



A Measurement of the Hubble Constant Using Galaxy Redshift Surveys

Yuting Wang^{1,2}, Lixin Xu³, and Gong-Bo Zhao^{1,2}

¹National Astronomy Observatories, Chinese Academy of Science, Beijing, 100012, China; ytwang@nao.cas.cn, lx Xu@dlut.edu.cn

²Institute of Cosmology and Gravitation, University of Portsmouth, Portsmouth, PO1 3FX, UK; gbzhao@nao.cas.cn

³Institute of Theoretical Physics, School of Physics & Optoelectronic Technology, Dalian University of Technology, Dalian, 116024, China
Received 2017 July 3; revised 2017 September 11; accepted 2017 September 20; published 2017 November 6

Abstract

We perform a measurement of the Hubble constant, H_0 , using the latest baryonic acoustic oscillation (BAO) measurements from galaxy surveys of 6dFGS, SDSS DR7 Main Galaxy Sample, BOSS DR12 sample, and eBOSS DR14 quasar sample, in the framework of a flat Λ CDM model. Based on the Kullback–Leibler divergence, we examine the consistency of H_0 values derived from various data sets. We find that our measurement is consistent with that derived from Planck and with the local measurement of H_0 using the Cepheids and type Ia supernovae. We perform forecasts on H_0 from future BAO measurements, and find that the uncertainty of H_0 determined by future BAO data alone, including complete eBOSS, DESI, and Euclid-like, is comparable with that from local measurements.

Key words: cosmological parameters – distance scale – large-scale structure of universe

1. Introduction

Determining the Hubble constant, H_0 , which is the present expansion rate of the Universe, with high precision plays a crucial role in cosmology, and H_0 can be measured locally, or derived cosmologically through measurements of the Cosmic Microwave Background (CMB) and Baryon Acoustic Oscillations (BAO; see Freedman & Madore 2010; Freedman 2017 for a recent review on astronomical methods of H_0 measurements and their significance in cosmology).

Recently, a direct measurement of H_0 led by Riess et al. (2016; R16) using Cepheids and type Ia supernovae finds $H_0 = 73.24 \pm 1.74 \text{ km s}^{-1} \text{ Mpc}^{-1}$, which is a 2.4% measurement. On the other hand, a recent CMB measurement of H_0 using the Planck satellite (PLC15) achieved a percent level precision, namely, $H_0 = 67.27 \pm 0.66 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (Planck Collaboration et al. 2016). Note that, unlike the local measurement, the CMB measurement of the Hubble constant is model-dependent as a cosmological model, which is Λ CDM used for the measurement we quote here, is needed to convert the observed angular diameter distance at $z \sim 1100$ and the sound horizon into a measurement of H_0 .

These two measurements are in apparent tension at a more than 3σ level (Riess et al. 2016). The tension may imply that the Λ CDM used in the CMB analysis needs to be extended (Wyman et al. 2014; Wang et al. 2015; Di Valentino et al. 2016; Poursidou & Tram 2016; Sola et al. 2017; Zhao et al. 2017a), or that the measurements were contaminated by systematics to an unknown level. In this situation, additional independent measurements of H_0 , e.g., using BAO distance measurements derived from galaxy surveys⁴ (Cheng & Huang 2015), can provide the critical information that we need.

The BAO distance measurements using galaxy redshift surveys play a key role in probing the cosmic expansion history. The BAO characteristic scale can be measured in both radial and transverse directions of the line of sight to provide estimates of the Hubble parameter, $H(z)$, and angular diameter distance, $D_A(z)$, respectively, at redshift z . Recently,

the collaboration of the Baryon Oscillation Spectroscopic Survey (BOSS), which is a part of the Sloan Digital Sky Survey (SDSS)-III, performed BAO measurements in the redshift range of $0.2 < z < 0.75$ using the completed Data Release 12 (DR12; Alam et al. 2017; Wang et al. 2017; Zhao et al. 2017b). The extended BOSS (eBOSS, part of SDSS-IV) detected a BAO signal at a 4% precision at $z \sim 1.5$ using the DR14 quasar sample (Ata et al. 2017). These new BAO measurements can provide a H_0 measurement, which is independent of CMB and local measurements, and thus can be highly informative.

In this paper, we determine the Hubble constant using the BOSS DR12 and eBOSS DR14 BAO measurements, combined with other measurements presently available, and investigate the consistency of H_0 values derived from different data sets using the Kullback–Leibler (KL) divergence (Kullback & Leibler 1951). We also perform a forecast for future BAO data for a feasibility study.

