A DEMATEL-Based Completion Method for Incomplete Pairwise Comparison Matrix in AHP

Xinyi Zhou · Yong Hu · Yong Deng · Felix T.S. Chan · Alessio Ishizaka

Abstract Pairwise Comparison Matrix (PCM) as a crucial component of Analytic Hierarchy Process (AHP) presents the preference relations among alternatives. However, in many cases, the PCM is difficult to be completed, which obstructs the subsequent operations of the classical AHP. In this paper, based on Decision-Making and Trial Evaluation Laboratory (DEMATEL) method which has ability to derive the total relation matrix from direct relation matrix, a new completion method for incomplete Pairwise Comparison Matrix (iPCM) is proposed. The proposed method provides a new perspective to estimate the missing values in iPCMs with explicit physical meaning, which is straightforward and flexible. Several experiments are implemented as well to present the completion ability of the proposed method and some insights into the proposed method and matrix consistency.

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1 Introduction

Analytic Hierarchy Process (AHP) is a multi-criteria decision-making method that helps the decision-makers facing a complex problem with multiple conflicting and subjective criteria (Ishizaka and Labib, 2011). AHP has been widely applied in various areas (Moreno-Jiménez et al, 2016). Based on a hierarchical structure, the priority of alternatives can be derived from the PCM. Pairwise Comparison Matrix (PCM) as a crucial component of AHP is commonly utilized to estimate the preference values of finite alternatives with respect to a given set of criteria (Mieza et al, 2017).

However, PCMs are always incomplete in the following cases, which obstructs the subsequent operations of the classical AHP.

- The experts lack the knowledge of one or more alternatives.
- Partial data in the PCM has been lost.
- Huge number of pairwise comparisons is required: $(\frac{m^2-m}{2} \cdot n + \frac{n^2-n}{2})$ for m alternatives and n criteria. For example, 8 alternatives and 6 criteria require 183 entries.

Hitherto, it has attracted widespread attention over the issue of incomplete Pairwise Comparison Matrix (iPCMs) in AHP. Quantities of methods have been developed. One of the most classical methods is proposed by Harker (1987), which is called geometric mean method based on the concept of "connecting path". Many state-of-the-art methods are inspired by Harker's approach. For example, Chen et al (2015) used the connecting path method to estimate missing judgements in iPCMs with minimal geometric consistency index. Ergu et al (2016) extended the geometric mean induced bias matrix to estimate the missing values. Based on graph theory, Bozóki et al (2010) proposed the optimal completion will be unique if and only if the graph associated with the partially deifined matrix is connected.

Besides, some other measures and methods are also proposed to estimate the missing values in iPCMs. For example, Fedrizzi and Giove (2007) completed the iPCMs by minimizing a measure of global inconsistency. Benítez et al (2015) provided a full matrix termination mechanism for an iPCM produced by an actor.

There is no doubt that we can handle iPCMs to assign priority weights directly by ignoring the missing values (*e.g.* Csató and Rónyai (2016); Jandova et al (2016); Vetschera (2017)). However, the methods that focus on matrix completion (*e.g.* Alonso et al (2008); Büyüközkan and Çifçi (2012); Gomez-Ruiz et al (2010); Liang et al (2017); Wang and Xu (2016); Wang and Li (2016); Zhang (2016)) are more functional since they "repair" the preference relations among alternatives/criteria, which avoids the loss of discarding important information.

Nevertheless, a key problem is hard to avoid in the estimation of missing values: how to evaluate completion methods? Since the initial complete PCMs are unknown, it is impossible to some extent to evaluate by the accuracy of the estimated values. It can be easily seen from the above review that many existing completion methods use consistency ratio/index as the measure of evaluation (*e.g.* Bozóki et al (2010); Chen et al (2015); Fedrizzi and Giove (2007)). In other words, if the matrix has the optimal consistency after completion, the method is the most effective. Though PCMs with optimal consistency is exactly what the decision-makers expect in decision-making, the key consideration in completing an iPCM should be how to "restore" it to its "original" state (even if the original matrix is not very consistent) instead to the "ideal" one. Hence, we suggest to separate the process of estimating missing values and consistency optimization. For many other completion methods supporting this idea, however, complexity (*e.g.* neural network) and unclear physical meaning (*e.g.* pure linear algebra) limit their extensibility and understandability.

Taking into account above issues, our motivation is to propose a new approach to estimate missing values in iPCMs. This method does not regard the optimal consistency as the measure of estimation and can be simple and flexible with explicit physical meaning. DEMATEL, which has ability to derive the Total Relation Matrix (TRM) from Direct Relation Matrix (DRM), is a powerful tool to satisfy all above requirements. Therefore, in this paper, a DEMATEL-based completion method is proposed and consists three simple steps:

Step 1: Convert the iPCM into DRM.

Step 2: Convert the DRM into TRM based on DEMATEL method.

Step 3: Transform the TRM into PCM with reciprocal preference relations.

The rest of this paper is organized as follows. Section 2 introduces some preliminaries of this work. Section 3 illustrates the procedure of the proposed method. In section 4, several experiments are designed and implemented to provide insights into the proposed method and matrix consistency. Section 5 ends the paper with the conclusion.

2 Preliminaries

2.1 AHP

AHP is a multi-criteria decision-making method developed by Saaty (Saaty, 1980) and aims at quantifying relative weights for a given set of criteria on a ratio scale (Xu, 2015). As a decision-making approach, it permits a hierarchical structure of criteria, which provides users with a better focus on specific criteria and sub-criteria when allocating the weights. Besides, AHP has ability to judge the consistency of a PCM to show its potential conflicts in decision-making process (Jiang et al, 2016, 2017). AHP has been widely used in supply chain management (Chan and Kumar, 2007; Chan et al, 2016; Zhang et al, 2016), energy planning (Ishizaka et al, 2016), healthcare analysis (Rouyendegh et al, 2016), risk analysis (Miccoli and Ishizaka, 2017; Zhou et al, 2017a), site selection (Erdogan and Kaya, 2016; Lee et al, 2015; Pourahmad et al, 2015) and performance appraisal (Ishizaka and Pereira, 2016; Wu et al, 2010; Zavadskas et al, 2015). AHP can be extended further by other theories and methods such as fuzzy theory (Rodríguez et al, 2016; Wang et al, 2008), choquet integral (Corrente et al, 2016), VIKOR (Büyüközkan and Görener, 2015) and TOPSIS (Erdogan and Kaya, 2016; Kahraman et al, 2016; Liu et al, 2016).

However, it should be pointed out AHP has some open issues especially in PCMs (Emrouznejad and Marra, 2017), such as missing values, inconsistency and rank reversal (Tomashevskii, 2015). Many state-of-the-art methods have been developed to address them (Brunelli and Fedrizzi, 2015; Brunelli et al, 2013).

Generally, the procedure of AHP consists three steps. Firstly, establish a hierarchical structure by recursively decomposing the decision problem. Secondly, construct PCMs to indicate the relative importance of alternatives/criteria. A numerical rating including nine rank scales is suggested, as shown in Table 1. Thirdly, verify the consistency of PCMs and calculate the priority weights of alternatives. For the completeness of explanation, several basic concepts and formulas are presented as follows.

Table 1 Numerical rating in AHP

Scale	Meaning
1	Equal importance
3	Moderate importance
5	Strong importance
7	Demonstrated importance
9	Extreme importance
2, 4, 6, 8	Intermediate values

Definition 1 *Assume* $\left\{E_1, E_2, \cdots, E_n\right\}$ *are* n *alternatives available for decision-making, the PCM is indicated as* $M = (m_{ij})_{n\times n}\,(i, j = 1, 2, \cdots, n)$ *and satisfies:*

$$m_{ij} = \begin{cases} \frac{1}{mji} & i \neq j \\ 1 & i = j \end{cases} \tag{1}$$

where m_{ij} *represents the relative importance of* E_i *over* E_j*.*

Consistency checking is introduced in AHP to verify the usability of PCMs.

Definition 2 *For a PCM* $M_{n\times n}$*, let* $\lambda_{\max}$ *denote the largest eigenvalue of* M*, consistency index (*CI*) is defined as*

$$CI = \frac{\lambda_{\max} - n}{n - 1} \tag{2}$$

Based on CI*, consistency ratio (*CR*) is defined as*

$$CR = \frac{CI}{RI} \tag{3}$$

where RI *is the random consistency index related to the dimension of matrices, listed in Table 2.*

Table 2 Random consistency index *RI*

n	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.52	0.89	1.12	1.26	1.36	1.41	1.46	1.49

If $CR < 0.1$, the constructed PCM is considered acceptable and the priority weights of alternatives can be obtained by (4). Otherwise, the PCM needs to be reconstructed.

