Shale Gas Production Decline Trend over Time in the Barnett Shale

Michael Kenomore\textsuperscript{a}, Mohamed Hassan, Reza Malakooti, Hom Dhakal, Amjad Shah

Address: University of Portsmouth, School of Engineering, Anglesea Building, Anglesea Road, Portsmouth, PO1 3DJ

Email: michael.kenomore@port.ac.uk, mohamed.hassan@port.ac.uk, reza.malakooti@port.ac.uk, hom.dhakal@port.ac.uk, amjad.shah@port.ac.uk

\textsuperscript{a} – Corresponding Author

Abstract

Natural gas produced from shale formations in the United States over the past decade have altered the oil and gas industry remarkably. The Barnett shale was at the forefront of the shale gas revolution in the United States and was considered to be the highest producing natural gas field in the United States until 2012, yielding the top producer spot to the Marcellus shale. Due to the uncertainty regarding the accurate determination of Estimated Ultimate Recoverable (EUR) in shale gas reservoirs, this paper aims to assess EUR values for the Barnett shale using empirical decline curve methods like the Arp’s hyperbolic, Modified Arp’s hyperbolic and Doung’s method. In addition, we investigated the economic viability of wells over time in the Barnett under various probabilities of success. Throughout this paper, reference is made to two key publications where a similar work was carried out for various shale plays in the United States, including the Barnett shale – though only the Arp’s hyperbolic decline was employed. The dataset in this paper consisted of more horizontal wells from covering more counties within the Barnett shale compared to other similar studies. We conclude that either the Arp’s hyperbolic or Doung’s method can be used to forecast EUR in the Barnett shale as only marginal differences were observed. This is on the basis that production history exceeds 10 months (a maximum of 80 months production history was used). We also obtained reliable and conservative estimates of EUR compared to previous studies.

Keywords

Estimated ultimate recovery, Barnett shale, Arp’s hyperbolic, Doung’s method, modified hyperbolic.
1. Introduction

The Barnett shale is located in the Forth-Worth basin of Texas and was the first modern commercial shale play in the US having been discovered in 1981 (Kennedy, Luo, Vello, 2016). Until 2012, it was the largest shale gas basin (Newark East Field) in the world before been replaced by Marcellus (Kennedy, Luo, Vello, 2016). It is split into the Upper and Lower Barnett which are separated by the Forestburg limestone with the thicker Lower Barnett contributing “about 70 to 80%” of the production (Kennedy, Luo, Vello, 2016). Overlying the Barnett is the Marble Falls, which is a barrier to hydraulic fractures’ growth (Figure 1). The lower Barnett boundary is either the Viola or the Ellenburger limestone (Figure 1). While the Viola can exhibit good reservoir characteristics, the Ellenburger often contains water so well trajectories and hydraulic fractures are designed to avoid the Ellenburger limestone (Kennedy, Luo, Vello, 2016) (Figure 1).

![Figure 1: North-South through the Newark East Field in the Barnett Shale (Bruner and Smosna, 2011)](image)

Unlike other major US shale gas basins like Fayetteville, Haynesville, Eagleford and Marcellus, the Barnett shale has a comparably higher silica content (Figure 2) which has an influence on the fracability because high silica contents makes the shale brittle and easier to fracture (Slatt, 2013).
Figure 2: Mineralogy variation among US shale resource plays (Slatt, 2013)

(Montgomery et al., 2005) also noted that the Barnett shale is generally rich in silica and relatively poor in clay content. The initial gas production rate in the Barnett shale was also found to be lower compared to other major US shale gas plays like the Eagleford, Marcellus and Haynesville shale (Rezaee, 2015). (Baihly et al., 2010, 2015) also observed the same results in initial gas production rate from the Barnett shale when compared to other major US shale gas plays like Haynesville, Marcellus, Eagleford, Woodford and Fayetteville. Figure 3 shows the variability in performance among the different shale plays (Baihly et al., 2015).

Figure 3: Variability in gas production rate among different shale plays in the United States (Baihly et al., 2015).
(Baihly et al., 2010), carried out an assessment of EUR on various shale plays, including the Barnett shale using horizontal wells with a date of first production (DOFP) from 2003 to a DOFP from 2009 (Figure 4).