This paper is organized as follows. In the next section, we present the method and data used in this work, followed by a section devoted to the results. We present our conclusion and discussions in Section 4.

2. Method and Data

In the spatially flat Λ CDM model, the Hubble parameter is

$$H(z) = H_0 \sqrt{\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda}, \quad (1)$$

where $\Omega_r + \Omega_m + \Omega_\Lambda = 1$. The present energy density of radiation⁵ $\Omega_r = \Omega_m/(1+z_{\text{eq}})$, with $z_{\text{eq}} = 2.5 \times 10^4 \Omega_m h^2 (T_{\text{CMB}}/2.7 \text{ K})^{-4}$ being the redshift of matter-radiation equality. We adopt $T_{\text{CMB}} = 2.7255 \text{ K}$. The angular diameter distance is

$$D_A(z) = \frac{1}{1+z} \int_0^z \frac{dz'}{H(z')}. \quad (2)$$

⁴ There are other methods to determine the Hubble constant using galaxies. See Chen et al. (2017) for an example.

⁵ Three species of massless neutrinos are included, and the energy density is given in terms of the photon density, ρ_γ , by $\rho_\nu = 3(7/8)(4/11)^{4/3} \rho_\gamma$.

The sound horizon, r_s , at the redshift of the drag epoch, z_d , can be calculated as

$$r_d \equiv r_s(z_d) = \int_0^{1+z_d} \frac{da'}{a'^2 H(a') \sqrt{3(1 + \bar{R}_b a')}}}, \quad (3)$$

where $\bar{R}_b = 3.15 \times 10^4 \Omega_b h^2 (T_{\text{CMB}}/2.7 \text{ K})^{-4}$. Note that z_d is well approximated analytically (Eisenstein & Hu 1998) as

$$z_d = \frac{1291(\Omega_m h^2)^{0.251}}{1 + 0.659(\Omega_m h^2)^{0.828}} [1 + b_1(\Omega_b h^2)^{b_2}], \quad (4)$$

where

$$b_1 = 0.31(\Omega_m h^2)^{-0.419} [1 + 0.607(\Omega_m h^2)^{0.674}], \quad (5)$$

$$b_2 = 0.238(\Omega_m h^2)^{0.223}. \quad (6)$$

We use a fixed value of the baryon density $\Omega_b h^2 = 0.02225$ from the Planck result (Planck Collaboration et al. 2016).⁶ The baryon density can also be accurately determined in a CMB-independent way, e.g., using the primordial deuterium abundance in the Big Bang Nucleosynthesis (BBN) theory (Riemer-Sørensen & Sem Jønsen 2017).

Note that the quantities $H(z)r_d$ and $D_A(z)/r_d$ can be estimated from anisotropic BAO measurements, while the quantity,

$$D_V(z)/r_d \equiv \left[(1+z)^2 D_A^2(z) \frac{z}{H(z)} \right]^{1/3} / r_d, \quad (7)$$

is determined by isotropic BAO measurements.

As shown above, the BAO distance measurements, $H(z)r_d$, $D_A(z)/r_d$ or $D_V(z)/r_d$, are two-variable functions of Ω_m and H_0 (once $\Omega_b h^2$ is known) in a flat Λ CDM cosmology, and therefore the Hubble constant can be, in principle, determined from the BAO distances with Ω_m marginalized over.

In what follows, we use isotropic or anisotropic BAO distance measurements to determine the Hubble constant with Ω_m marginalized over, i.e., our parameter space is simply (assuming a flatness of the Universe)

$$P \equiv \{\Omega_m, H_0\}. \quad (8)$$

The sound horizon at the drag redshift r_d is calculated using Equation (3).⁷ We perform a Monte Carlo Markov Chain (MCMC) global fitting for parameter estimation using a modified version of `COSMOMC` (Lewis & Bridle 2002).⁸

The BAO data sets used in this work include

1. the isotropic BAO measurements using the 6dFRS (6dF; Beutler et al. 2011) and SDSS main galaxy sample (MGS) (Ross et al. 2015) at effective redshifts $z_{\text{eff}} = 0.106$ and $z_{\text{eff}} = 0.15$, respectively;
2. the BOSS DR12 anisotropic BAO measurements at three effective redshifts (BOSS 3zbin) in Alam et al. (2017) or at nine effective redshifts (BOSS 9zbin) in Wang et al. (2017) and Zhao et al. (2017b);

3. the eBOSS DR14 isotropic BAO measurement at $z_{\text{eff}} = 1.52$ (Ata et al. 2017); and
4. a combination of 6dF + MGS + BOSS 3zbin + eBOSS DR14 (All 3zbin), or a combination of 6dF + MGS + BOSS 9zbin + eBOSS DR14 (All 9zbin).