Definition 3 *For a PCM $M_{n\times n}$ with acceptable consistency, suppose $\mathbf{w} = (w_1, w_2, \cdots, w_n)^T$ is the eigenvector of M whose $w_i (i = 1, 2, \cdots, n)$ is indicated as the priority weight of the ith alternative and calculated by*

$$M\mathbf{w} = \lambda_{\max}\mathbf{w} \tag{4}$$

2.2 DEMATEL method

How to model real and complex system is still an open issue (Dong et al, 2017; Zhang et al, 2017). Many methods are presented to addree this issue, including DEMATEL. DEMATEL method was originally developed by Battelle Memorial Institute of Geneva Research Center (Fontela and Gabus, 1976; Gabus and Fontela, 1972). DEMATEL can help managers measure the importance and causal relationship of system components through assessing their direct and indirect relations and constructing a map (Tzeng et al, 2007). Reviewing the former studies, DEMATEL has been successfully applied in many diverse areas such as emergency management (Zhou et al, 2017b), environmental performance (Liou, 2015; Tsai et al, 2015), risk assessment (Mentes et al, 2015), stock selection (Shen and Tzeng, 2015) and supply chain management (Liou et al, 2016; Wu and Chang, 2015; Wu et al, 2017). DEMATEL can be further extended by other theories and methods such as evidence theory (Jiang and Zhan, 2017; Liu et al, 2017), fuzzy theory (Fei et al, 2017), grey theory (Su et al, 2016) and ANP (Tsai and Chou, 2009; Tzeng and Huang, 2012). The procedure of DEMATEL consists five steps:

Step 1: *Define quality feature and establish measurement scale.*

Quality feature is a set of influential characteristics that impact the sophisticated system, which can be determined by literature review, brainstorming and expert evaluation. After defining the influential characteristics in researching system, establish the measurement scale for the causal relationships and pairwise comparisons among influential characteristics. Four levels 0,1,2,3 are suggested, respectively meaning "no impact", "low impact", "high impact" and "extreme high impact". In this step, factors and their direct relations are displayed by a weighted and directed graph.

Step 2: *Extract the DRM of influential factors.*

In this step, transformation from the weighted and directed graph into DRM has been carried out. For n influential characteristics $F_1, F_2, \cdots, F_n$, DRM is denoted as $D = (d_{ij})_{n\times n}\,(i, j = 1, 2, \cdots, n)$, where d_{ij} is the direct relation of F_i over F_j based on the measurement scale.

Step 3: *Normalize the DRM.*

Normalized direct relations of factors are a mapping from d_{ij} to $[0, 1]$, which is calculated by (5).

Definition 4 *For the framework of n influential characteristics* $\{F_1, F_2, \cdots, F_n\}$*, normalized matrix* N *of DRM* $D = (d_{ij})_{n\times n}$ $(i, j = 1, 2, \cdots, n)$ *is obtained by*

$$N = \frac{D}{\max(\sum_{j=1}^{n} d_{ij}, \sum_{i=1}^{n} d_{ij})} \tag{5}$$

Step 4: *Calculate the TRM.*

TRM contains direct and indirect relations among factors. The calculation of TRM through the normalized DRM is shown in Definition 5. There is an interesting view to explain (6): The increase of k from 1 to ∞ can be seen as the process that each pair of elements in DRM gradually finds their indirect relations based on all the known direct relations. Hence, $k = 1$ corresponds to the (normalized) DRM and $k = \infty$ corresponds to the TRM. Based on this viewpoint, in Section 4, we plot the consistency trend with different k to test whether the process that alternatives with unknown direct relations gain their indirect relations is "friendly" to consistency ratio.

Definition 5 *For the framework of n influential characteristics* $\{F_1, F_2, \cdots, F_n\}$*, assume* N *is the normalized DRM, TRM* T *is computed by*

$$\begin{aligned} T &= \lim_{k\to\infty}(N + N^2 + \cdots + N^k) \\ &= \lim_{k\to\infty} N(E - N^k)(E - N)^{-1} \\ &= N(E - N)^{-1} \end{aligned} \tag{6}$$

where O *is a* $n \times n$ *null matrix and* E *is a* $n \times n$ *identity matrix.*

Step 5: *Classify influential factors.*

Based on the sum of each row $R_i (i = 1, 2, \cdots, n)$ and column C_i $(i = 1, 2, \cdots, n)$ of the TRM $T_{n\times n}$, $R_i + C_i$ and $R_i - C_i$ can be obtained. $R_i + C_i$ is defined as the prominence, indicating the importance of the i^{th} influential factor. $R_i - C_i$ classifies the i^{th} influential factor as the cause $(R_i - C_i > 0)$ or effect $(R_i - C_i < 0)$ factor in researching system.

3 The DEMATEL-based completion method for iPCMs in AHP

In this section, a DEMATEL-based completion method for iPCMs in AHP is proposed first. Then an example is presented to show the procedure of the proposed method.

3.1 Procedure of the DEMATEL-based completion method

Assume an iPCM $M = (m_{ij})_{n\times n}$. The procedure of the DEMATEL-based completion method (see Figure 1 and Algorithm 1) consists three steps as below.

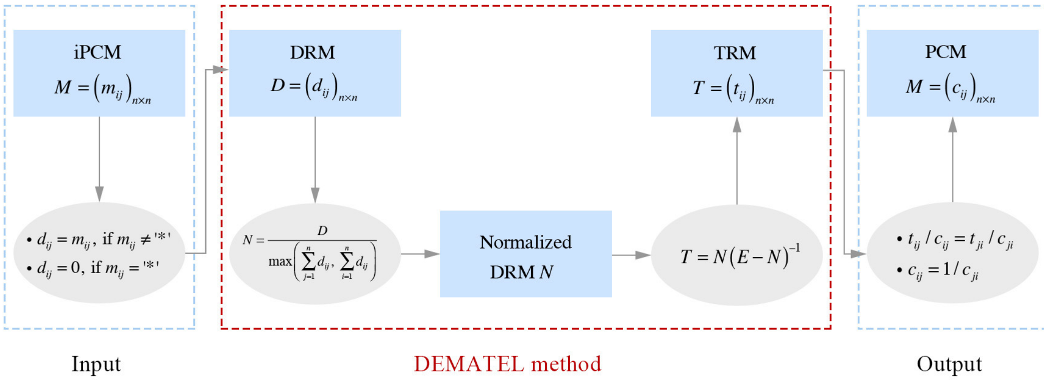


Fig. 1 Procedure of DEMATEL-based completion method

Algorithm 1 The DEMATEL-based completion method for iPCMs in AHP

Input: An iPCM $M = (m_{ij})_{n\times n}(i, j = 1, 2, \cdots, n)$ in AHP;

Output: A complete PCM $M_c = (c_{ij})_{n\times n}(i, j = 1, 2, \cdots, n)$ corresponding to M in AHP;

1: Compute DRM $D = (d_{ij})_{n\times n}$ where $d_{ij} = m_{ij}$ if m_{ij} is a known value and $d_{ij} = 0$ if m_{ij} is an unknown value;

2: Compute the sum of each row $\sum_{j=1}^{n} d_{ij}$ and column $\sum_{i=1}^{n} d_{ij}$ of DRM D;

3: Find the maximum of $\sum_{j=1}^{n} d_{ij}$ and $\sum_{i=1}^{n} d_{ij}$ of D;

4: Compute the normalized DRM $N = \frac{D}{\max(\sum_{j=1}^{n} d_{ij}, \sum_{i=1}^{n} d_{ij})}$;

5: Compute TRM $T = N(E - N)^{-1}$ where E is a $n \times n$ identity matrix;

6: Compute PCM $M_c = (c_{ij})_{n\times n}$ via TRM $T = (t_{ij})_{n\times n}$ where $\frac{t_{ij}}{c_{ij}} = \frac{t_{ji}}{c_{ji}}$ and $c_{ij} = \frac{1}{c_{ji}}$;

Step 1: *Convert the iPCM into DRM.*

PCM reflects the preference relations between each pair of factors in matrix. For example, for a PCM $M_c = (c_{ij})_{n\times n}(i, j = 1, 2, \cdots, n)$, c_{ij} indicates the relative importance (*i.e.* direct relation) of factor i over factor j. Hence the known values in iPCM M can be put into the DRM $D = (d_{ij})_{n\times n}(i, j = 1, 2, \cdots, n)$ directly. For unavailable ones, we use 0 to substitute for them (see (7)).

$$\begin{cases} d_{ij} = m_{ij} & m_{ij} \text{ is a known value} \\ d_{ij} = 0 & m_{ij} \text{ is an unknown value} \end{cases} \tag{7}$$

Step 2: *Convert the DRM into TRM based on DEMATEL.*

(5) and (6) show the calculation from DRM to TRM (see Algorithm 2).

Step 3: *Transform the TRM into PCM.*

In fact, the TRM T obtained in last step has completed the unavailable/missing values in iPCM M. Nevertheless, the PCMs in AHP satisfy the multiplication of each pair of

Algorithm 2 Transformation of TRM from DRM in DEMATEL

Input: A DRM $D = (d_{ij})_{n\times n}(i, j = 1, 2, \cdots, n)$ in DEMATEL;
Output: A TRM $T = (t_{ij})_{n\times n}(i, j = 1, 2, \cdots, n)$ of D in DEMATEL;

Initial $E = \text{eye}(n)$; % Initialize E to a $n \times n$ identity matrix.
for all i **do**
  $SR(i, 1) = \sum_{j=1}^{n} d_{ij}$; % Calculate the sum of each row of D
  $SC(i, 1) = \sum_{j=1}^{n} d_{ji}$; % Calculate the sum of each column of D
end for
$maxsr = \max(SR)$;
$maxsc = \max(SC)$;
$maxsum = \max(maxsr, maxsc)$; % Find the maximum sum of each row and column of D
$N = D/maxsum$; % Normalize the DRM D
$T = (E - N) \setminus N$; % Calculate the TRM T where $T = N(E - N)^{-1}$

symmetric values along the diagonal line is required to equal 1, yet the values in TRM are between 0 and 1. Based on (8), it is feasible to accomplish this transformation from the TRM $T = (t_{ij})_{n\times n}$ to PCM $M_c = (c_{ij})_{n\times n}$ $(i, j = 1, 2, \cdots, n)$ (see Algorithm 3).

$$\begin{cases} \frac{t_{ij}}{c_{ij}} = \frac{t_{ji}}{c_{ji}} \\ c_{ij} = \frac{1}{c_{ji}} \end{cases} \tag{8}$$

Algorithm 3 Transformation of PCM in AHP from TRM in DEMATEL

Input: A TRM $T = (t_{ij})_{n\times n}(i, j = 1, 2, \cdots, n)$ in DEMATEL;
Output: A PCM $M_c = (c_{ij})_{n\times n}(i, j = 1, 2, \cdots, n)$ in AHP;

for $i = 1; i < n; i++$ **do**
  for $j = i; j < n; j++$ **do**
    $c_{ij} = \text{sqrt}(t_{ij}/t_{ji})$;
    $c_{ji} = 1/c_{ij}$;
  end for
end for

Finally, through the complete PCM M_c, the unavailable/missing values in iPCM M are estimated as $m_{ij} = c_{ij}(i, j = 1, 2, \cdots, n)$ where m_{ij} are unknown values in M.