In this paper, horizontal Barnett shale wells with a DOFP from 2008 to 2014 were used to understand decline trends and forecast EUR. The EUR for DOFP 2008 and 2009 was calculated in the (Baihly et al., 2010) study and these were compared to results from this paper. The area of interest in the (Baihly et al., 2010) study was located in the Tarrant, Wise, Denton, and Parker counties of the Barnett shale. In contrast, data from this paper included more counties with exception to the Archer, Comanche, Cooke, Coryell, Hamilton, Shakleford and Stephens non-core counties (Table 1).

Table 1: Core and Non-Core Counties in the Barnett Shale (Railroad Commission of Texas, 2017)

<table>
<thead>
<tr>
<th>Core Counties</th>
<th>Non-Core Counties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Denton, Johnson, Tarrant, Wise</td>
<td>Archer, Bosque, Clay, Comanche, Cooke, Coryell, Dallas, Eastland, Ellis, Erath, Hamilton, Hill</td>
</tr>
<tr>
<td></td>
<td>Hood, Jack, Montague, Palo Pinto, Parker, Shakleford, Somervell, Stephens</td>
</tr>
</tbody>
</table>
Baihly et al., 2015) revisited their previous study in 2010 and carried out a similar work on assessing decline trends and forecasting EUR on various shale plays including the Barnett. The horizontal wells used in (Baihly et al., 2015) were from a DOFP 2003 to DOFP 2013 resulting a total of 1,138 wells (Figure 5) compared to 10237 wells in this paper representing an 800% increase in well count compared to (Baihly et al., 2015). The areas of study for Barnett shale wells in the (Baihly et al., 2015) study was also located in the Tarrant, Wise, Denton and Parker counties.

![Figure 5: Barnett shale average daily gas production rate per well in MSCF/day against time grouped by DOFP (Baihly et al., 2015).](image)

(Baihly et al., 2015) observed that the Barnett shale showed no production improvement over time across all the time groups despite well lateral lengths increasing by 50% and fracture proppant per horizontal well increasing by 33% over the five year period. This means that wells are becoming tightly spaced and interfering with older offset wells. The production trends in the Barnett were consistent over the five-year period which asserts that the impact of reservoir quality is significantly greater than that of completion quality on the productivity of the Barnett play (Baihly et al., 2010, 2015). In addition, The Barnett also exhibited the lowest percentage change (41.2%) in EUR after five years of production compared to other shale gas basins (Fayetteville, Woodford, Haynesville, Eagle Ford and Marcellus). In contrast, Haynesville shale showed the highest change in EUR of 64.2% (Baihly et al., 2015).
A number of techniques are available to model the reservoir performance and predict the EUR, this includes analogs, decline curve analysis (DCA), volumetric material balance and numerical simulations (Cauter, 2013). Among these methods, DCA is widely used to estimate EUR in unconventional shale reservoirs due to its simplicity and lower costs (Lee and Sidle, 2010). DCA is based on only wells production data exhibiting reservoir characteristics dominated by drained reservoir volume (Golan and Whitson, 1986). A common DCA methodology is the Arp’s empirical models. However, the use of Arp’s models (Exponential, Hyperbolic and Harmonic) in DCA is subject to certain assumptions, which are; constant bottom hole pressure, boundary dominated flow (BDF), unchanging drainage area and a fixed skin factor (Lee and Sidle, 2010). The DCA model needs to be updated to improve the accuracy of the EUR upon availability of new production data either through existing old wells or additional new infill drilled wells.

The objective of this paper is to carry out a technical and economic assessment of EUR variations over time in the Barnett shale and compare the results with the work in ((Baihly et al., 2010, 2015)) instead of critically evaluating the effectiveness of various empirical forecasting methods which has been carried out extensively in literature such as (Robertson, 1988; Ilk et al., 2008; Duong, 2011; Kanfar and Wattenbarger, 2012; Cauter, 2013; Joshi and Lee, 2013). In this paper, Duong’s method (Duong, 2011), Modified hyperbolic decline method (Robertson, 1988) in addition to Arp’s hyperbolic decline method (Arps, 1945) were used to carry out an assessment of EUR in the Barnett Shale. With the surge in exploiting unconventional shale gas, the need to forecast production and EUR to improve investment decisions has grown (Cauter, 2013).