To check the consistency of the H_0 values determined from different data sets within the Λ CDM model, we compute the tension, T , based on the KL divergence (Kunz et al. 2006; Paykari & Jaffe 2013; Amara & Refregier 2014; Seehars et al. 2014, 2016; Verde et al. 2014; Grandis et al. 2016; Raveri et al. 2016), which quantifies the distance between two probability density functions (PDFs), p_1 , and p_2 . If both p_1 and p_2 are assumed to be Gaussian, the relative entropy in bits between the two PDFs can be evaluated as

$$D(p_2||p_1) = \frac{1}{2 \log 2} \left[\text{Tr}(\mathcal{C}_1^{-1} \mathcal{C}_2) - d - \log \frac{\det \mathcal{C}_2}{\det \mathcal{C}_1} + (\theta_2 - \theta_1)^T \mathcal{C}_1^{-1} (\theta_2 - \theta_1) \right], \quad (9)$$

where θ_i is the best-fit parameter vector, \mathcal{C}_i is the corresponding covariance matrix, and d denotes the dimensions of the parameter space (e.g., $d=2$ in our case where both H_0 and Ω_m are free parameters). If data are assumed to be more informative than the priors, one can compute the expected relative entropy, $\langle D \rangle$, with its standard deviation, Σ , via

$$\langle D \rangle \simeq \frac{1}{\log 2} \left[\text{Tr}(\mathcal{C}_2 \mathcal{C}_1^{-1}) - \frac{1}{2} \log \frac{\det \mathcal{C}_2}{\det \mathcal{C}_1} \right], \quad (10)$$

$$\Sigma(D) \simeq \frac{1}{\sqrt{2} \log 2} \sqrt{\text{Tr}(\mathcal{C}_1^{-1} \mathcal{C}_2 + \mathbb{I})^2}, \quad (11)$$

$$S \equiv D(p_2||p_1) - \langle D \rangle, \quad (12)$$

where the Surprise, S , is defined as the difference between the relative entropy and its expectation value. The tension, T , is defined as the signal-to-noise ratio of the Surprise, i.e.,

$$T \equiv S/\Sigma. \quad (13)$$

If $T \lesssim 1$, then p_1 and p_2 are consistent with each other, while otherwise the two PDFs are in tension (Seehars et al. 2016).

We also perform forecasts on the uncertainty of H_0 using ongoing and upcoming redshift surveys, including eBOSS⁹ (Dawson et al. 2016; Zhao et al. 2016), Dark Energy Spectroscopic Instrument (DESI¹⁰; DESI Collaboration et al. 2016a, 2016b), and ESA's Euclid satellite¹¹ (Laureijs et al. 2011). We use a flat, Λ CDM cosmology derived from the Planck mission as our fiducial model (Planck Collaboration et al. 2016), take the forecasted BAO data for galaxy surveys of a complete eBOSS from Zhao et al. (2016; i.e., the BAO result from eBOSS luminous red galaxies, high-density emission-line galaxies, and clustering quasars in Table 4 from Zhao et al. 2016), DESI (i.e., the BAO result from DESI luminous red galaxies, emission-line galaxies, and clustering quasars in Table 2.3 from DESI Collaboration et al. (2016a) and DESI bright galaxies in Table 2.5 from DESI Collaboration

⁶ We have tested to marginalize over $\Omega_b h^2$ with a Gaussian prior derived from the Planck measurement and find that the result is largely unchanged.

⁷ Except for 6dF, the value of r_d is rescaled by a factor r_d/\tilde{r}_d , where \tilde{r}_d value is calculated from CAMB (Lewis et al. 2000, available at <http://camb.info/>) in the same fiducial cosmology (Bennett et al. 2014).

⁸ <http://cosmologist.info/cosmomc/>

⁹ We use eBOSS throughout for the complete 5-year eBOSS sample, while we use eBOSS DR14 to denote the eBOSS DR14 quasar sample. More information on the eBOSS survey is available at <http://www.sdss.org/surveys/eboss/>.