3.2 Example of the DEMATEL-based completion method

In this section, an example is given to demonstrate the procedure of the proposed method (see Example 1).

Example 1 *Assume a 4×4 iPCM M and the unavailable/missing values of M are displayed by '*'.*

$$M = \begin{bmatrix} 1 & * & 4 & 8 \\ * & 1 & 2 & 4 \\ 0.25 & 0.50 & 1 & 2 \\ 0.13 & 0.25 & 0.50 & 1 \end{bmatrix}$$

Firstly, obtain the DRM D through iPCM M, as

$$D = \begin{bmatrix} 1 & 0 & 4 & 8 \\ 0 & 1 & 2 & 4 \\ 0.25 & 0.50 & 1 & 2 \\ 0.13 & 0.25 & 0.50 & 1 \end{bmatrix}$$

Secondly, normalize the DRM D via (5) and then calculate the TRM T via (6), as:

$$N = \begin{bmatrix} 0.07 & 0 & 0.27 & 0.53 \\ 0 & 0.07 & 0.13 & 0.27 \\ 0.02 & 0.03 & 0.07 & 0.13 \\ 0.01 & 0.02 & 0.03 & 0.07 \end{bmatrix}, \ T = \begin{bmatrix} 0.08 & 0.02 & 0.34 & 0.67 \\ 0.01 & 0.08 & 0.17 & 0.34 \\ 0.02 & 0.04 & 0.09 & 0.18 \\ 0.01 & 0.02 & 0.04 & 0.09 \end{bmatrix}$$

Thirdly, transform the TRM T into PCM M_c via (8), as

$$M_c = \begin{bmatrix} 1 & 2 & 4 & 8 \\ 0.50 & 1 & 2 & 4 \\ 0.25 & 0.50 & 1 & 2 \\ 0.13 & 0.25 & 0.50 & 1 \end{bmatrix}$$

Finally, based on the proposed method, the completion for iPCM M has been accomplished as:

$$M = \begin{bmatrix} 1 & * & 4 & 8 \\ * & 1 & 2 & 4 \\ 0.25 & 0.50 & 1 & 2 \\ 0.13 & 0.25 & 0.50 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \underline{\mathbf{2}} & 4 & 8 \\ \underline{\mathbf{0.50}} & 1 & 2 & 4 \\ 0.25 & 0.50 & 1 & 2 \\ 0.13 & 0.25 & 0.50 & 1 \end{bmatrix}$$

4 Empirical study

In this section, a further analysis of the proposed method is presented. We collect a bunch of iPCMs first. Then three experiments are designed and implemented to test the completion ability of the proposed method. Consistency evaluation and some insights into the proposed method are provided as well.

4.1 Matrix collection and processing

We collect 20 initial iPCMs based on the existing literature (see Appendix A). These matrices are from order four to order eight with one pair of missing values initially. Each order has four matrices: three of them (Examples 1-3) are not perfectly consistent ($CR > 0$) and one (*Example 4) is perfectly consistent ($CR = 0$). For each instance (matrix), we use the following removing strategy to generate more iPCMs for experiments: the position of n missing pairs is determined by the known position of $n-1$ missing pairs and the random position of the n^{th} pair, instead of any n pairs of random position. For example, to generate the second iPCM with two pairs of missing values based on $M = (m_{ij})_{4\times4}(i,j = 1,2,3,4)$ (see below), m_{23} and m_{32} should be also removed. The second missing pair can be one of any other pairs randomly.

$$M = \begin{bmatrix} 1 & 0.80 & 1.55 & 1 \\ 1.25 & 1 & * & 3.65 \\ 0.65 & * & 1 & 1.93 \\ 1 & 0.27 & 0.52 & 1 \end{bmatrix}$$

The next considering problem is what the maximum missing entries are tested in experiment (*i.e.* generate how many matrices based on each initial instance). It is related to answering which iPCMs can/cannot be handled by the proposed method. We set the maximum missing entries as 40% of the total entries of matrix. Hence, the statistics of iPCMs for experiments are shown in Table 3.

Table 3 Statistics of iPCMs for experiments

Order	Number of examples		Number of missing pairs	References
	$CR > 0$	$CR = 0$		
4	3	1	1 to 3	Alonso et al (2008); Benítez et al (2015); Bozóki et al (2010);
5	3	1	1 to 4	Chen et al (2015); Cheng et al (2016); Dey and Cheffi (2013);
6	3	1	1 to 6	Ergu et al (2013); Ergu et al (2016); Govindan et al (2014);
7	3	1	1 to 8	Miccoli and Ishizaka (2017); Pinto et al (2017); Saaty (2004);
8	3	1	1 to 11	Santos et al (2017)

4.2 Experiment design

Experiments are designed to mainly answer the following questions:

1. Which iPCMs can/cannot be handled by the proposed method?
2. How is the effect of the proposed method respectively addressing matrices with/without perfect consistency?
3. How does the consistency change with different number of missing values within the same matrix?

4. We have mentioned the interesting view to explain (6): the increase of the k can be seen as the process that each pair of elements in (normalized) DRM gradually finds their indirect relations. Therefore, how does the consistency change with different k? In other words, whether this process is "friendly" to consistency?

Focusing on these problems and based on the collected iPCMs, we design the following three experiments.

Experiment 1: The proposed method is used to estimate the missing values in each iPCM.

Experiment 2: To explore the relation between the number of missing values in iPCMs and their corresponding consistency (CR) after being completed by the proposed method, we divide the whole iPCMs into five groups – the iPCMs with the same order are in the same group. Each group contains four iPCMs – three of them without perfect consistency and one with perfect consistency. Based on the removing strategy mentioned in last subsection, each initial iPCM generates a set of iPCMs with different number of missing pairs. The maximum number of missing pairs in one matrix is 40% of its total entries. Then, we plot a figure for each group – x-axis is the number of missing pairs of iPCMs and y-axis is CR. Each line in a figure represents an example.

Experiment 3: To explore the relation between k in (6) and CR, we divide the whole iPCMs into five groups in the same way. Each group contains four sets of iPCMs with different missing pairs. We plot a figure for each set of iPCMs – x-axis is k and y-axis is CR.

4.3 Experiment results and discussion

Experiment 1: Due to the limited space, only partial completion results are displayed below: Example 2 (represents iPCMs without perfect consistency) and Example 4 (represents iPCMs with perfect consistency) for each order are selected; For each example, the "worst" case of initial iPCMs are selected – all iPCMs have 40% entries removed.

Order 4

Example 2:

$$\begin{bmatrix} 1 & 0.80 & 1.55 & 1 \\ 1.25 & 1 & \underline{\mathbf{1.93}} & \underline{\mathbf{1.25}} \\ 0.65 & \underline{\mathbf{0.52}} & 1 & \underline{\mathbf{0.65}} \\ 1 & \underline{\mathbf{0.80}} & \underline{\mathbf{1.54}} & 1 \end{bmatrix}$$

$CR = 0$

*Example 4:

$$\begin{bmatrix} 1 & \underline{\mathbf{2}} & 4 & \underline{\mathbf{8}} \\ \underline{\mathbf{0.50}} & 1 & 2 & \underline{\mathbf{4}} \\ 0.25 & 0.50 & 1 & 2 \\ \underline{\mathbf{0.13}} & \underline{\mathbf{0.25}} & 0.50 & 1 \end{bmatrix}$$

$CR = 0$

Order 5

Example 2:

$$\begin{bmatrix} 1 & \underline{\mathbf{0.93}} & \underline{\mathbf{1.69}} & 5 & 8 \\ \underline{\mathbf{1.08}} & 1 & 3 & 5 & \underline{\mathbf{9}} \\ \underline{\mathbf{0.59}} & 0.33 & 1 & \underline{\mathbf{1.69}} & 5 \\ 0.20 & 0.20 & \underline{\mathbf{0.59}} & 1 & 3 \\ 0.13 & \underline{\mathbf{0.11}} & 0.20 & 0.33 & 1 \end{bmatrix}$$

$CR = 0.0127$

*Example 4:

$$\begin{bmatrix} 1 & 2 & 2 & 4 & 8 \\ 0.50 & 1 & \underline{\mathbf{1}} & 2 & \underline{\mathbf{4}} \\ 0.50 & \underline{\mathbf{1}} & 1 & 2 & \underline{\mathbf{4}} \\ 0.25 & 0.50 & 0.50 & 1 & \underline{\mathbf{2}} \\ 0.13 & \underline{\mathbf{0.25}} & \underline{\mathbf{0.25}} & \underline{\mathbf{0.50}} & 1 \end{bmatrix}$$