2. Theory

2.1. Hyperbolic, Harmonic and Exponential Decline Method

The general Arp’s hyperbolic decline (Arps, 1945) (Equation 1) is described as;

$$q = q_i \frac{1}{(1 + bD_t t)^{(b/b)}}$$

Equation 1 (Arps, 1945)

Where 0<b<1. The decline rate, D (Equation 2) decreases continuously in the Arp’s hyperbolic equation (Seshadri and Mattar, 2010). When D becomes very small over time, the gas rate no longer declines significantly thus reserves can be over-estimated (Seshadri and Mattar, 2010).
Equation 2 \textit{(Seshadri and Mattar, 2010)}

(Baihly \textit{et al.}, 2010, 2015) used only the Arp’s hyperbolic decline model (Arps, 1945) to estimate the EUR. While the Arp’s hyperbolic can often be used in shale gas reservoirs, the assumptions for its use in unconventional shale is not valid. For example, the low-permeability nature of unconventional shale reservoirs creates a long transient flow regime (Lee and Sidle, 2010) with an expanding drainage area resulting in high b values and optimistic forecasts (Cauter, 2013).

Nevertheless, the hyperbolic decline equation with b-values greater than one can provide a reasonable fit for the long transient linear-flow regime observed in shale gas wells despite having to exceed the limit of the Arp’s equation (Duong, 2011). However, values of b greater than one (indicating transient flow) are often observed which is physically impossible as cumulative production increases to infinity (Cauter, 2013). If there is sufficient data to reach boundary dominated or stabilized flow regime, the b-values of less than one would match the production decline data (Lee and Sidle, 2010). Without stabilized data, forecasting future production with the Arp’s hyperbolic can result in an overestimation of reserves. The Arp’s equation becomes harmonic in a special case when b=1 (Equation 3);

\[
D = \frac{1}{\frac{1}{D_i} + bt}
\]

Equation 3 \textit{(Lee and Sidle, 2010)}

When b=0, the general hyperbolic equation becomes an exponential equation (Lee and Sidle, 2010) (Equation 4).

\[
q = q_i \left(1 + D_i t\right)^{-1}
\]

Equation 5 \textit{(Lee and Sidle, 2010)}

In exponential decline, the decline rate (D) is constant and this method is straightforward. However, it has a poor fit to shale gas reservoirs because they tend to show large initial declines but become smaller over time (Seshadri and Mattar, 2010). The exponential decline typically under-predicts reserves (Seshadri and Mattar, 2010). Other empirical models have been proposed to resolve the long term decline behaviour in Arp’s the modified hyperbolic (Robertson, 1988) describes the practise of switching the initial hyperbolic to an exponential tail at a predetermined fixed decline rate (typically 5%)\textit{(Cauter, 2013)}. \textit{(Valko, 2009) and (Ilk \textit{et al.}, 2008)} have proposed models specifically to model transient and flow and have a finite EUR. Doung’s method (Duong, 2011) was proposed to specifically model infinite-lasting linear flow.
which is dominant in shale gas reservoirs. Plots showing the historical production data match with Doung’s model and diagnostic plots showing that the data is still in transient flow are shown on Appendix A. (Joshi and Lee, 2013), proposed the modified Doung method to account for later BDF since the Doung method is designed to model only transient flow – it simply involves switching from Doung to Arp’s hyperbolic at an arbitrary decline rate (typically 5% in shales).

2.2. Modified Hyperbolic Method
The modified hyperbolic method attempts to resolve Arp’s hyperbolic long term decline behaviours where D becomes too small (Cauter, 2013). Proposed by (Robertson, 1988), it works by replacing the initial hyperbolic decline to an exponential tail at a predetermined fixed decline rate when the decline rate is achieved (Cauter, 2013). In a nutshell, It works by imposing a decline limit $D_{\text{limit}}$ below which D is not allowed to decline (Seshadri and Mattar, 2010). Once the D reaches $D_{\text{limit}}$, Equation 6 switches to exponential decline (Seshadri and Mattar, 2010).

\[
q = \begin{cases} 
\frac{q_1}{(1 + bD_t)^{1/b}} & D > D_{\text{limit}} \\
q_1e^{D_{\text{limit}}}, & D \leq D_{\text{limit}}
\end{cases}
\]

Equation 6 (Seshadri and Mattar, 2010)

Typically a 5% terminal decline rate is often used in shales (Joshi and Lee, 2013). The exponential tail mathematically represents systems of low compressibility, BDF and stable operating conditions (Cauter, 2013). Fitting the decline curve to transient data provides no help for selecting $D_{\text{limit}}$, therefore the choice of $D_{\text{limit}}$ is based on experience, a best-guess, or through agreement between production companies and reserves assessors (Seshadri and Mattar, 2010).