¹⁰ <http://desi.lbl.gov/>

¹¹ <http://www.euclid-ec.org/>

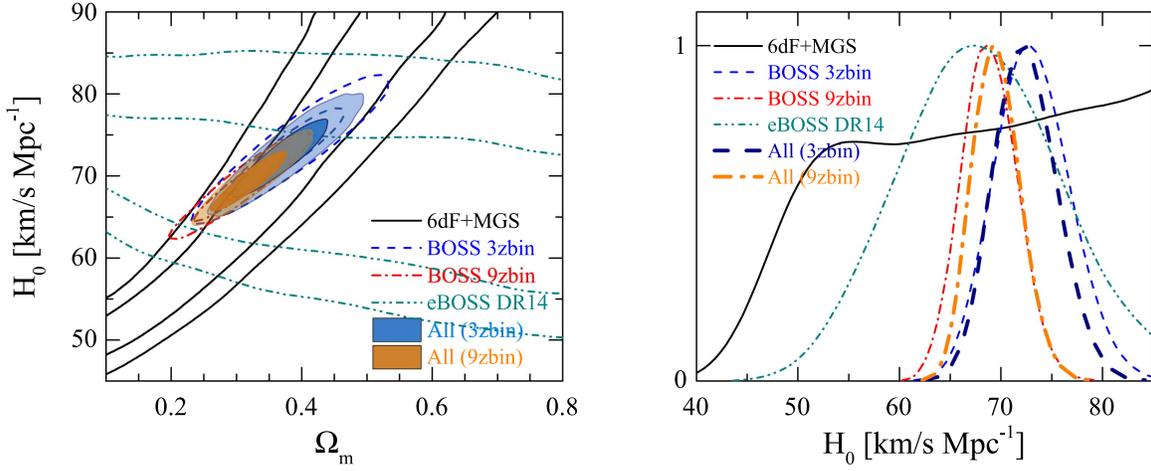


Figure 1. Left panel: the 68% and 95% confidence level (CL) contour plots of Ω_m and H_0 derived from various BAO measurements. Right panel: the probability distribution of H_0 derived from various BAO measurements.

et al. 2016a), and Euclid-like Font-Ribera et al. (2014; i.e., Table 6 in Font-Ribera et al. 2014), respectively, and perform parameter estimation using the MCMC method, in the same way as we did for the current data sets.

3. Results

We present the joint constraint on H_0 and Ω_m , and the posterior probability distribution of H_0 from various BAO data sets, including the latest eBOSS DR14 quasar sample, in Figure 1. As shown, the contours derived from different data sets show different degeneracy between H_0 and Ω_m . This is expected as the degeneracy is largely determined by the effective redshift at which the BAO measurement is performed. Hence having tomographic BAO measurements at a large number of redshifts helps to break the degeneracy. This can be seen by comparing the “All 3zbin” to the “All 9zbin” results. The only difference in these two data sets is that the BOSS DR12 galaxies were subdivided into more redshift slices in the 9zbin sample to gain more light-cone information. As shown in the upper part of Table 1, the improvement on the uncertainty of H_0 is significant, namely, the error of H_0 reduces from 3.05 to 2.34 $\text{km s}^{-1} \text{Mpc}^{-1}$, which is a 23% improvement.

We quantify the (in)consistency among the derived Ω_m and H_0 from BAO data and PLC15 using the quantity defined in Equation (13). We also calculate the KL divergence between the PDFs for H_0 with Ω_m marginalized over from various data sets, including those from PLC15 and R16. The result is presented in Table 2, including the relative entropy, D , its expected value, $\langle D \rangle$, the Surprise, S , in bits, and the tension, T , with 1σ error. As shown, except for the PLC15 and R16 pair, where T is larger than 1 at about a 2σ level, all others are consistent with each other (the tension T are all less than unity).

Given that the best measurement of H_0 to date using BAO alone (i.e., the “All 9zbin” result) has a worse precision than that from R16 or PLC15, we investigate the constraining capability of future BAO surveys, including the complete eBOSS, DESI, and Euclid-like, on H_0 . The joint constraint on H_0 and Ω_m , and the marginalized constraint on H_0 , from these surveys are shown in Figure 2 and in the lower part of Table 1, respectively. As shown, future galaxy surveys, especially for DESI or Euclid-like alone, are able to provide a better constraint on H_0 than that of the current CMB constraint, which is promising.