$CR = 0$

Order 6

Example 2:

*Example 4:

$$\begin{bmatrix} 1 & 5 & \underline{\mathbf{3.02}} & 3 & 6 & \underline{\mathbf{1.27}} \\ 0.20 & 1 & 0.33 & \underline{\mathbf{0.39}} & 3 & \underline{\mathbf{0.26}} \\ \underline{\mathbf{0.33}} & 3 & 1 & 0.50 & \underline{\mathbf{3.38}} & 0.33 \\ 0.33 & \underline{\mathbf{2.56}} & 2 & 1 & 5 & \underline{\mathbf{0.80}} \\ 0.17 & 0.33 & \underline{\mathbf{0.30}} & 0.20 & 1 & 0.20 \\ \underline{\mathbf{0.79}} & \underline{\mathbf{3.87}} & 3 & \underline{\mathbf{1.25}} & 5 & 1 \end{bmatrix}$$

$CR = 0.0335$

$$\begin{bmatrix} 1 & \underline{\mathbf{1.50}} & 2.25 & \underline{\mathbf{3.38}} & 5.06 & 7.60 \\ \underline{\mathbf{0.67}} & 1 & 1.50 & \underline{\mathbf{2.25}} & 3.38 & 5.06 \\ 0.44 & 0.67 & 1 & \underline{\mathbf{1.50}} & 2.25 & 3.38 \\ \underline{\mathbf{0.30}} & \underline{\mathbf{0.44}} & \underline{\mathbf{0.67}} & 1 & \underline{\mathbf{1.50}} & 2.25 \\ 0.20 & 0.30 & 0.44 & \underline{\mathbf{0.67}} & 1 & \underline{\mathbf{1.50}} \\ 0.13 & 0.20 & 0.30 & 0.44 & \underline{\mathbf{0.67}} & 1 \end{bmatrix}$$

$CR = 0$

Order 7

Example 2:

$$\begin{bmatrix} 1 & 0.25 & 5 & \underline{\mathbf{0.24}} & 0.33 & 0.50 & \underline{\mathbf{1.20}} \\ 4 & 1 & \underline{\mathbf{6.11}} & 0.33 & \underline{\mathbf{0.42}} & 0.25 & \underline{\mathbf{1.10}} \\ 0.20 & \underline{\mathbf{0.16}} & 1 & 0.14 & 0.14 & \underline{\mathbf{0.18}} & 0.33 \\ \underline{\mathbf{4.13}} & 3 & 7 & 1 & \underline{\mathbf{1.01}} & 2 & 3 \\ 3 & \underline{\mathbf{2.39}} & 7 & \underline{\mathbf{0.99}} & 1 & 2 & 3 \\ 2 & 4 & \underline{\mathbf{5.62}} & 0.50 & 0.50 & 1 & \underline{\mathbf{1.59}} \\ \underline{\mathbf{0.83}} & \underline{\mathbf{0.91}} & 3 & 0.33 & 0.33 & \underline{\mathbf{0.63}} & 1 \end{bmatrix}$$

$CR = 0.0566$

*Example 4:

$$\begin{bmatrix} 1 & \underline{\mathbf{1}} & \underline{\mathbf{1}} & \underline{\mathbf{2}} & 4 & \underline{\mathbf{8}} & \underline{\mathbf{8}} \\ \underline{\mathbf{1}} & 1 & \underline{\mathbf{1}} & \underline{\mathbf{2}} & \underline{\mathbf{4}} & 8 & 8 \\ \underline{\mathbf{1}} & \underline{\mathbf{1}} & 1 & 2 & 4 & 8 & 8 \\ \underline{\mathbf{0.50}} & \underline{\mathbf{0.50}} & 0.50 & 1 & 2 & 4 & 4 \\ 0.25 & \underline{\mathbf{0.25}} & 0.25 & 0.50 & 1 & 2 & 2 \\ \underline{\mathbf{0.13}} & 0.13 & 0.13 & 0.25 & 0.50 & 1 & 1 \\ \underline{\mathbf{0.13}} & 0.13 & 0.13 & 0.25 & 0.50 & 1 & 1 \end{bmatrix}$$

$CR = 0$

Order 8

Example 2:

$$\begin{bmatrix} 1 & \underline{\mathbf{1.78}} & \underline{\mathbf{1.93}} & 7 & 6 & 6 & \underline{\mathbf{0.69}} & \underline{\mathbf{0.81}} \\ \underline{\mathbf{0.56}} & 1 & \underline{\mathbf{1.08}} & 5 & \underline{\mathbf{2.67}} & 3 & \underline{\mathbf{0.40}} & 0.14 \\ \underline{\mathbf{0.52}} & \underline{\mathbf{0.93}} & 1 & 4 & 3 & 3 & 0.17 & \underline{\mathbf{0.42}} \\ 0.14 & 0.2 & 0.25 & 1 & 1 & \underline{\mathbf{0.81}} & 0.11 & 0.13 \\ 0.17 & \underline{\mathbf{0.37}} & 0.33 & 1 & 1 & 1 & \underline{\mathbf{0.11}} & 0.11 \\ 0.17 & 0.33 & 0.33 & \underline{\mathbf{1.23}} & 1 & 1 & 0.11 & 0.17 \\ \underline{\mathbf{1.45}} & \underline{\mathbf{2.52}} & 6 & 9 & \underline{\mathbf{9}} & 9 & 1 & \underline{\mathbf{1.29}} \\ \underline{\mathbf{1.23}} & 7 & \underline{\mathbf{2.37}} & 8 & 9 & 6 & \underline{\mathbf{0.78}} & 1 \end{bmatrix}$$

$CR = 0.0237$

*Example 4:

$$\begin{bmatrix} 1 & 2 & 0.50 & 2 & \underline{\mathbf{0.50}} & 2 & 0.50 & \underline{\mathbf{2}} \\ 0.50 & 1 & \underline{\mathbf{0.25}} & 1 & 0.25 & 1 & \underline{\mathbf{0.25}} & \underline{\mathbf{1}} \\ 2 & \underline{\mathbf{4}} & 1 & 4 & \underline{\mathbf{1}} & 4 & 1 & \underline{\mathbf{4}} \\ 0.50 & 1 & 0.25 & 1 & 0.25 & \underline{\mathbf{1}} & \underline{\mathbf{0.25}} & 1 \\ \underline{\mathbf{2}} & 4 & \underline{\mathbf{1}} & 4 & 1 & 4 & 1 & 4 \\ 0.50 & 1 & 0.25 & \underline{\mathbf{1}} & 0.25 & 1 & 0.25 & \underline{\mathbf{1}} \\ 2 & \underline{\mathbf{4}} & 1 & \underline{\mathbf{4}} & 1 & 4 & 1 & \underline{\mathbf{4}} \\ \underline{\mathbf{0.50}} & \underline{\mathbf{1}} & \underline{\mathbf{0.25}} & 1 & 0.25 & \underline{\mathbf{1}} & \underline{\mathbf{0.25}} & 1 \end{bmatrix}$$

$CR = 0$

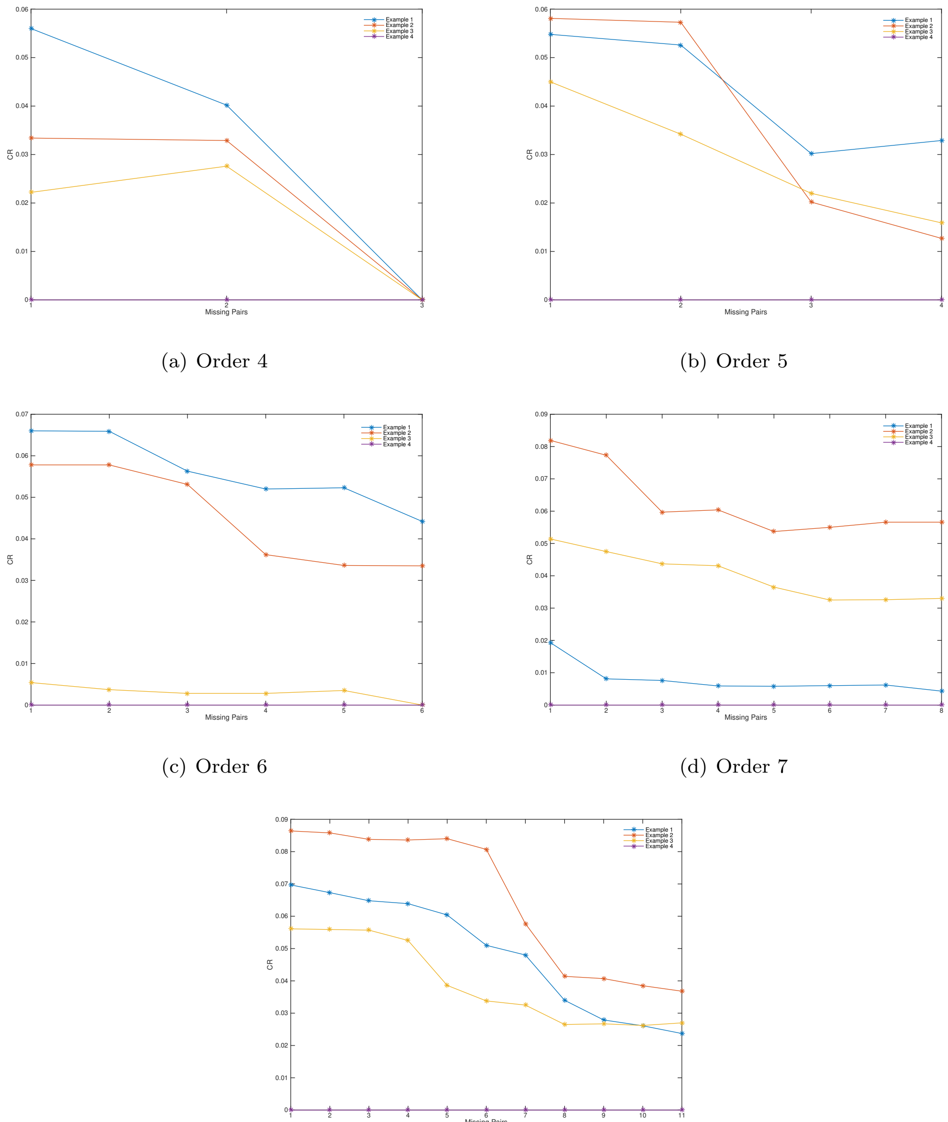
Note: Bold values with underline are estimated values.