2.3. Doung’s Method
Doung’s method works on the basis that a log-log plot of $\frac{q}{q_p}$ vs time will form a straight line with a negative slope, $-m$, and an intercept of $a$ (Duong, 2011) (Equation 7). The slope is negative but $m$ is always positive.

\[
\frac{q}{q_p} = at^{-m}
\]

Equation 7 (Duong, 2011)
According to (Duong, 2011), $m$ is always greater than one for shale reservoirs. When $m$ is less than one, it may indicate a conventional tight well. The log-log straight line is indicative of transient linear flow conditions for which the Doung’s method accurately predicts (Figure 6) (Kanfar and Wattenbarger, 2012). The production trend will deviate from a log-log straight line when BDF is reached (Kanfar and Wattenbarger, 2012). After removing any abrupt rate data from surface choking effects, skin effects and comparably low well counts, the schematics (from Figure 6 to Figure 9) illustrates the procedure for the Doung’s model.

**Step 1: Plot the log of $q/Gp$ vs time on a linear scale**

![Figure 6: Step 1](image)

**Step 2: On a log-log plot of $q/Gp$ vs time, determine the values of $a$ & $m$. The value of “a” would be $10^{0.3262}$ from Figure 6**
Step 3: $q_t$ determination

After determining $a$ *and* $m$ from the log-log plot of $\frac{q}{ \frac{d}{dP} }$ vs. time, $v_1$ and $v_w$ need to be determined from Figure 8 by plotting a graph of $q$ vs. $t(a,m)$ in the form of Equation 8.
\[ q = q_1 c(a, m) + q_\infty \]

Where,

\[ c(a, m) = e^{m \exp \left( \frac{m-1}{1-m} \left( \frac{m-a}{m-1} \right) \right)} \]

Equation 7 can then be used for step 4.

**Step 4: Validation and Forecasts**

![Figure 9: Step 4](image-url)
The term $q_{\infty}$, which can be zero, positive, or negative, was added to provide a better fit to some field data that showed an intercept instead of a straight line to the origin when plotting $q$ vs $t$ (Kanfar and Wattenbarger, 2012). It is important to note that (Joshi and Lee, 2013) observed in their research on a Barnett shale well that using a non-zero $q_{\infty}$ can lead to unrealistic results for especially when only 6-12 months of historical production data are available. (Joshi and Lee, 2013) concluded that, for a well with production history greater than 24 months, forcing the straight line through the origin i.e. $q_{\infty} = 0$ leads to the minimum error in estimating remaining production (Joshi, 2012).

Typically, none of the methods used in this paper would be effective for prediction when less than six months of production data are available irrespective of the shale play (Joshi and Lee, 2013).

3. Methodology

The methodology applied in this study was similar to that used in ((Baihly et al., 2010, 2015)) and the steps are outlined below:

1. Monthly gas production data (in MScf/day) from the selected horizontal wells in the Barnett shale producing from 2008 to at most 2015 were arranged in order of DOFP. Wells with a 2015 DOFP were excluded from the analysis due to having only four months of gas production which was insufficient for appropriate conclusions to be made.

2. The production data was quality checked on a well by well basis by removing abrupt changes in production data from each well. Possible reasons can be associated with pipeline constraints and re-stimulation (Baihly et al., 2015). The rate data (in MScf/day) were shifted with respect to time such that all wells were normalized to start at “time zero”.

