\frac{1+2\alpha}{5}} H} \frac{dk}{k} \frac{d\langle\rho_E\rangle}{d\ln k}, \\
 &\simeq \frac{(1+2\alpha)^2}{200\pi^2} H^4 \exp\left[\frac{6(2-\alpha)}{5}(N_{\text{tot}} - N_{\text{ef}})\right], \tag{A.32}
 \end{aligned}$$

where in the last equality we only wrote the dominant contribution to the integral in the range $\alpha > -1/2$, and recall that N_{ef} corresponds to the number of e-folds since the beginning of inflation. An analogous calculation gives the total energy stored in the magnetic field:

$$\begin{aligned}
 \rho_B &= \int_{a_i^{1+\alpha-\beta} H}^{a^{1+\alpha-\beta} H} \frac{dk}{k} \frac{d\langle\rho_B\rangle}{d\ln k}, \\
 &= \frac{9 N_{\text{ef}} (1+2\alpha)^5}{12500\pi^2} H^4 \exp\left[\frac{6(2-\alpha)}{5}(N_{\text{tot}} - N_{\text{ef}})\right]. \tag{A.33}
 \end{aligned}$$

In both cases, we have an exponential dependence on the number of efolds: given that $(N_{\text{tot}} - N_{\text{ef}}) > 0$, to avoid large backreaction we need to impose that the coefficient of $(N_{\text{tot}} - N_{\text{ef}})$ in the exponent is negative, leading to the requirement

$$\alpha \geq 2. \tag{A.34}$$

We impose this condition on this work. At this point, we need to avoid large backreaction towards the end of inflation, when $N_{\text{ef}} \simeq N_{\text{tot}}$ and the energy stored in the electromagnetic field is maximal. The requirement that $\rho_{B,E}$ are less than the inflationary energy density ρ_{inf} imposes the following condition

$$\frac{9 N_{\text{tot}} (1+2\alpha)^5}{12500\pi^2} H^4 \lesssim 3H^2 M_{\text{Pl}}^2 \quad \Rightarrow \quad H \lesssim \frac{100}{\sqrt{N_{\text{tot}}(1+2\alpha)^2}} M_{\text{Pl}}. \tag{A.35}$$

This requirement can be easily satisfied for relevant values of the Hubble parameter, as the ones discussed after eq. (A.29).

B A fast rolling scalar field with no large backreaction during inflation

In section 3, we learned that, by including derivative couplings of a scalar field φ to fermions, it is possible to avoid the strong coupling problem [8] for scenarios of primordial magnetogenesis based on Ratra's idea of coupling a scalar to electromagnetism during inflation. The simplest realization of our set-up requires that the *velocity* φ' of the scalar field is large during the early epochs of inflationary era, so that the scalar is in a fast-roll regime at this stage. With eq (3.12) we established the following requirement for the homogeneous part of the scalar profile

$$\varphi \propto a^{-\frac{\alpha}{2}}, \quad \alpha \geq 2 \tag{B.1}$$

with a constant of proportionality that depends on the parameters of the model.

In this sense, then, we are moving the strong coupling issue for magnetogenesis to the challenging problem of finding a scenario where the time-derivative of a scalar field is large during the inflationary quasi-de Sitter expansion. At first sight, this condition seems hard to satisfy, since in conventional inflationary models scalar fields *slowly roll* during the inflationary epoch, in order to avoid that its kinetic energy spoils the inflationary dynamics. On the other hand, this requirement is not strictly necessary: we review here a scenario, elaborated in [32], where a scalar field can acquire a large velocity during inflation, yet avoiding strong backreaction problems.

We assume that φ is *not* the inflaton; the inflaton is denoted with Φ and has its own dynamics that we do not need to discuss. The action for our scalar φ is given by

$$\mathcal{S}_\varphi = \int d^4x \sqrt{-g} \mathcal{N}^2 [\Phi(\eta)] \left[-\frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - \frac{m^2}{2} \varphi^2 \right]. \quad (\text{B.2})$$

The function $\mathcal{N} [\Phi(\eta)]$ depends on time since it is a function of the inflaton field Φ that acts as clock. We still assume that the metric is well approximated by a conformally flat de Sitter space during inflation, with Hubble parameter H . By selecting a suitable inflaton homogeneous profile $\Phi(\eta)$, we assume that the function $\mathcal{N}(\eta)$ is proportional to a power of the de Sitter scale factor

$$\mathcal{N} = a^\gamma, \quad (\text{B.3})$$

with γ a constant parameter that we choose to be positive, to avoid strong coupling problems in the scalar sector during inflation [32]. We make the hypothesis that the scalar φ does not interfere with the inflationary dynamics characterized by an (almost) constant Hubble parameter H , so that the profile of \mathcal{N} does not change during inflation. It is straightforward to solve the homogeneous equation of motion for φ in a de Sitter geometry, obtaining the following solution

$$\varphi = \varphi_0 a^{-\left(\frac{3}{2} + \gamma\right)} \left[1 - \sqrt{1 - \frac{4m^2}{H^2(3+2\gamma)^2}} \right] \quad (\text{B.4})$$

for a constant parameter φ_0 , that we can imagine to select with an appropriate choice of initial conditions. By tuning the quantities γ and m/H we can obtain the preferred exponent of the scale factor in the previous solution, to match the condition (B.1) with the preferred value for α .

The energy density associated with the configuration (B.4) is

$$\rho = -T_0^0 = \frac{\varphi_0^2 H^2}{4} (3+2\gamma)^2 \left[1 - \sqrt{1 - \frac{4m^2}{H^2(3+2\gamma)^2}} \right] a^3 \left[\sqrt{\left(1 + \frac{2\gamma}{3}\right)^2 - \frac{4m^2}{9H^2}} - 1 \right]. \quad (\text{B.5})$$

In order to avoid a large backreaction of the scalar energy density at the early stages of inflation, when $a \simeq e^{-N_{\text{eff}}}$, we demand that the power of the scale factor in the previous expression is larger or equal than zero, requiring

$$\left(1 + \frac{2}{3}\gamma \right) \geq \sqrt{1 + \frac{4m^2}{9H^2}}. \quad (\text{B.6})$$

This ensures us that our scalar configuration does not strongly backreact on the geometry during inflation, for values of φ_0 smaller than M_{Pl} .

This particular realization of a scalar action modulated by an appropriate function of the inflaton, as in eq (B.2), allows us to obtain the preferred homogeneous profile for

φ to match the conditions to impose for a consistent magnetogenesis scenario. Having a concrete example at hands, like this one, can allow one to ask whether our ideas pass more refined requirements to have consistent magnetogenesis, as for example the full dynamics of inflationary fluctuations. This is a topic that goes well beyond the scope of this work, but let us emphasize that the dynamics of fluctuations can impose interesting constraints on a given inflationary model, as for example provide an upper bound on the scale of inflation, or can lead to some additional requirements as completing the theory in the UV with further operators that control the fluctuation dynamics. This is a broad subject that we intend to pursue in the future.

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