The completion results indicate that the proposed method has ability to address the iPCMs within 40% missing values of its total entries (even more). Only one condition is excepted: the (normalized) DRM transformed by iPCM is a singular matrix that cannot be inversed (*i.e.* (6) is inapplicable). However, for these special iPCMs, we come up with a doable solution based on the empirical result of Experiment 3 (conclude in the result of Experiment 3).

Furthermore, we find that if a PCM is perfectly consistent ($CR = 0$), the proposed method can accurately estimate the missing values when one (or more) pair(s) of values is (are) removed from this PCM – nonsingular on the premises.

Experiment 2: The relations are plotted in Figure 2 between the number of missing values in iPCMs and their corresponding consistency (CR) after being completed by the proposed method.

Figure 2 shows a pattern between these two attributes. For matrices without perfect consistency (see Examples 1-3 in each subfigure), their consistency always increases (*i.e.* CR decreases) with the increase number of missing values. For matrices with perfect consistency (see Example 4 in each subfigure), their consistency is always zero no matter how many values are removed. It proves that the estimation of missing values by the proposed method is unique and accurate when the matrix is perfectly consistent (conclude in the result of Experiment 1). This empirical



(a) Order 4

(b) Order 5

(c) Order 6

(d) Order 7

(e) Order 8

Fig. 2 Missing pairs *v.s.* CR

result also shows that based on the proposed method, iPCMs tend to be more consistent with less information.

Besides, we notice the change of slope in each line, which means the change of consistency when an additional pair of values is removed from the matrix. The slope can be helpful to evaluate the contribution of each pair of values to the matrix consistency. Take a PCM $M_c =$

$(c_{ij})_{4\times4}(i,j=1,2,3,4)$ as an example (see below), we respectively remove each pair of values, then use the proposed method to complete the matrix and obtain the corresponding consistency:

$$M_c = \begin{bmatrix} 1 & 0.50 & 4 & 8 \\ 2 & 1 & 2 & 4 \\ 0.25 & 0.50 & 1 & 2 \\ 0.13 & 0.25 & 0.50 & 1 \end{bmatrix}$$

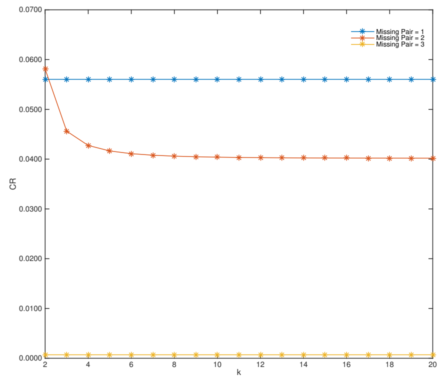
Initial consistency:	$CR = 0.0933$
Remove c_{12} and c_{21}:	$CR' = 0$
Remove c_{13} and c_{31}:	$CR' = 0.0695$
Remove c_{14} and c_{41}:	$CR' = 0.0695$
Remove c_{23} and c_{32}:	$CR' = 0.0695$
Remove c_{24} and c_{42}:	$CR' = 0.0695$
Remove c_{34} and c_{43}:	$CR' = 0.0933$

Since removing c_{12} and c_{21} helps to obtain the largest change of CR ($\Delta = CR - CR'$), c_{12} and c_{21} should be the pair of values contributing most to the inconsistency of the initial PCM M_c. To some extent, this discovery is valuable for consistency optimization of PCMs and we will peruse it as part of our future work.

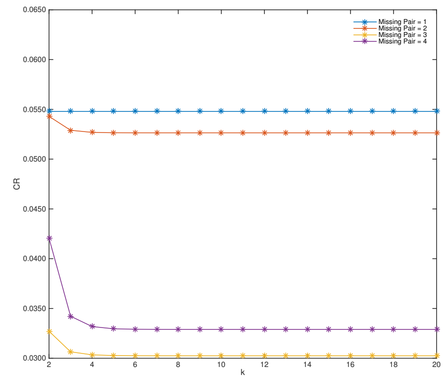
Experiment 3: Due to the limited space, only partial results are displayed in Figure 3: Example 1 (represents iPCMs without perfect consistency) and Example 4 (represents iPCMs with perfect consistency) for each order are selected. The others can be seen in Appendix B.

The results have two indications. First, with the increase of k, the CR of the iPCMs completed by the proposed method decreases and converges. It means the consistency of the iPCMs completed by the proposed method is optimizing in the process of obtaining the indirect relations among alternatives whose relative importance (*i.e.* direct relation) is unknown. In other words, the proposed method is friendly to the matrix consistency. Second, the speed of converge is very quick – converge with $k \geq 5$ in most cases. Therefore, if an iPCM is singular (*i.e.* it does not have inverse matrix and its corresponding TRM cannot be calculated by (6)), it is feasible to let k be a certain integer (*e.g.* $k = 5$) and calculate the approximate TRM by $T = N + N^2 + \cdots + N^k$. Note that though hypothesis of nonsingular matrices has been freed, the proposed method (even every completion method) can do nothing for one condition of missing values in iPCMs: one (or more) row(s)/column(s) of values are all missing (without considering the diagonal elements). Hence for a $n \times n$ iPCM, the proposed method must be able to forecast all the missing values when the number of missing values is less than $2n - 1$. When the number of missing values is higher than or equal to $2n - 1$, we cannot promise the proposed method must be in a position to complete the matrix.

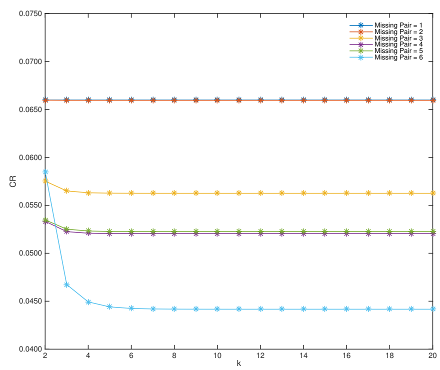
In addition, an additional advantage of the proposed method should be mentioned: it is sufficiently flexible. In this paper, the proposed method is utilized to address iPCMs with reciprocal (multiplicative) preference relations. Since the existence of fuzzy DEMATEL (Wu and Lee, 2007), the proposed method can be extended to address iPCMs with fuzzy preference relations. To handle iPCMs with additive preference relations, the extension of the proposed method can merely concentrate on the last step (*i.e.* transforming the TRM into PCM). These are part of our future work.



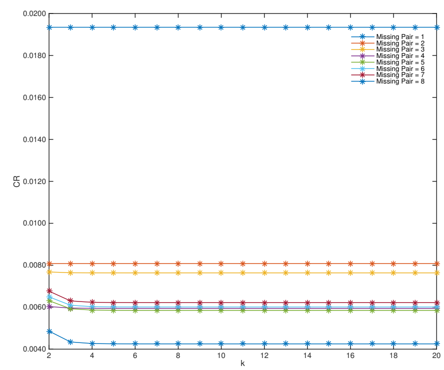
(a) Order 4: Example 1



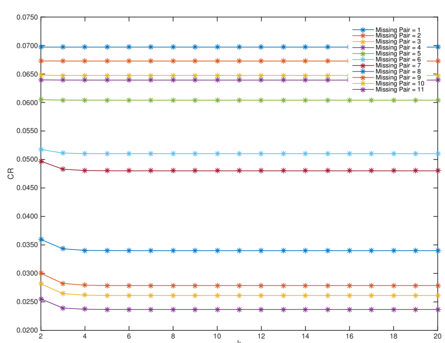
(b) Order 5: Example 1



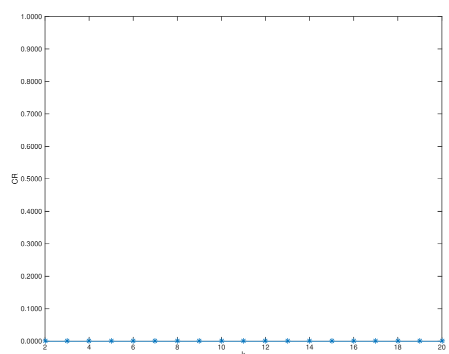
(c) Order 6: Example 1



(d) Order 7: Example 1



(e) Order 8: Example 1



(f) Order 4-8: Example 4

Fig. 3 *k v.s. CR*

We should also mention the limitation of the proposed method. Since it involves matrix multiplication, the general time complexity of the proposed method is $O(n^3)$ where n is the matrix order. Divide-and-conquer strategy (Strassen, 1969) is suggested to reduce the time complexity when n is large. Besides, since the proposed method separate the process of estimating missing values and consistency optimization (*i.e.* optimal consistency is not the measure of estimation), some iPCMs will be inconsistent after completion. For example,

$$M = \begin{bmatrix} 1 & 6 & 8 & 2 \\ 0.17 & 1 & * & 3 \\ 0.13 & * & 1 & 2 \\ 0.50 & 0.33 & 0.50 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 6 & 8 & 2 \\ 0.17 & 1 & \underline{\mathbf{1.41}} & 3 \\ 0.13 & \underline{\mathbf{0.71}} & 1 & 2 \\ 0.50 & 0.33 & 0.50 & 1 \end{bmatrix} (CR = 0.2323)$$

Note that since we have proved the proposed method is friendly to the matrix consistency, the cause of this problem is not the proposed method but the known values in iPCM M have many conflicts. For this problem, many existing methods (*e.g.* Ergu et al (2016)) can be used to optimize consistency of matrix. It is not in the scope of this paper.