3. A final production type curve was generated by applying a cut-off to the initial production type curve at a point in time when the well count in each data group began decreasing drastically (Figure 10). This is because at this point, the calculated average rate no longer represent the average behaviour of the original group of wells hence the production data for those months were excluded to prevent erroneous decisions been made about the production decline.
4. The cut-off time was selected at the point when the onset of a sharp decline in well count was observed for each year group (Figure 10). The cut-off times were at the following months: 80 months for DOFP 2008, 59 months for DOFP 2009, 60 months for DOFP 2010, 46 months for DOFP 2011, 35 months for DOFP 2012, 24 months for DOFP 2013, and 10 months for DOFP 2014 (Figure 10).

5. DCA was performed on each type curve for individual and combined time groups using three approaches including Arp’s hyperbolic method, modified hyperbolic method and Doung’s method. Detailed procedure for the Arp’s hyperbolic method is presented in Appendix C.

6. After obtaining a match with the production data, the EUR at 30 years was then computed from the three methods and compared to similar work carried out in ([Baihly et al., 2010, 2015]). Detailed historical match using the Doung’s method is presented in Appendix A.

7. Finally, an economic analysis was carried out to investigate the gas prices required for Barnett shale wells to yield a rate of return of 10% at different probabilities of success.

4. Results

4.1. Historical Production Data Match with Decline Models
A total of 10237 historical gas well production data was used in this paper across all dates of first production (Figure 11). Using DOFP 2008 and DOFP 2009 as an example, the historical data match with the decline models is shown in Figure 12. Detailed historical production data match with the Doung’s model and Arp’s hyperbolic model is presented in Appendix A and B respectively.

Figure 11: Historical production data from this paper
4.1.1. Root Mean Square Error Analysis

The root mean square error (RMSE) between the model and historical rates was used to further evaluate the forecasting performance for the Arp’s hyperbolic and Doung’s decline models (Table 2). This was performed across all time groups using the excluded historical production data which corresponded to the months where a drastic drop in well counts was observed (see Figure 10). The total production history comprises of the excluded production data; for DOFP2008, the decline models were developed over 80 months and then used to forecast an additional 15 months. Detailed plots of the forecasts is presented in Appendix D.

Table 2: RMSE Analysis over Time

<table>
<thead>
<tr>
<th>Time Group (forecast range)</th>
<th>Arp’s Hyperbolic</th>
<th>Doung’s Method</th>
<th>Total Production History</th>
</tr>
</thead>
<tbody>
<tr>
<td>DOFP 2008 (last 15 months)</td>
<td>11.8</td>
<td>21.4</td>
<td>95 months</td>
</tr>
<tr>
<td>DOFP 2009 (last 23 months)</td>
<td>9.1</td>
<td>21.9</td>
<td>72 months</td>
</tr>
<tr>
<td>DOFP 2010 (last 10 months)</td>
<td>23.2</td>
<td>25.6</td>
<td>70 months</td>
</tr>
<tr>
<td>DOFP 2011 (last 11 months)</td>
<td>17.8</td>
<td>16.9</td>
<td>57 months</td>
</tr>
<tr>
<td>DOFP 2012 (last 11 months)</td>
<td>40.7</td>
<td>26.3</td>
<td>46 months</td>
</tr>
<tr>
<td>DOFP 2013 (last 11 months)</td>
<td>39.3</td>
<td>41.4</td>
<td>35 months</td>
</tr>
<tr>
<td>DOFP 2014 (last 11 months)</td>
<td>47.4</td>
<td>32.4</td>
<td>21 months</td>
</tr>
</tbody>
</table>
4.2. EUR Variation over Time

Figure 13, presents the variation in EUR over time in the Barnett shale and compares estimated EUR in this study with available data from (Baihly et al., 2010, 2015). Detailed results of the EUR is in Appendix A and Appendix E.

![EUR variation over time in the Barnett Shale](image)

Figure 13: EUR variation over time in the Barnett Shale (see Appendix E for tabular results)

Table 3 summarises calculated EUR at 30 years using three methods of Arp’s hyperbolic, modified hyperbolic and Doung along with results from ((Baihly et al., 2010, 2015)).