Table 1

The Mean and 68% CL Constraint on H_0 Using Various Data Sets

Data Set	H_0 ($\text{km s}^{-1} \text{Mpc}^{-1}$)	Precision
All 3zbin	71.75 ± 3.05	4.25%
All 9zbin	69.13 ± 2.34	3.38%
R16	73.24 ± 1.74	2.38%
PLC15	67.27 ± 0.66	0.98%
eBOSS	67.27 ± 1.55	2.30%
DESI	67.27 ± 0.33	0.49%
Euclid-like	67.27 ± 0.21	0.31%

Note. The upper part of the table (above the horizontal line) is for current data sets, while the lower part shows the forecast result based on a fiducial model derived from PLC15.

Table 2

Quantification of the (in) Consistency among Various Data Sets

Data Set	D	$\langle D \rangle$	S	T	σ_T
2D: $\{\Omega_m, H_0\}$					
All 3zbin \rightarrow All 9zbin	0.87	2.46	-1.59	-0.71	0.14
All 3zbin \rightarrow PLC15	7.91	7.87	0.04	0.02	1.12
All 9zbin \rightarrow PLC15	8.58	8.31	0.27	0.09	1.54
1D: $\{H_0\}$					
All 3zbin \leftrightarrow All 9zbin	0.62	1.23	-0.61	-0.38	0.19
All 3zbin \leftrightarrow PLC15	3.08	2.28	0.80	0.75	1.74
All 9zbin \leftrightarrow PLC15	1.62	1.94	-0.32	-0.29	0.94
All 3zbin \leftrightarrow R16	0.50	1.28	-0.78	-0.58	0.26
All 9zbin \leftrightarrow R16	2.33	1.23	1.10	0.70	0.42
PLC15 \leftrightarrow R16	61.91	8.63	53.29	6.57	2.73

Note. Top section: the KL divergence between the PDFs for Ω_m and H_0 using BAO data and PLC15. Bottom Section: the KL divergence between the PDFs for H_0 with Ω_m marginalized over from various data sets. The tension, $T \lesssim 1$, illustrates the relevant pairs of data sets that are consistent with each other.

4. Conclusion and Discussions

In this paper, we determine the Hubble constant using BAO measurements from galaxy redshift surveys in a flat Λ CDM cosmology. A combination of recent BAO measurements from

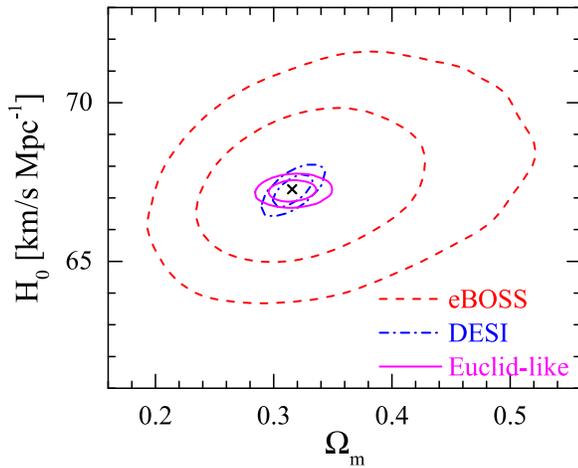


Figure 2. The 68% and 95% CL contour plots of parameters Ω_m and H_0 in a flat Λ CDM model, derived from the complete eBOSS (red dashed), DESI (blue dash-dotted), and Euclid-like (magenta solid). The black cross corresponds to the fiducial model.

6dF, MGS, BOSS DR12 (with 9 redshift slices), and eBOSS DR14 quasar samples yield a measurement of the Hubble constant, namely, $H_0 = 69.13 \pm 2.34 \text{ km s}^{-1} \text{ Mpc}^{-1}$, which is a 3.4% measurement. Given the level of the uncertainty, this measurement is consistent with both R16 and PLC15, which are in tension between themselves.

Based on a forecast, we find that future galaxy surveys, including DESI and Euclid-like, will be able to provide competitive constraints on H_0 , compared with current local or CMB measurements.

We thank Will Percival for discussions and comments. We also thank Chao Liu for discussions. Y.W. is supported by the NSFC grant No. 11403034, and by the Young Researcher Grant of National Astronomical Observatories, Chinese Academy of Sciences. L.X. is supported by the NSFC grant No. 11275035, Grant No.11675032, and by “the Fundamental Research Funds for the Central Universities” under grant No. DUT16LK31. G.B.Z. is supported by an NSFC grant No. 11673025, and by a Royal Society-Newton Advanced Fellowship.