5 Conclusion

PCMs play a pivotal role in AHP. However, in many cases, only partial information in a PCM is available, which obstructs the subsequent operations of the classical AHP. In this paper, a DEMATEL-based completion method for iPCMs in AHP is proposed. The proposed method provides a new perspective to estimate the missing values in iPCMs with explicit physical meaning: calculate the indirect relation of two alternatives/criteria if their relative importance (*i.e.* direct relation) is unknown. Experimental simulation proves the proposed method can well address the matrices especially with perfect consistency on the premises that one (or more) row(s)/column(s) of values are not all missing (without considering the diagonal elements). Besides, the proposed method is simple, flexible and friendly to the matrix consistency. In our further study, we would extend the proposed method for iPCMs with various preference relations such as additive, fuzzy, interval-valued and linguistic preference relations.

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References

Alonso S, Chiclana F, Herrera F, Herrera-Viedma E, Alcalá-Fdez J, Porcel C (2008) A consistency-based procedure to estimate missing pairwise preference values. International Journal of Intelligent Systems 23(2):155–175

Benítez J, Delgado-Galván X, Izquierdo J, Pérez-García R (2015) Consistent completion of incomplete judgments in decision making using AHP. Journal of Computational and Applied Mathematics 290:412–422

Bozóki S, Fülöp J, Rónyai L (2010) On optimal completion of incomplete pairwise comparison matrices. Mathematical and Computer Modelling 52(1):318–333

Brunelli M, Fedrizzi M (2015) Boundary properties of the inconsistency of pairwise comparisons in group decisions. European Journal of Operational Research 240(3):765–773

Brunelli M, Canal L, Fedrizzi M (2013) Inconsistency indices for pairwise comparison matrices: a numerical study. Annals of Operations Research 211(1):493–509

Büyüközkan G, Çifçi G (2012) A new incomplete preference relations based approach to quality function deployment. Information Sciences 206:30–41

Büyüközkan G, Görener A (2015) Evaluation of product development partners using an integrated AHP-VIKOR model. Kybernetes 44(2):220–237

Chan FT, Kumar N (2007) Global supplier development considering risk factors using fuzzy extended AHP-based approach. Omega 35(4):417–431

Chan FT, Jha A, Tiwari MK (2016) Bi-objective optimization of three echelon supply chain involving truck selection and loading using NSGA-II with heuristics algorithm. Applied Soft Computing 38:978–987

Chen K, Kou G, Tarn JM, Song Y (2015) Bridging the gap between missing and inconsistent values in eliciting preference from pairwise comparison matrices. Annals of Operations Research 235(1):155–175

Cheng B, Hu S, Wang JW, Dong YY (2016) A goal programming approach for group decision making with four incomplete preference relations. In: Intelligent Networking and Collaborative Systems (INCoS), 2016 International Conference on, IEEE, pp 435–438

Corrente S, Greco S, Ishizaka A (2016) Combining analytical hierarchy process and choquet integral within non-additive robust ordinal regression. Omega 61:2–18

Csató L, Rónyai L (2016) Incomplete pairwise comparison matrices and weighting methods. Fundamenta Informaticae 144(3-4):309–320

Dey PK, Cheffi W (2013) Green supply chain performance measurement using the analytic hierarchy process: a comparative analysis of manufacturing organisations. Production Planning & Control 24(8-9):702–720

Dong Y, Wang J, Chen F, Hu Y, Deng Y (2017) Location of facility based on simulated annealing and "ZKW" algorithms. Mathematical Problems in Engineering 2017:Article ID 4628,501, DOI 10.1155/2017/4628501

Emrouznejad A, Marra M (2017) The state of the art development of AHP (1979–2017): a literature review with a social network analysis. International Journal of Production Research pp 1–23

Erdogan M, Kaya I (2016) A combined fuzzy approach to determine the best region for a nuclear power plant in Turkey. Applied Soft Computing 39:84–93

Erdogan M, Kaya I (2016) Evaluating Alternative-Fuel Busses for Public Transportation in Istanbul Using Interval Type-2 Fuzzy AHP and TOPSIS. Journal of Multiple-Valued Logic and Soft Computing 26(6):625–642

Ergu D, Kou G, Peng Y, Shi Y, Shi Y (2013) The analytic hierarchy process: task scheduling and resource allocation in cloud computing environment. The Journal of Supercomputing pp

1–14

Ergu D, Kou G, Peng Y, Zhang M (2016) Estimating the missing values for the incomplete decision matrix and consistency optimization in emergency management. Applied Mathematical Modelling 40(1):254–267

Fedrizzi M, Giove S (2007) Incomplete pairwise comparison and consistency optimization. European Journal of Operational Research 183(1):303–313

Fei L, Wang H, Chen L, Deng Y (2017) A new vector valued similarity measure for intuitionistic fuzzy sets based on OWA operators. Iranian Journal of Fuzzy Systems accepted

Fontela E, Gabus A (1976) The DEMATEL observer. Battelle Geneva Research Centre, Geneva

Gabus A, Fontela E (1972) World problems, an invitation to further thought within the framework of DEMATEL. Battelle Geneva Research Center, Geneva, Switzerland

Gomez-Ruiz JA, Karanik M, Peláez JI (2010) Estimation of missing judgments in AHP pairwise matrices using a neural network-based model. Applied Mathematics and Computation 216(10):2959–2975

Govindan K, Kaliyan M, Kannan D, Haq AN (2014) Barriers analysis for green supply chain management implementation in indian industries using analytic hierarchy process. International Journal of Production Economics 147:555–568

Harker PT (1987) Incomplete pairwise comparisons in the analytic hierarchy process. Mathematical Modelling 9(11):837–848

Ishizaka A, Labib A (2011) Review of the main developments in the analytic hierarchy process. Expert Systems with Applications 38(11):14,336–14,345

Ishizaka A, Pereira VE (2016) Portraying an employee performance management system based on multi-criteria decision analysis and visual techniques. International Journal of Manpower 37(4):628–659

Ishizaka A, Siraj S, Nemery P (2016) Which energy mix for the UK (United Kingdom)? An evolutive descriptive mapping with the integrated GAIA (graphical analysis for interactive aid)–AHP (analytic hierarchy process) visualization tool. Energy 95:602–611

Jandova V, Krejci J, Stoklasa J, Fedrizzi M (2016) Computing interval weights for incomplete pairwise-comparison matrices of large dimension - a weak consistency based approach. IEEE Transactions on Fuzzy Systems PP(99):1–1

Jiang W, Zhan J (2017) A modified combination rule in generalized evidence theory. Applied Intelligence 46(3):630–640

Jiang W, Wei B, Zhan J, Xie C, Zhou D (2016) A visibility graph power averaging aggregation operator: A methodology based on network analysis. Computers & Industrial Engineering 101:260–268

Jiang W, Wei B, Tang Y, Zhou D (2017) Ordered visibility graph average aggregation operator: An application in produced water management. Chaos: An Interdisciplinary Journal of Nonlinear Science 27(2):023,117

Kahraman C, Suder A, Bekar ET (2016) Fuzzy multiattribute consumer choice among health insurance options. Technological and Economic Development of Economy 22(1):1–20

Lee AH, Kang HY, Lin CY, Shen KC (2015) An Integrated Decision-Making Model for the Location of a PV Solar Plant. Sustainability 7(10):13,522–13,541

Liang Q, Liao X, Liu J (2017) A social ties-based approach for group decision-making problems with incomplete additive preference relations. Knowledge-Based Systems 119:68–86

Liou JJH (2015) Building an effective system for carbon reduction management. Journal of Cleaner Production 103:353–361

Liou JJH, Tamosaitiene J, Zavadskas EK, Tzeng GH (2016) New hybrid COPRAS-G MADM Model for improving and selecting suppliers in green supply chain management. International Journal of Production Research 54(1, SI):114–134

Liu S, Chan FT, Ran W (2016) Decision making for the selection of cloud vendor: An improved approach under group decision-making with integrated weights and objective/subjective attributes. Expert Systems with Applications 55:37–47

Liu T, Deng Y, Chan F (2017) Evidential supplier selection based on DEMATEL and game theory. International Journal of Fuzzy Systems pp DOI: 10.1007/s40,815–017–0400–4