<table>
<thead>
<tr>
<th>Method</th>
<th>EUR at 30 years – Barnett Shale</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Arp’s Hyperbolic (BCF)</strong>&lt;br&gt;b=1.477;Di=0.1362 (This Work)</td>
<td><strong>Doung (BCF) (This Work)</strong>&lt;br&gt;Di=0.0766; b=1.5933 (Baihly et al., 2010)</td>
</tr>
<tr>
<td>2.229&lt;br&gt;(60 months, 10237 wells)</td>
<td>2.169&lt;br&gt;(60 months, 10237 wells)</td>
</tr>
</tbody>
</table>

4.3. Economic Analysis

An economic study was performed to investigate the gas prices ($/MMScf) required for Barnett shale wells (both individual and combined time groups) to result in a rate of return of 10% at...
various probabilities of success (100%, 90%, 50%, and 10%) (Table 5). The probability of success was applied on the gross production every month to obtain the risked gross production (Madani and Holditch, 2011). The gas price was applied to the net production after royalties have been deducted to obtain the net revenue. The gathering and compression costs was calculated by:

\[
\text{Net production} \times \text{gathering and compression costs} \times (0.7) \times (1 - \text{cost escalations}) \times \text{time}
\]

\[\text{Equation 9: Cost due to fuel and shrinkage (Madani and Holditch, 2011)}\]

After taking all the cost considerations and investments, the net cash flow was obtained from the net revenue. Changes in the probability of success affects the risked gross production and changes in gas price affects the net revenue which has an effect on the final net cash flows from which the rate of return is determined.

The gas production rates and EUR (at 30 years) used were calculated from the Arp’s hyperbolic decline model. Key parameters used in the economic model are shown in Table 4 and the results obtained are shown in Table 5.

| Well costs | $3 million |
| Operating costs – gathering and compression costs | $0.7 million/MScf |
| Fuel and Shrinkage\(^1\) | 6% |
| Discount rate | 10% |
| Royalty Burden | 22% |

| Table 5: Detailed results of gas prices at various probabilities of success |
|-------------------|-------------|-------------|-------------|-------------|
|                   | 10% PoS     | 50% PoS     | 90% PoS     | 100% PoS    |
| DOFP 2008         | $35.73      | $7.71       | $4.59       | $4.20       |
| DOFP 2009         | $36.73      | $7.76       | $4.64       | $4.24       |
| DOFP 2010         | $32.72      | $7.09       | $4.27       | $3.91       |
| DOFP 2011         | $30.51      | $6.66       | $4.03       | $3.68       |
| DOFP 2012         | $30.77      | $6.7        | $4.05       | $3.72       |
| DOFP 2013         | $30.46      | $6.86       | $4.02       | $3.68       |
| DOFP 2014         | $38.72      | $8.33       | $4.94       | $4.51       |
| Combined DOFP     | $36.67      | $7.9        | $4.7        | $4.30       |

\(^1\) The mechanical compression of natural gas along the interstate pipeline results in a volume loss called shrinkage or fuel cost (Scana Energy, 2017).
5. Discussion

1. Evidence from RMSE analysis across all time groups suggests that the Arp’s hyperbolic decline model had a better forecast compared to the Doung’s model for the top three longest production histories i.e. DOFP 2008 (by 45%), 2009 (by 58%) and 2010 (by 9%) (Table 2). For the rest of the time groups, Doung’s model performed better with the exception of wells with a DOFP2013 where the Arp’s hyperbolic performed better by a margin of approximately 5%. Though there was no evidence of BDF which Arp’s is specifically suited for, results showed that for longer production histories i.e. >70 months, Arp’s hyperbolic decline may be a better choice for forecasting compared to Doung’s model which is specifically for transient flow as evident in the Doung’s diagnostic plot (Figure 6) which shows no deviation from straight line at late time to suggest the presence of BDF. This may suggest why Doung’s model performed better at relatively shorter production histories.

2. When considering the EUR at 30 years for each time group, only marginal differences in EUR was observed for the Barnett shale irrespective of the decline method considered with exception to wells with a DOFP2014 which only had a 10 month production history (Figure 13). (Baihly et al., 2015) also observed that the Barnett shale showed the lowest percentage change in EUR over a five year period compared to the Eagleford, Marcellus, Haynesville, Woodford and Fayetteville. This was attributed to lower clay contents and lower reservoir pressures of the Barnett shale compared to other shale formations (Baihly et al., 2015). Figure 11 also confirms the very close decline rates resulting from all DOFP groups over the time. Either methods may be used in the Barnett shale providing historical data exceeds 10 months based on results in this paper.