This research used resources of the SCIAMA cluster supported by University of Portsmouth, and the ZEN cluster supported by NAOC.

References

- Alam, S., Ata, M., Bailey, S., et al. 2017, *MNRAS*, **470**, 2617
Amara, A., & Refregier, A. 2014, *PhRvD*, **89**, 083501
Ata, M., Baumgarten, F., Bautista, J., et al. 2017, arXiv:1705.06373
Bennett, C. L., Larson, D., Weiland, J. L., & Hinshaw, G. 2014, *ApJ*, **794**, 135
Beutler, F., Blake, C., Colless, M., et al. 2011, *MNRAS*, **416**, 3017
Chen, Y., Kumar, S., & Ratra, B. 2017, *ApJ*, **835**, 86
Cheng, C., & Huang, Q. 2015, *SCPMA*, **58**, 5684
Dawson, K. S., Kneib, J.-P., Percival, W. J., et al. 2016, *AJ*, **151**, 44
DESI Collaboration, Aghamousa, A., Aguilar, J., et al. 2016a, arXiv:1611.00036
DESI Collaboration, Aghamousa, A., Aguilar, J., et al. 2016b, arXiv:1611.00037
Di Valentino, E., Melchiorri, A., & Silk, J. 2016, *PhLB*, **761**, 242
Eisenstein, D. J., & Hu, W. 1998, *ApJ*, **496**, 605
Font-Ribera, A., McDonald, P., Mostek, N., et al. 2014, *JCAP*, **5**, 023
Freedman, W. L. 2017, *NatAs*, **1**, 0169
Freedman, W. L., & Madore, B. F. 2010, *ARA&A*, **48**, 673
Grandis, S., Seehars, S., Refregier, A., Amara, A., & Nicola, A. 2016, *JCAP*, **5**, 034
Kullback, S., & Leibler, R. A. 1951, *Ann. Math. Stat.*, **22**, 79, <https://projecteuclid.org/euclid.aoms/1177729694>
Kunz, M., Trotta, R., & Parkinson, D. R. 2006, *PhRvD*, **74**, 023503
Laureijs, R., Amiaux, J., Arduini, S., et al. 2011, arXiv:1110.3193
Lewis, A., & Bridle, S. 2002, *PhRvD*, **66**, 103511
Lewis, A., Challinor, A., & Lasenby, A. 2000, *ApJ*, **538**, 473
Paykari, P., & Jaffe, A. H. 2013, *MNRAS*, **433**, 3523
Planck Collaboration, Ade, P. A. R., Aghanim, N., et al. 2016, *A&A*, **594**, A13
Poursidou, A., & Tram, T. 2016, *PhRvD*, **94**, 043518
Raveri, M., Martinelli, M., Zhao, G., & Wang, Y. 2016, arXiv:1606.06273
Riemer-Sørensen, S., & Sem Jensen, E. 2017, arXiv:1705.03653
Riess, A. G., Macri, L. M., Hoffmann, S. L., et al. 2016, *ApJ*, **826**, 56
Ross, A. J., Samushia, L., Howlett, C., et al. 2015, *MNRAS*, **449**, 835
Seehars, S., Amara, A., Refregier, A., Paranjape, A., & Akeret, J. 2014, *PhRvD*, **90**, 023533
Seehars, S., Grandis, S., Amara, A., & Refregier, A. 2016, *PhRvD*, **93**, 103507
Sola, J., Gomez-Valent, A., & de Cruz Perez, J. 2017, arXiv:1705.06723
Verde, L., Protopapas, P., & Jimenez, R. 2014, *PDU*, **5**, 307
Wang, Y., Zhao, G.-B., Chuang, C.-H., et al. 2017, *MNRAS*, **469**, 3762
Wang, Y., Zhao, G.-B., Wands, D., Pogolian, L., & Crittenden, R. G. 2015, *PhRvD*, **92**, 103005
Wyman, M., Rudd, D. H., Vanderveld, R. A., & Hu, W. 2014, *PhRvL*, **112**, 051302
Zhao, G.-B., Raveri, M., Pogolian, L., et al. 2017a, *NatAs*, **1**, 627
Zhao, G.-B., Wang, Y., Ross, A. J., et al. 2016, *MNRAS*, **457**, 2377
Zhao, G.-B., Wang, Y., Saito, S., et al. 2017b, *MNRAS*, **466**, 762