Mentes A, Akyildiz H, Yetkin M, Turkoglu N (2015) A FSA based fuzzy DEMATEL approach for risk assessment of cargo ships at coasts and open seas of Turkey. Safety Science 79:1–10

Miccoli F, Ishizaka A (2017) Sorting municipalities in Umbria according to the risk of wolf attacks with AHPSort II. Ecological Indicators 73:741–755

Mieza IZ, Yepes V, Moreno-Jiménez JM (2017) A systematic review of application of multi-criteria decision analysis for aging-dam management. Journal of Cleaner Production 147:217–230

Moreno-Jiménez JM, Salvador M, Gargallo P, Altuzarra A (2016) Systemic decision making in AHP: a Bayesian approach. Annals of Operations Research 245(1-2):261–284

Pinto D, Shrestha S, Babel MS, Ninsawat S (2017) Delineation of groundwater potential zones in the Comoro watershed, Timor Leste using GIS, remote sensing and analytic hierarchy process (AHP) technique. Applied Water Science 7(1):503–519

Pourahmad A, Hosseini A, Banaitis A, Nasiri H, Banaitiene N, Tzeng GH (2015) Combination of fuzzy-AHP and DEMATEL-ANP with GIS in a new hybrid MCDM model used for the selection of the best space for leisure in a blighted urban site. Technological and Economic Development of Economy 21(5):773–796

Rodríguez RM, Bedregal B, Bustince H, Dong Y, Farhadinia B, Kahraman C, Martínez L, Torra V, Xu Y, Xu Z, et al (2016) A position and perspective analysis of hesitant fuzzy sets on information fusion in decision making. Towards high quality progress. Information Fusion 29:89–97

Rouyendegh BD, Oztekin A, Ekong J, Dag A (2016) Measuring the efficiency of hospitals: a fully-ranking DEA–FAHP approach. Annals of Operations Research pp doi:10.1007/s10,479–016–2330–1

Saaty TL (1980) The analytic hierarchy process: planning, priority setting, resources allocation. New York: McGraw

Saaty TL (2004) Decision makingthe analytic hierarchy and network processes (AHP/ANP). Journal of systems science and systems engineering 13(1):1–35

Santos LFdOM, Osiro L, Lima RHP (2017) A model based on 2-tuple fuzzy linguistic representation and Analytic Hierarchy Process for supplier segmentation using qualitative and quantitative criteria. Expert Systems with Applications 79:53–64

Shen KY, Tzeng GH (2015) Combined soft computing model for value stock selection based on fundamental analysis. Applied Soft Computing 37:142–155

Strassen V (1969) Gaussian elimination is not optimal. Numerische mathematik 13(4):354–356

Su CM, Horng DJ, Tseng ML, Chiu AS, Wu KJ, Chen HP (2016) Improving sustainable supply chain management using a novel hierarchical grey-DEMATEL approach. Journal of Cleaner Production 134:469–481

Tomashevskii I (2015) Eigenvector ranking method as a measuring tool: Formulas for errors. European Journal of Operational Research 240(3):774–780

Tsai SB, Chien MF, Xue Y, Li L, Jiang X, Chen Q, Zhou J, Wang L (2015) Using the Fuzzy DEMATEL to Determine Environmental Performance: A Case of Printed Circuit Board Industry in Taiwan. PloS one 10(6):e0129,153

Tsai WH, Chou WC (2009) Selecting management systems for sustainable development in SMEs: A novel hybrid model based on DEMATEL, ANP, and ZOGP. Expert Systems with Applications 36(2):1444–1458

Tzeng GH, Huang CY (2012) Combined DEMATEL technique with hybrid MCDM methods for creating the aspired intelligent global manufacturing & logistics systems. Annals of Operations Research 197(1):159–190

Tzeng GH, Chiang CH, Li CW (2007) Evaluating intertwined effects in e-learning programs: A novel hybrid MCDM model based on factor analysis and DEMATEL. Expert Systems with Applications 32(4):1028–1044

Vetschera R (2017) Deriving rankings from incomplete preference information: a comparison of different approaches. European Journal of Operational Research 258(1):244–253

Wang H, Xu Z (2016) Interactive algorithms for improving incomplete linguistic preference relations based on consistency measures. Applied Soft Computing 42:66–79

Wang YM, Luo Y, Hua Z (2008) On the extent analysis method for fuzzy AHP and its applications. European Journal of Operational Research 186(2):735–747

Wang ZJ, Li KW (2016) Group decision making with incomplete intuitionistic preference relations based on quadratic programming models. Computers & Industrial Engineering 93:162–170

Wu CR, Lin CT, Tsai PH (2010) Evaluating business performance of wealth management banks. European Journal of Operational Research 207(2):971–979

Wu HH, Chang SY (2015) A case study of using DEMATEL method to identify critical factors in green supply chain management. Applied Mathematics and Computation 256:394–403

Wu KJ, Liao CJ, Tseng ML, Lim MK, Hu J, Tan K (2017) Toward sustainability: using big data to explore the decisive attributes of supply chain risks and uncertainties. Journal of Cleaner Production 142:663–676

Wu WW, Lee YT (2007) Developing global managers competencies using the fuzzy DEMATEL method. Expert Systems with Applications 32(2):499–507

Xu Z (2015) Deviation square priority method for distinct preference structures based on generalized multiplicative consistency. IEEE Transactions on Fuzzy Systems 23(4):1164–1180

Zavadskas EK, Turskis Z, Antucheviciene J (2015) Selecting a Contractor by Using a Novel Method for Multiple Attribute Analysis: Weighted Aggregated Sum Product Assessment with Grey Values (WASPAS-G). Studies in Informatics and Control 24(2):141–150

Zhang Q, Li M, Deng Y (2017) Measure the structure similarity of nodes in complex networks based on relative entropy. Physica A: Statistical Mechanics and its Applications p 10.1016/j.physa.2017.09.042

Zhang X, Deng Y, Chan FTS, Adamatzky A, Mahadevan S (2016) Supplier selection based on evidence theory and analytic network process. Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture 230(3):562–573

Zhang Z (2016) Logarithmic least squares approaches to deriving interval weights, rectifying inconsistency and estimating missing values for interval multiplicative preference relations. Soft Computing pp doi:10.1007/s00,500–016–2049–6

Zhou X, Deng X, Deng Y, Mahadevan S (2017a) Dependence assessment in human reliability analysis based on D numbers and AHP. Nuclear Engineering and Design 313:243–252

Zhou X, Shi Y, Deng X, Deng Y (2017b) D-DEMATEL: A new method to identify critical success factors in emergency management. Safety Science 91:93–104

A Initial iPCMs

Order 4

Example 1:

1	1	5	2
1	1	3	4
0.20	0.33	1	*
0.50	0.25	*	1

Example 2:

1	0.80	1.55	1
1.25	1	*	3.65
0.65	*	1	1.93
1	0.27	0.52	1

Example 3:

1	0.33	0.25	0.11
3	1	*	0.14
4	*	1	0.25
9	7	4	1

*Example 4:

1	2	4	*
0.50	1	2	4
0.25	0.50	1	2
*	0.25	0.50	1

Order 5

Example 1:

1	3	5	5	9
0.33	1	3	4	6
0.20	0.33	1	*	5
0.20	0.25	*	1	5
0.11	0.17	0.20	0.20	1

Example 2:

1	*	3	5	8
*	1	3	5	7
0.33	0.33	1	0.50	5
0.20	0.20	2	1	3
0.13	0.14	0.20	0.33	1

Example 3:

1	0.20	3	0.50	5
5	1	*	1	7
0.33	*	1	0.25	3
2	1	4	1	7
0.20	0.14	0.33	0.14	1

*Example 4:

1	2	2	4	8
0.50	1	1	2	*
0.50	1	1	2	4
0.25	0.50	0.50	1	2
0.13	*	0.25	0.50	1

Order 6

Example 1:

1	4	0.33	4	7	0.25
0.25	1	*	2	5	0.33
3	*	1	6	7	1
0.25	0.50	0.17	1	3	0.25
0.14	0.20	0.14	0.33	1	0.14
4	3	1	4	7	1

Example 2:

1	5	*	3	6	2
0.20	1	0.33	0.33	3	0.25
*	3	1	0.50	5	0.33
0.33	3	2	1	5	2
0.17	0.33	0.20	0.20	1	0.20
0.50	4	3	0.50	5	1

Example 3:

1	3	2	1	3	3
0.33	1	0.50	0.33	2	1
0.50	2	1	0.50	*	2
1	3	2	1	3	2
0.25	0.50	*	0.33	1	0.50
0.33	1	0.50	0.50	2	1

*Example 4:

1	1.50	2.25	*	5.06	7.60
0.67	1	1.50	2.25	3.38	5.06
0.44	0.67	1	1.50	2.25	3.38
*	0.44	0.67	1	1.50	2.25
0.20	0.30	0.44	0.67	1	1.50
0.13	0.20	0.30	0.44	0.67	1

Order 7

Example 1:

1	9	5	2	1	1	0.50
0.11	1	0.33	0.11	0.11	0.11	0.11
0.20	3	1	0.33	*	0.33	0.11
0.50	9	3	1	0.50	1	0.33
1	9	*	2	1	2	0.50
1	9	3	1	0.50	1	0.33
2	9	9	3	2	3	1

Example 2:

1	0.25	5	0.14	0.33	0.50	0.50
4	1	*	0.33	0.33	0.25	0.33
0.20	*	1	0.14	0.14	0.17	0.33
7	3	7	1	0.50	2	3
3	3	7	2	1	2	3
2	4	6	0.50	0.50	1	2
2	3	3	0.33	0.33	0.50	1

Example 3:

1	3	0.33	*	0.25	0.33	3
0.33	1	0.14	0.14	0.17	0.33	2
3	7	1	0.50	2	3	3
*	7	2	1	2	3	5
4	6	0.50	0.50	1	2	5
3	3	0.33	0.33	0.50	1	3
0.33	0.50	0.33	0.20	0.20	0.33	1

*Example 4:

1	0	1	2	4	8	8
0	1	1	2	4	8	8
1	1	1	2	4	8	8
0.50	0.50	0.50	1	2	4	4
0.25	0.25	0.25	0.50	1	2	2
0.13	0.13	0.13	0.25	0.50	1	1
0.13	0.13	0.13	0.25	0.50	1	1

Order 8

Example 1:

1	5	5	0.14	0.33	0.50	0.50	2
0.20	1	3	0.33	0.33	0.25	0.33	3
0.20	0.33	1	0.14	0.14	0.17	0.33	2
7	3	7	1	0.50	2	3	3
3	3	7	2	1	2	3	5
2	4	6	0.50	0.50	1	2	*
2	3	3	0.33	0.33	0.50	1	3
0.50	0.33	0.50	0.33	0.20	*	0.33	1

Example 2:

1	5	3	7	6	6	0.33	0.25
0.20	1	0.50	5	*	3	0.14	0.14
0.33	2	1	4	3	3	0.17	0.17
0.14	0.20	0.25	1	1	0.25	0.11	0.13
0.17	*	0.33	1	1	1	0.20	0.11
0.17	0.33	0.33	4	1	1	0.11	0.17
3	7	6	9	5	9	1	0.50
4	7	6	8	9	6	2	1

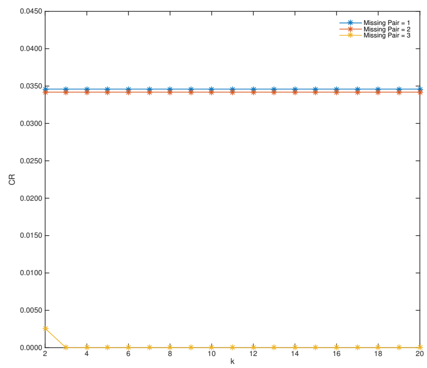
Example 3:

$$\begin{bmatrix} 1 & 0.78 & 2.73 & 0.66 & 2.48 & 3.65 & 7.78 & 9 \\ 1.28 & 1 & 2.89 & 3.70 & 2.89 & 5 & 8 & 8.12 \\ 0.37 & 0.35 & 1 & 1.66 & 2 & 2.65 & 7.37 & 8.85 \\ 1.52 & 0.27 & 0.60 & 1 & * & 3.18 & 8.81 & 7.22 \\ 0.40 & 0.35 & 0.50 & * & 1 & 1.42 & 4 & 7.75 \\ 0.27 & 0.20 & 0.38 & 0.31 & 0.70 & 1 & 3 & 5 \\ 0.13 & 0.13 & 0.14 & 0.11 & 0.25 & 0.33 & 1 & 4 \\ 0.11 & 0.12 & 0.11 & 0.14 & 0.13 & 0.20 & 0.25 & 1 \end{bmatrix}$$

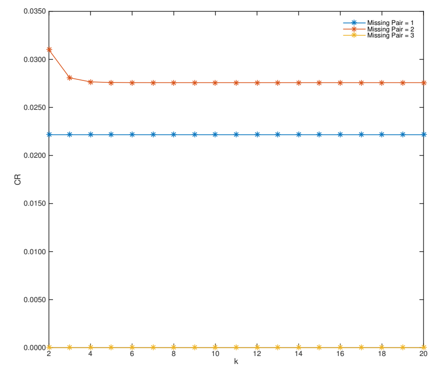
*Example 4:

$$\begin{bmatrix} 1 & 2 & 0.50 & 2 & 0.50 & 2 & 0.50 & 2 \\ 0.50 & 1 & 0.25 & 1 & 0.25 & 1 & 0.25 & 1 \\ 2 & 4 & 1 & 4 & 1 & 4 & 1 & 4 \\ 0.50 & 1 & 0.25 & 1 & 0.25 & 1 & 0.25 & 1 \\ 2 & 4 & 1 & 4 & 1 & 4 & 1 & 4 \\ 0.50 & 1 & 0.25 & 1 & 0.25 & 1 & 0.25 & 1 \\ 2 & 4 & 1 & 4 & 1 & 4 & 1 & * \\ 0.50 & 1 & 0.25 & 1 & 0.25 & 1 & * & 1 \end{bmatrix}$$

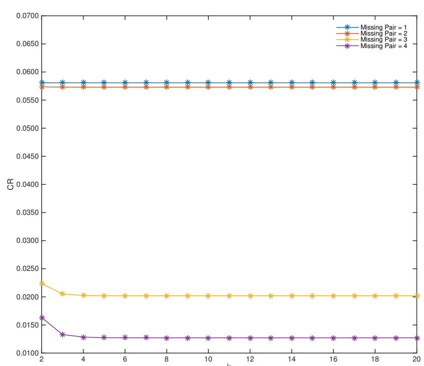
B Relation between k and CR



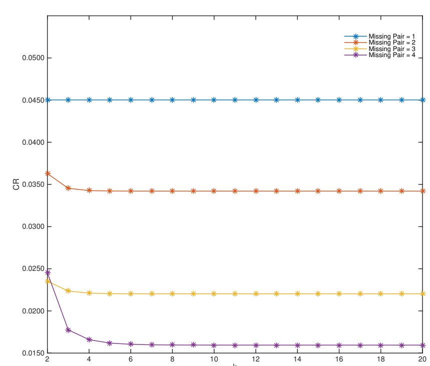
(a) Order 4: Example 2



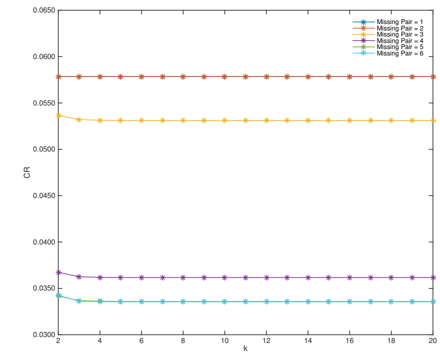
(b) Order 4: Example 3



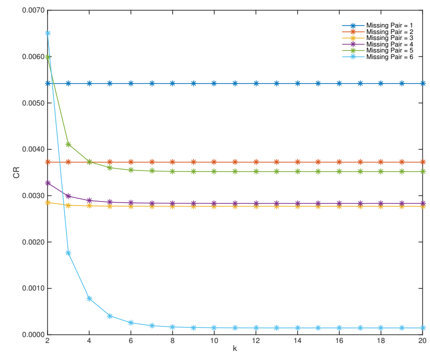
(c) Order 5: Example 2



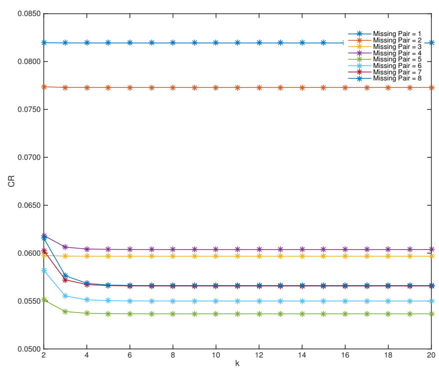
(d) Order 5: Example 3



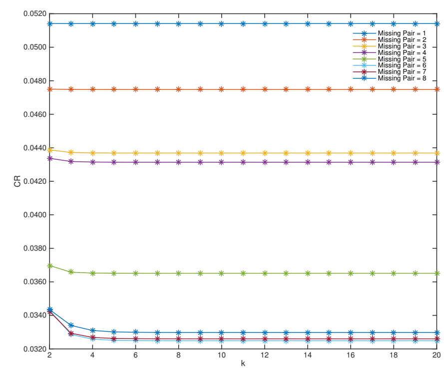
(e) Order 6: Example 2



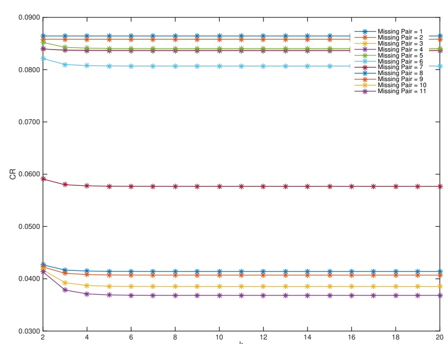
(f) Order 6: Example 3



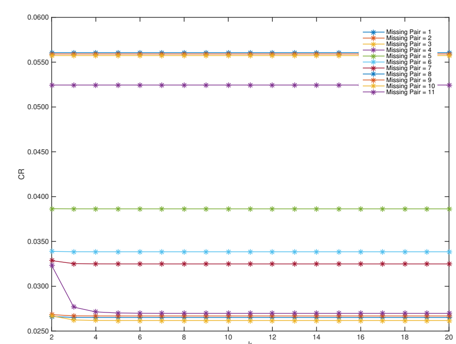
(g) Order 7: Example 2



(h) Order 7: Example 3



(i) Order 8: Example 2



(j) Order 8: Example 3

Fig. 4 k *v.s.* CR