3. (Baihly et al., 2010) observed a 25% and 26% increase in EUR for wells with a 2008 and 2009 DOFP respectively compared to this paper when using Arp’s hyperbolic decline model (Figure 13). The EUR for the combined type curve in this paper was 25% less compared to (Baihly et al., 2010) and 29% less compared to (Baihly et al., 2015) (Table 3). These differences can be attributed to an increase in the number of wells studied (up to a 1000% increase in DOFP 2008), covering more counties compared to ((Baihly et al., 2010, 2015)). This provided a better representation of the Barnett shale performance (wells covered eighteen counties in this paper compared to only four in (Baihly et al.,
The availability of large volume of production data has resulted in prediction of more accurate DCA model in Barnett shale.

4. Generally, the economic analysis showed that as the PoS decreased, higher gas prices are needed to keep the development viable (Table 5).

6. Conclusion

The following conclusions were evident in this study;

1. Results from (Baihly et al., 2010, 2015) for the combined time groups overestimated EUR for the Barnett shale in comparison to all the DCA methods used in this paper. This study showed a more reliable and conservative estimate of EUR as more wells covering a wider acreage were used in the analysis.

2. The use of Arp’s hyperbolic or Doung’s model only showed marginal differences in the long term forecast of EUR for the Barnett shale hence either methods can be used in the Barnett providing historical data exceeds 10 months. This does not mean that these conditions apply to other shale plays because shales are highly heterogeneous.

3. The RMSE analysis for the Arp’s hyperbolic was lower compared to Doung’s model for the top three longest production histories despite no evidence of BDF.

Nomenclature

\[ q = \text{gas production rate at time } t \, (\text{Mscf/day}) \]

\[ q_0 = \text{production rate at time zero} \, (\text{Mscf/day}) \]

\[ b = \text{Arp's hyperbolic decline constant, dimensionless} \]

\[ a = \text{Arp's constant decline rate in exponential decline, dimensionless} \]

\[ D_t = \text{Arp's initial decline rate} \, (1/\text{time}) \]

\[ t = \text{time (months)} \]

\[ C_p = \text{cumulative gas production (Bcf)} \]

\[ a = \text{Doung's intercept constant (months)} \]

\[ t(a.m) = \text{Doung's time constant, dimensionless} \]
m = slope

$q_{\infty}$ = production rate at infinite time (MScf/day)

$q_1$ = production rate at day 1 (MScf/day)

PoS = probability of success

**Acknowledgements**

We would like to thank Anh Doung for his technical input on the Doung’s method via email communication. We would also like to thank IHS Markit for providing the data for this work at a discounted price. This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

**Appendix A**

**Match Production Data with Doung’s Model**
DOFP 2008 – 80 month production history
DOPP 2009 – 59 month production history
DOFP 2010 – 60 month production history
DOFP 2011 - 46 month production history
DOFP 2012 – 35 month production history
DOFP 2013 – 24 month production history
DOFP 2014 - 10 month production history
Appendix B

Match Production Data with Arp’s Hyperbolic Model

![Graph showing match production data with Arp's Hyperbolic Model for DOFP 2008 and DOFP 2009.]
DOFP 2010  
\( b=1.6479, \; D_i=0.1637 \)  
1792 wells

DOFP 2011  
\( b=1.6187, \; D_i=0.1418 \)  
1693 wells

DOFP 2012  
\( b=1.9254, \; D_i=0.1633 \)  
1184 wells

DOFP 2013  
\( b=1.7374, \; D_i=0.1287 \)  
738 wells
Appendix C

Newton Raphson method to determine \( D_i \) and \( b \) from historical production data

**Step 1:** Plot \( q \) vs \( t \) on a Semi-log scale and draw a smooth curve through the points.

**Step 2:** Extend the curve to intercept the y-axis at \( t=0 \) and read \( q_i \).

**Step 3:** Select the end point on the smooth curve and record the coordinates as \((t_2, q_2)\).

**Step 4:** Determine the coordinates of the middle point on the smooth curve that corresponds to \((t_1, q_1)\) with the value of \( q_1 \) obtained from:

\[
q_1 = \sqrt{q_1 q_2}
\]

The corresponding value of \( t_1 \) is read from the smooth curve at \( q_1 \).

**Step 5:** Solve the following equation iteratively for \( b \):

\[
f(b) = t_2 \left( \frac{q_1}{q_1} \right)^b - t_1 \left( \frac{q_1}{q_2} \right)^b - (t_2 - t_1) = 0
\]

The Newton-Raphson iterative method can be used to solve the above non-linear function by using the equation:

\[
b^{k+1} = b^k - \frac{f(b^k)}{f'(b^k)}
\]

Where the derivative \( f'(b^k) \) is given by;
\[ f'(b^k) = t_2 \left( \frac{q_t}{q_1} \right)^{b^k} \ln \left( \frac{q_t}{q_1} \right) - t_1 \left( \frac{q_t}{q_2} \right)^{b^k} \ln \left( \frac{q_t}{q_2} \right) \]

Starting with an initial value of \( b=0.5 \), i.e. \( b^k = 0.5 \), the method will converge after 4-5 iterations.

**Step 6:** Solve for \( D_i \) by using the expression below and using the calculated value of \( b \).

\[ D_i = \frac{(q_{t1}/q_2)^b - 1}{bt_2} \]

Appendix D
## APPENDIX E

### EUR Results

<table>
<thead>
<tr>
<th>EUR at 30 years – Barnett Shale</th>
<th>Arp’s Hyperbolic (BCF) (This Work)</th>
<th>Doung (BCF) (This Work)</th>
<th>Modified Hyperbolic @5% switch (BCF) (This Work)</th>
<th>Arp’s Hyperbolic (BCF) (Baihly et al., 2010)</th>
<th>Arp’s Hyperbolic (BCF) (Baihly et al., 2015)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DOFP 2008</td>
<td>2.307 (b=1.646;Di=0.15322)</td>
<td>2.184</td>
<td>2.207</td>
<td>2.895</td>
<td>n/a</td>
</tr>
<tr>
<td>DOFP 2009</td>
<td>2.280 (b=1.7550;Di=0.2138)</td>
<td>2.489</td>
<td>2.182</td>
<td>2.867</td>
<td>n/a</td>
</tr>
<tr>
<td>DOFP 2010</td>
<td>2.665 (b=1.6479;Di=0.1638)</td>
<td>2.582</td>
<td>2.595</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>DOFP 2011</td>
<td>2.980 (b=1.6188;Di=0.1419)</td>
<td>2.960</td>
<td>2.875</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>Year</td>
<td>DOFP</td>
<td>2.929 (b=1.9254; Di=0.1633)</td>
<td>2.842</td>
<td>2.763</td>
<td>n/a</td>
</tr>
<tr>
<td>------</td>
<td>-------</td>
<td>---------------------------</td>
<td>-------</td>
<td>-------</td>
<td>-----</td>
</tr>
<tr>
<td>2012</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2013</td>
<td></td>
<td>2.984 (b=1.7374; Di=0.1287)</td>
<td>2.766</td>
<td>2.853</td>
<td>n/a</td>
</tr>
<tr>
<td>2014</td>
<td></td>
<td>2.024 (b=1.1458; Di=0.1091)</td>
<td>2.694</td>
<td>2.006</td>
<td>n/a</td>
</tr>
</tbody>
</table>

References


Cauter, F. Van (2013) *Predicting Decline in Unconventional Reservoirs Using Analytical and Empirical Methods*. Available at: https://spiral.imperial.ac.uk/handle/10044/1/24264.


Kennedy, Luo, Vello, K. (2016) *The Unconventional Basins and Plays - North America, the rest*
of the World and Emerging Basins. Taylor and Francis.


Journal Highlights

- We provide in-depth analysis of gas production data to obtain the estimated ultimate recoverable (EUR) from 10237 horizontal shale gas wells over 18 counties in the Barnett shale from 2008 to 2014.

- Wells were grouped individually according to their date of first production (DOFP) from 2008 to 2014 and in combination.

- Wells with a DOFP of 2011 and 2013 required the lowest gas price ($3.68/Mscf) to obtain a ROR of 10% at 100% probability of success.

- We obtained a more representative outlook on the Barnett shale EUR compared to previous studies.

- Either the Arp’s hyperbolic decline model or the Doung’s decline model can be used to forecast EUR in the Barnett shale as only minimal differences were observed.